ECE 463/663 - Homework #2

State-Space, Eigenvalues, Eigenvectors. Due Monday, Jan 25th

Please make the subject "ECE 463/663 HW#2" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

1) For the following RLC circuit

Specify the dynamics for the system (write N coupled differential equations)

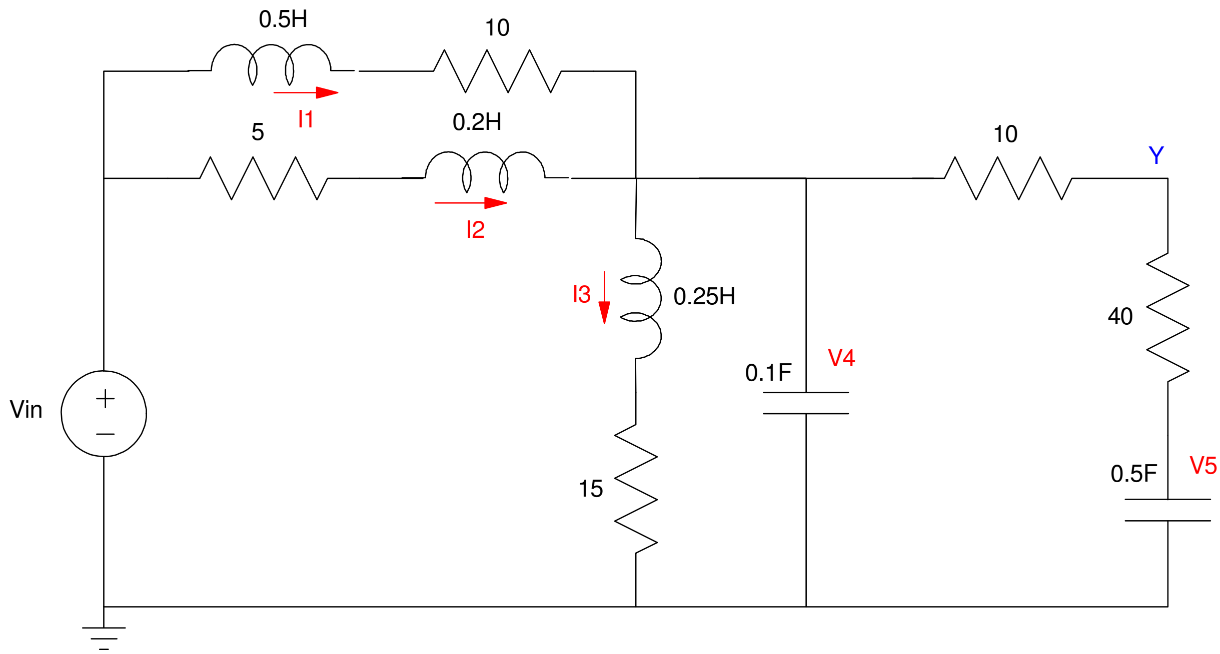
$$V_1 = 0.5sI_1 = V_{in} - 10I_1 - V_4$$

$$V_2 = 0.2sI_2 = V_{in} - 5I_2 - V_4$$

$$V_3 = 0.25sI_3 = V_4 - 15I_3$$

$$I_4 = 0.1sV_4 = I_1 + I_2 - I_3 - \left(\frac{V_3 - V_4}{50}\right)$$

$$I_5 = 0.5sV_5 = \left(\frac{V_4 - V_5}{50}\right)$$



Simplify

$$sI_1 = 2V_{in} - 20I_1 - 2V_4$$

$$sI_2 = 5V_{in} - 25I_2 - 5V_4$$

$$sI_3 = 4V_4 - 60I_3$$

$$sV_4 = 10I_1 + 10I_2 - 10I_3 - 0.2V_3 + 0.2V_4$$

$$sV_5 = 0.04V_4 - 0.04V_5$$

Express these dynamics in state-space form

$$s\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} -20 & 0 & 0 & -2 & 0 \\ 0 & -25 & 0 & -5 & 0 \\ 0 & 0 & -60 & 4 & 0 \\ 10 & 10 & -10 & -0.2 & 0.2 \\ 0 & 0 & 0 & 0.04 & -0.04 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V_4 \\ V_5 \end{bmatrix} + \begin{bmatrix} 2 \\ 5 \\ 0 \\ 0 \\ 0 \end{bmatrix} V_{in}$$

