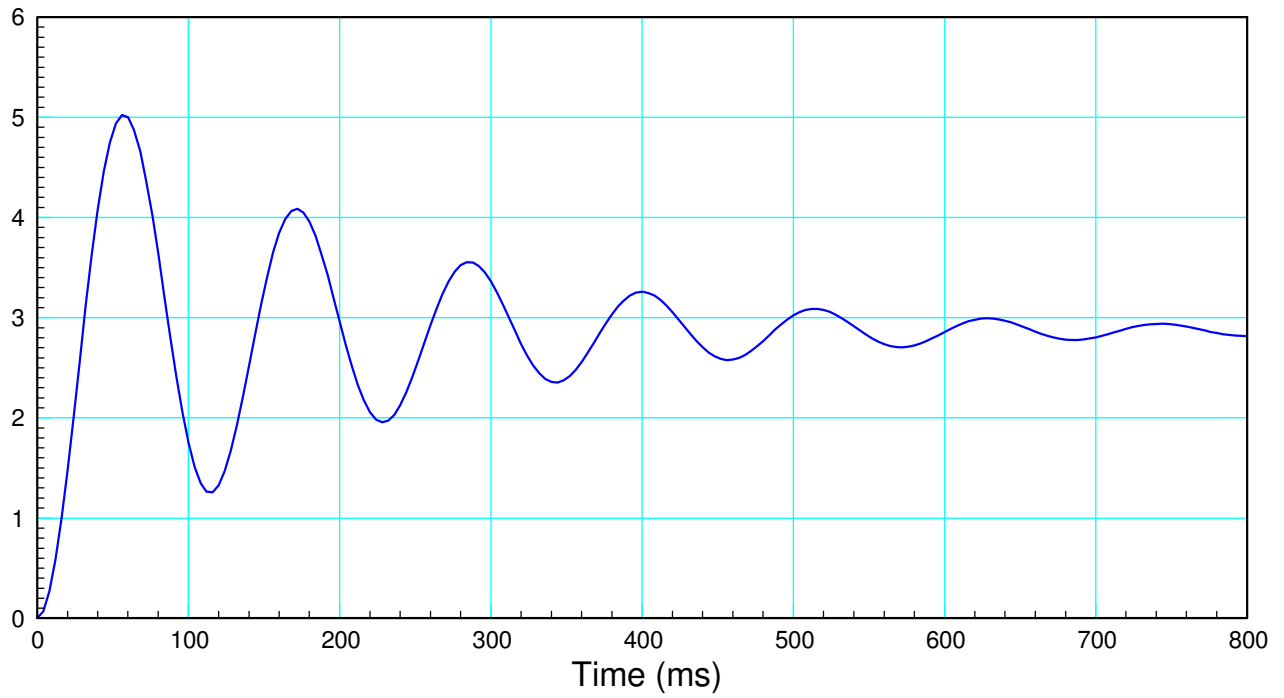


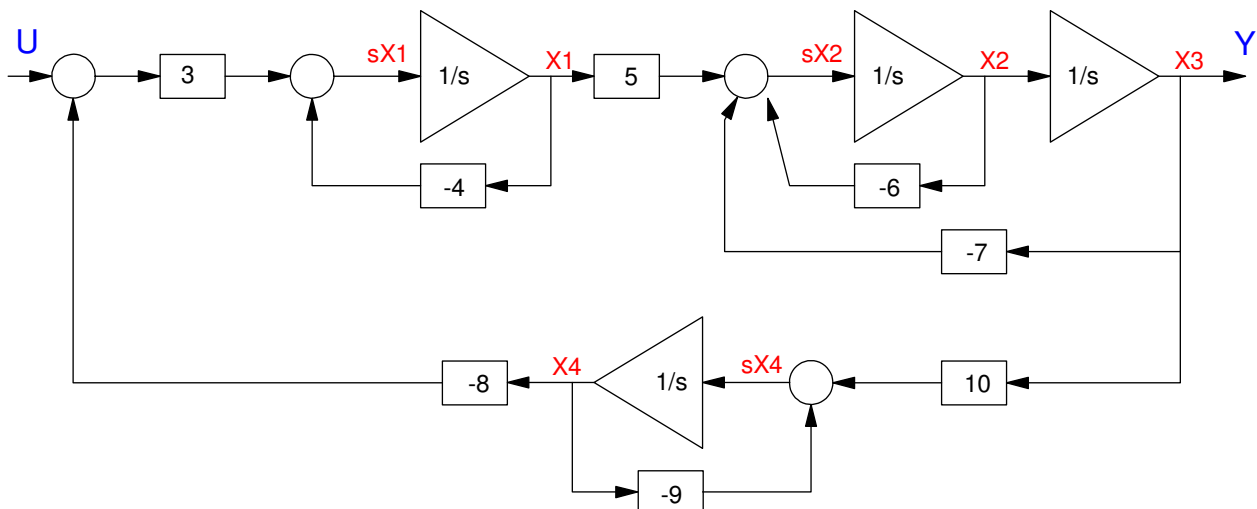
# CE 463/663: Test #1. Name \_\_\_\_\_

Spring 2021. Open Book, Open Notes. Calculators & Matlab allowed. Individual Effort

