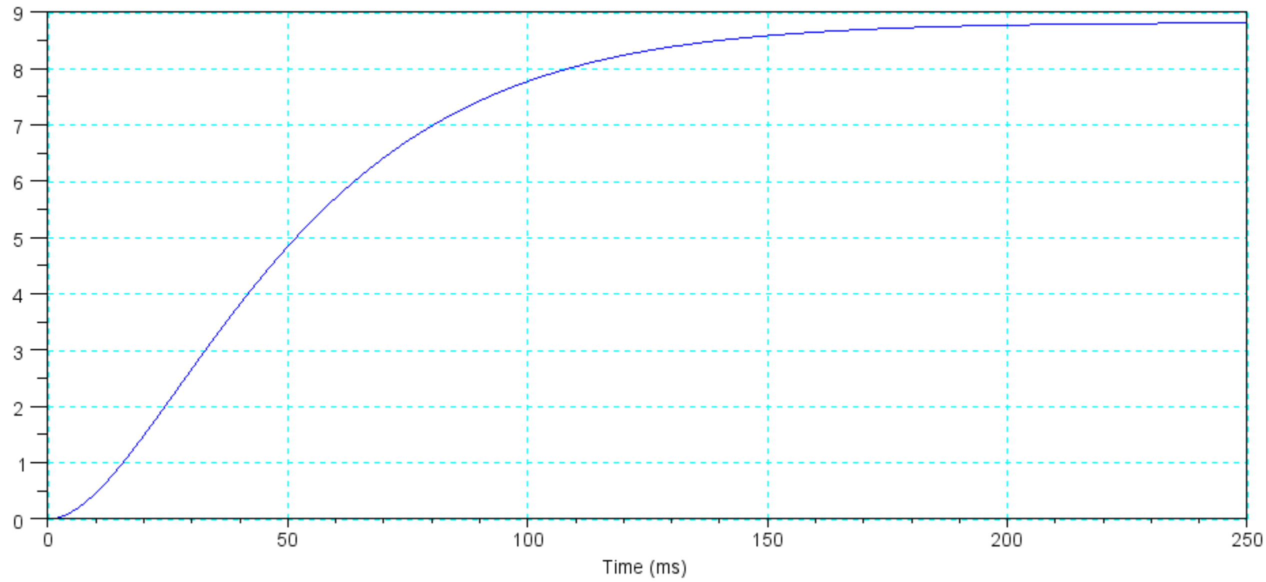
ECE 463/663 - Homework #1

LaPlace Transforms and Dominant Poles. Due Wednesday, Jan 20th

Please make the subject "ECE 463/663 HW#1" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

1) Name That System! Give the transfer function for a system with the following step response.



This is a 1st-order system (no overshoot), meaning

$$G(s) = \left(\frac{a}{s+b}\right)$$

DC gain = 8.8

$$\frac{a}{b} = 8.8$$

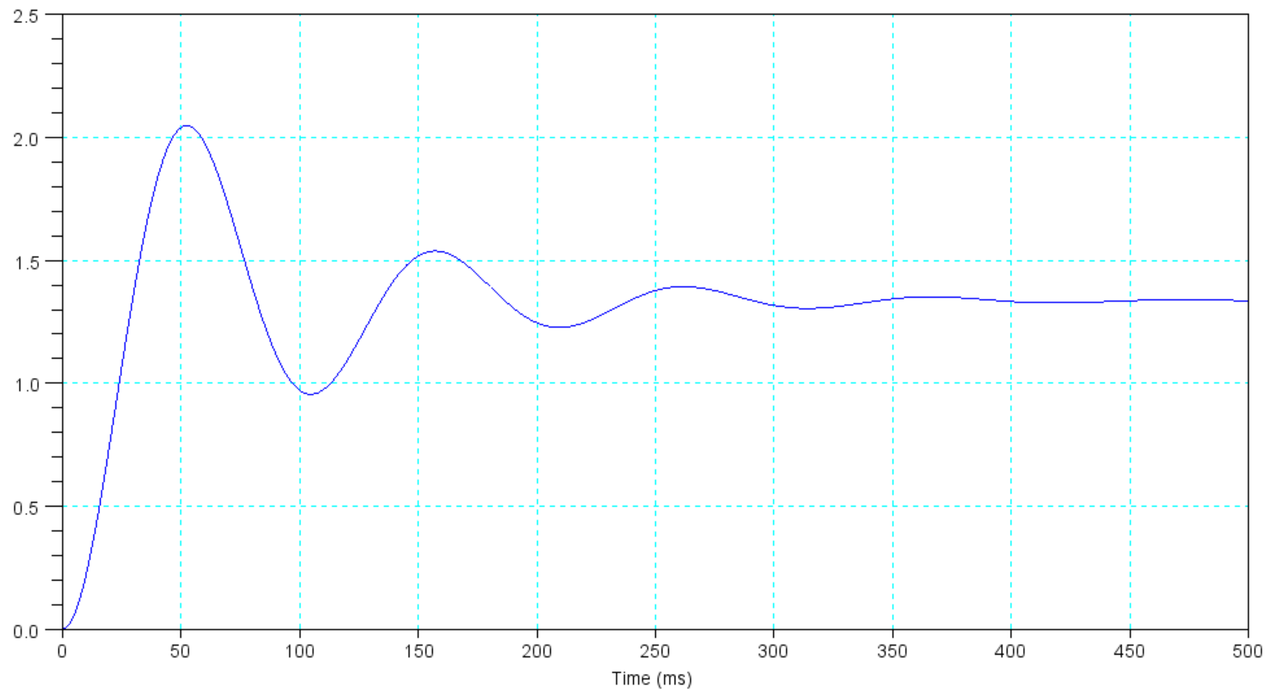
2% Settling Time = 200ms (approx)