The output equation (Y): write the voltage node equation

$$\left(\frac{Y-V_4}{10}\right) + \left(\frac{Y-V_5}{40}\right) = 0$$

$$\left(\frac{1}{10} + \frac{1}{40}\right) Y = \left(\frac{1}{10}\right) V_4 + \left(\frac{1}{40}\right) V_5$$

$$Y = 0.8V_4 + 0.2V_5$$

$$Y = \begin{bmatrix} 0 & 0 & 0 & 0.8 & 0.2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V_4 \\ V_5 \end{bmatrix}$$

Determine the transfer function from Vin to Y

```
>> A = [-20,0,0,-2,0;0,-25,0,-5,0;0,0,-60,4,0;10,10,-10,-0.2,0.2;0,0,0,0.04,-0.04]

A =

  -20.0000         0         0   -2.0000         0
         0  -25.0000         0   -5.0000         0
         0         0  -60.0000    4.0000         0
   10.0000   10.0000  -10.0000   -0.2000    0.2000
         0         0         0    0.0400   -0.0400

>> B = [2;5;0;0;0];
>> C = [0,0,0,0.8,0.2];
>> G = ss(A,B,C,0);
>> zpk(G)


           56 (s+21.43) (s+60) (s+0.05)
--------------------------------------------------
(s+59.35) (s+23.22) (s+17.94) (s+4.694) (s+0.03791)

>>
```

2) For the transfer function from Vin to Y

Determine a 1st or 2nd-order approximation for this trasfer function

```
>> zpk(G)

            56 (s+21.43) (s+60) (s+0.05)
---------------------------------------------------
(s+59.35) (s+23.22) (s+17.94) (s+4.694) (s+0.03791)

>> DC = evalfr(G,0)

DC =    0.8182
```

Keep

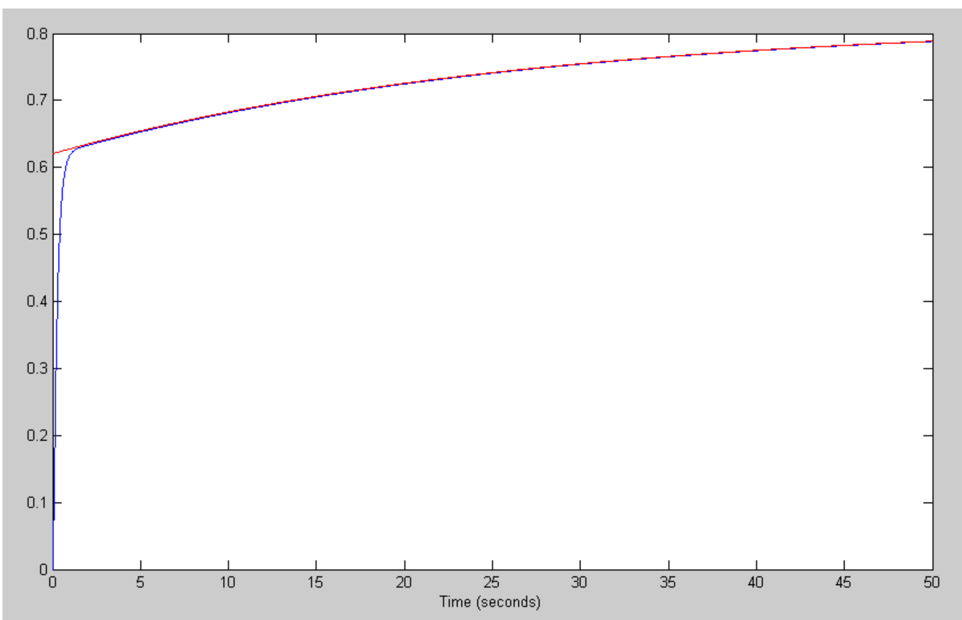
- The dominant pole at s = -0.03791
- The zero at s = -0.05 is within 10x the pole
- The DC gain is 0.8182

$$G(s) \approx \left(\frac{0.6204(s+0.05)}{s+0.03791} \right)$$

Note: This system has a non-zero D matrix

```
>> ss(G1)

a = -0.03791
b =  0.11
c =  0.06822
d =  0.6204
```

