# ECE 463/663 - Test #3: Name _____

Calculus of Variations. Optimal Control. Spring 2021

Open Book, Open Notes. Calculators, Matlab permitted - just not other people.

Calculus of Variations: Fixed endpoints

0) What is your birth date (month and day)

| m<br>birth month: 1..12 | d<br>birth date: 1..31 |
|---|---|
| | |

1) Find the function which minimizes the following funcitonal:

$$J = \int_0^5 \left( m \cdot x^2 + d \cdot \dot{x}^2 \right) dt$$

where m is your birth month (1..12) and d is your birth date (1..31) subject to the constraints

x(0) = 6

x(5) = 4

2) Calculus of Variations: Free Endpoint.

Find the function which minimizes the following funcitonal:

$$J = \int_0^5 \left( m \cdot x^2 + d \cdot \dot{x}^2 \right) dt$$

where m is your birth month (1..12) and d is your birth date (1..31) subject to the constraints

x(0) = 6

x(5) = free

3) Optimal Control: Find the optimal path, x(t), to minimuze the cost function

$$J = \int_0^5 \left( m \cdot x^2 + d \cdot u^2 \right) dt$$

where m is your birth month (1..12) and d is your birth date (1..31) subject to the constraints

- $\dot{x} = 0.2x + u$
- $x(0) = 6$
- $x(5) = 4$

4) Optimal Control: Non-Quadratic Cost Function.

Find the optimal path, x(t), to minize the following cost function

$$J = \int_0^5 \left( x^4 + 5u^2 \right) dt$$

subject to the constraint

- $\dot{x} = 0.2x + u$
- $x(0) = 6$
- $x(5) = 4$

Note: It is sufficient to give the differential equation that x(t) must satisfy to be optimal.

Bonus: (5pt); Determine x(t) for problem #4 using whatever method you like (except for having someone else solve the problem for you).