$$b = \frac{4}{200ms} = 20$$

or

$$G(s) \approx \left(\frac{176}{s+20}\right)$$

2) Name That System! Give the transfer function for a system with the following step response.



This is a second-order system (oscillates, meaning sine and cosine terms).

DC gain = 1.35

Frequency of oscillation

$$\omega_d = \left(\frac{3 \text{ cycles}}{310\text{ms}}\right) 2\pi = 60.8 \ \frac{rad}{\text{sec}}$$

2% settling time = 310ms

$$\sigma = \frac{4}{310ms} = 12.9$$

meaning

$$G(s) \approx \left(\frac{5215}{(s+12.9+j60.8)(s+12.9-j60.8)}\right)$$

(The numerator is whatever it takes to make the DC gain 1.35)

Problem 3 - 6) Assume

$$Y = \left(\frac{300}{(s+3)(s+9)(s+10)}\right) X$$

3) What is the differential equation relating X and Y?

Multiply out and cross multiply

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>> poly([-3,-9,-10])

ans =      1     22    147    270
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$$(s^3 + 22s^2 + 147s + 270)Y = 300X$$

'sY' means 'the derivative of Y'

$$y''' + 22y'' + 147y' + 270y = 300x$$

4) Determine y(t) assuming x(t) is a sinusoidal input:

$$x(t) = 2\cos(3t) + 4\sin(3t)$$

Use phasor analysis

$$s = j3$$

$$X = 2 - j4 \qquad \textit{real = cosine, -imag = sine}$$

$$Y = \left(\frac{300}{(s+3)(s+9)(s+10)}\right)_{s=j3} (2 - j4)$$

$$Y = -2.569 - j1.896$$

meaning

$$y(t) = -2.569\cos(3t) + 1.896\sin(3t)$$

real = cosine, -imag = sine

5) Determine y(t) assuming x(t) is a step input:

$$x(t) = u(t)$$

Since x(t) = 0 for t<0, use LaPlace transforms

$$Y = \left(\frac{300}{(s+3)(s+9)(s+10)}\right)X$$

$$Y = \left(\frac{300}{(s+3)(s+9)(s+10)}\right)\left(\frac{1}{s}\right)$$

use partial fractions

$$Y = \left(\frac{1.111}{s}\right) + \left(\frac{-2.381}{s+3}\right) + \left(\frac{5.555}{s+9}\right) + \left(\frac{-4.386}{s+10}\right)$$

meaning

$$y(t) = 1.111 - 2.381e^{-3t} + 5.555e^{-9t} - 4.386e^{-10t} \qquad \text{for } t > 0$$

6a) Determine a 1st-order approximation for this system

$$Y \approx \left(\frac{a}{s+b}\right)X$$

Keep the dominant pole and the DC gain

$$Y = \left(\frac{300}{(s+3)(s+9)(s+10)}\right)X \approx \left(\frac{3.333}{s+3}\right)X$$

6b) Compare the step response of your 1st-order model to the actual 3rd-order system

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>> G1 = zpk([],[-3],[3.333]);
>> G3 = zpk([],[-3,-9,-10],300);
>> t = [0:0.01:3]';
>> y1 = step(G1,t);
>> y3 = step(G3,t);
>> plot(t,y1,'b',t,y3,'r');
>> xlabel('Time (seconds)');
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