Plot the step response of the actual 5th-order system and its approximation shows the two match pretty well



3) For this circuit...

- What initial condition will the energy in the system decay as slowly as possible?
- What initial condition will the energy in the system decay as fast as possible?

This is an eigenvalue / eigenvetctor problem

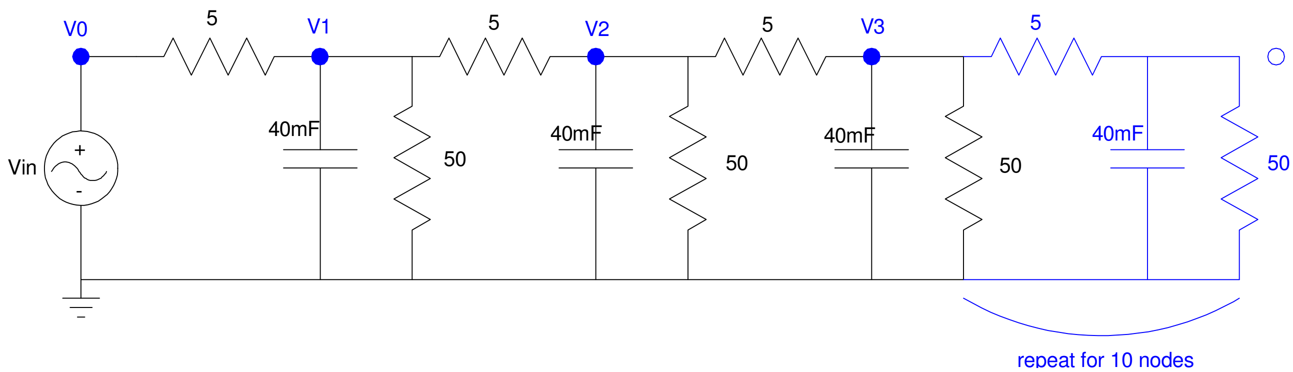
```
>> [M,V] = eig(A)

M =
     fast                                           slow
   -0.0082   -0.2033    0.6198    0.1256   -0.0052
   -0.0235    0.9220    0.4522    0.2366   -0.0104
   -0.9866   -0.0356   -0.0607   -0.0695    0.0035
   -0.1614   -0.3276   -0.6385   -0.9609    0.0521
    0.0001    0.0006    0.0014    0.0083    0.9986


V =

  -59.3454         0         0         0         0
         0  -23.2232         0         0         0
         0         0  -17.9395         0         0
         0         0         0   -4.6940         0
         0         0         0         0   -0.0379
```

If the intial conditions are proportional to the fast eigenvector, the transients decay as exp(-59.34t)

If the initial conditions are proportional to the slow eigenvector, the transients decay as exp(-0.0379t)

4) For the following 10-stage RC circuit

- Specify the dynamics for the system (write N coupled differential equations)
  - note: Nodes 1..9 have the same form. Just write the node equation for node 1 and node 10.
- Express these dynamics in state-space form
- Determine the transfer function from Vin to V10