1) Find the transfer function for a system with the following step response



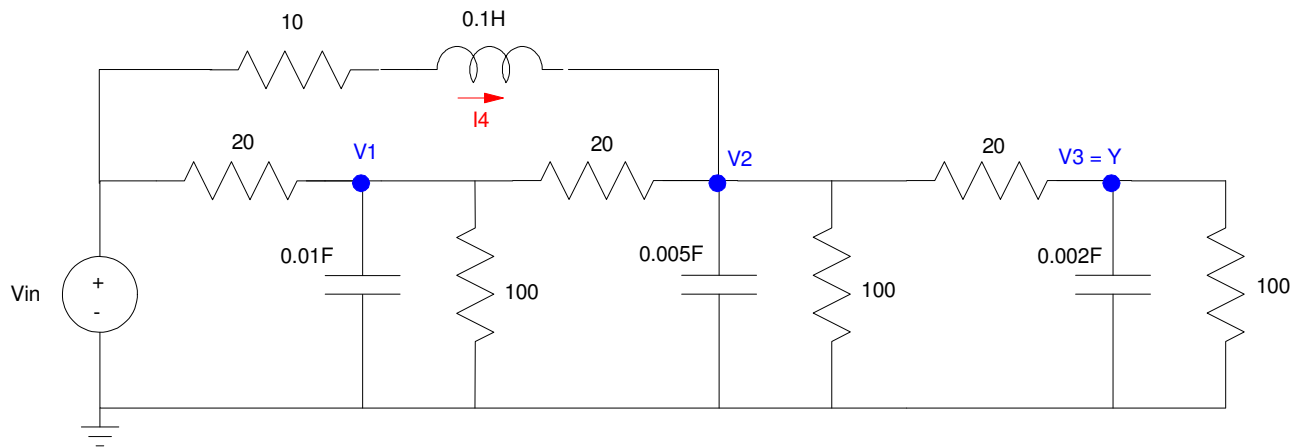
2) Give {A and B} for the the state-space model for the following system



$$\begin{bmatrix} sX_1 \\ sX_2 \\ sX_3 \\ sX_4 \end{bmatrix} = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} \\ \\ \\ \end{bmatrix} U$$

Problem 3) Option #1 (you may work either problem #3 - electrical or mechanical)

3a) Write four coupled differential equations to describe the following circuit

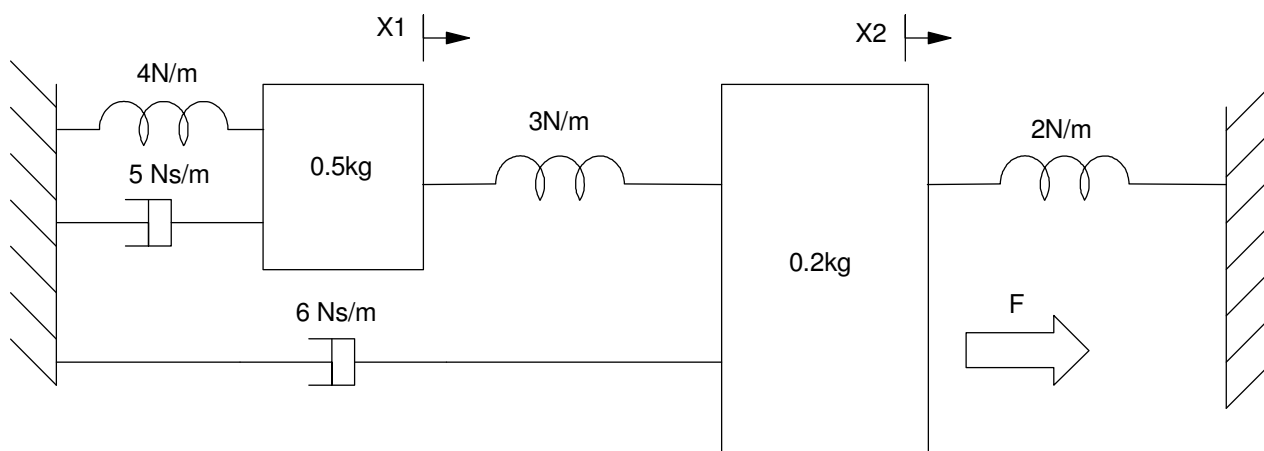


3b) Express the A and B matrices for the dynamics in state-space form

$$\begin{bmatrix} sV1 \\ sV2 \\ sV3 \\ sI4 \end{bmatrix} = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} V1 \\ V2 \\ V3 \\ I4 \end{bmatrix} + \begin{bmatrix} \\ \\ \\ \end{bmatrix} V_{in}$$

Problem 3) Option #2 (you may work either problem #3 - electrical or mechanical)

3a) Write two coupled differential equations to describe the following mass-spring system



3b) Express the A and B matrices for the dynamics in state-space form

$$\mathbf{s} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \mathbf{A} \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \mathbf{B} F$$

4) A ball with a mass of 1kg is rolling on a surface with the shape

$$y = 3 + x \cdot \cos(x)$$

Determine the potential and kinetic energy of the ball in terms of x:

4a)  $PE = mgy = f(x)$

4b)  $KE = 0.7m(\dot{x}^2 + \dot{y}^2) = g(x, \dot{x})$

5) Assume the LaGrangian is:

$$L = 0.7\dot{x}^2\dot{\theta} + 0.5\dot{x}\dot{\theta}\sin(\theta) - gx\cos\theta$$

Determine

$$F = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \left(\frac{\partial L}{\partial x}\right)$$