Node V2:

$$0.04sV_2 = \left(\frac{V_1 - V_2}{5}\right) + \left(\frac{V_3 - V_2}{5}\right) - \left(\frac{V_2}{50}\right)$$

$$sV_2 = 5V_1 - 10.5V_2 + 5V_3$$

ditto for nodes 1..9. Node #10 is slightly different

$$0.04sV_{10} = \left(\frac{V_9 - V_{10}}{5}\right) - \left(\frac{V_{10}}{50}\right)$$

$$sV_{10} = 5V_9 - 5.5V_{10}$$

In state-space form, the {A, B, C, D} matricies and transfer function are:

```
A = zeros(10,10);
for i=1:9
   A(i,i) = -10.5;
   A(i+1,i) = 5;
   A(i,i+1) = 5;
   end
A(10,10) = -5.5

A =

  -10.5000    5.0000         0         0         0         0         0         0         0         0
    5.0000  -10.5000    5.0000         0         0         0         0         0         0         0
         0    5.0000  -10.5000    5.0000         0         0         0         0         0         0
         0         0    5.0000  -10.5000    5.0000         0         0         0         0         0
         0         0         0    5.0000  -10.5000    5.0000         0         0         0         0
         0         0         0         0    5.0000  -10.5000    5.0000         0         0         0
         0         0         0         0         0    5.0000  -10.5000    5.0000         0         0
         0         0         0         0         0         0    5.0000  -10.5000    5.0000         0
         0         0         0         0         0         0         0    5.0000  -10.5000    5.0000
         0         0         0         0         0         0         0         0    5.0000   -5.5000

B = zeros(10,1);
B(1) = 5

     5
     0
     0
     0
     0
     0
     0
```

```
      0
      0
      0

C = zeros(1,10);
C(10) = 1

      0      0      0      0      0      0      0      0      0      1

D = 0;
G = ss(A,B,C,D);
DC = evalfr(G,0)

    DC = 0.0741

zpk(G)

                                             9765625
-------------------------------------------------------------------------------------------------
(s+20.06) (s+18.76) (s+16.73) (s+14.15) (s+11.25) (s+8.275) (s+5.5) (s+3.169) (s+1.49) (s+0.6117)
```

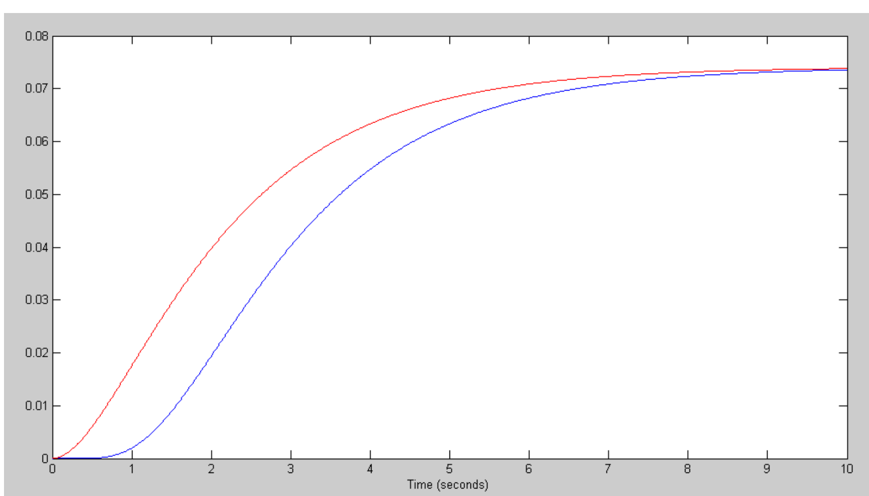
5) For the transfer function for problem #4

- Determine a 2nd-order approximation for this trasfer function
- Plot the step response of the actual 10th-order system and its 2nd-order approximation

Keep the two most dominant poles

Match the DC gain

```
>> DC = evalfr(G,0)

DC =    0.0741

>> G2 = zpk([],[-0.6117,-1.49],DC*0.6117*1.49)

      0.067522
-------------------
(s+0.6117) (s+1.49)

>> t = [0:0.01:10]';
>> y10 = step(G,t);
>> y2 = step(G2,t);
>> plot(t,y10,'b',t,y2,'r');
>> xlabel('Time (seconds)');
```



6) For the circuit for problem #4

- What initial condition will decay as slowly as possible?
- What initial condition will decay as fast as possible?

This is an eigenvector problem.

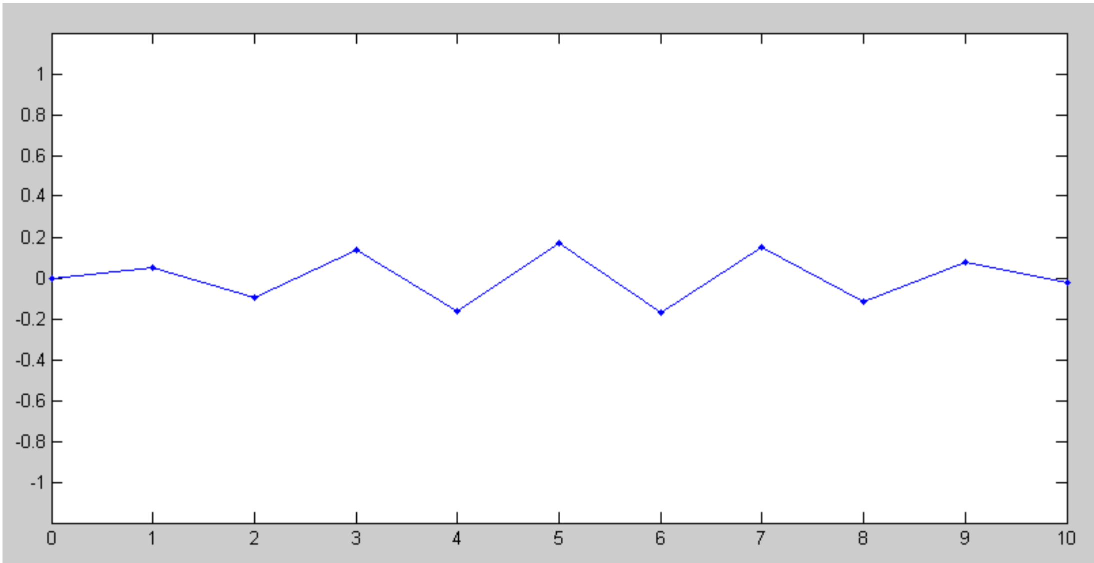
```
>> [M,V] = eig(A)

M =
   fast                                                                                      slow
  -0.1286   -0.2459    0.3412    0.4063    0.4352    0.4255    0.3780    0.2969   -0.1894    0.0650
   0.2459    0.4063   -0.4255   -0.2969   -0.0650    0.1894    0.3780    0.4352   -0.3412    0.1286
  -0.3412   -0.4255    0.1894   -0.1894   -0.4255   -0.3412   -0.0000    0.3412   -0.4255    0.1894
   0.4063    0.2969    0.1894    0.4352    0.1286   -0.3412   -0.3780    0.0650   -0.4255    0.2459
  -0.4352   -0.0650   -0.4255   -0.1286    0.4063    0.1894   -0.3780   -0.2459   -0.3412    0.2969
   0.4255   -0.1894    0.3412   -0.3412   -0.1894    0.4255    0.0000   -0.4255   -0.1894    0.3412
  -0.3780    0.3780    0.0000    0.3780   -0.3780   -0.0000    0.3780   -0.3780   -0.0000    0.3780
   0.2969   -0.4352   -0.3412    0.0650    0.2459   -0.4255    0.3780   -0.1286    0.1894    0.4063
  -0.1894    0.3412    0.4255   -0.4255    0.3412   -0.1894   -0.0000    0.1894    0.3412    0.4255
   0.0650   -0.1286   -0.1894    0.2459   -0.2969    0.3412   -0.3780    0.4063    0.4255    0.4352


>> eig(A)'

  -20.0557  -18.7624  -16.7349  -14.1534  -11.2473   -8.2748   -5.5000   -3.1695   -1.4903   -0.6117

>>
```

If the initial condition is proportional to the fast eigenvector, the transient decays as exp(-20.055t)

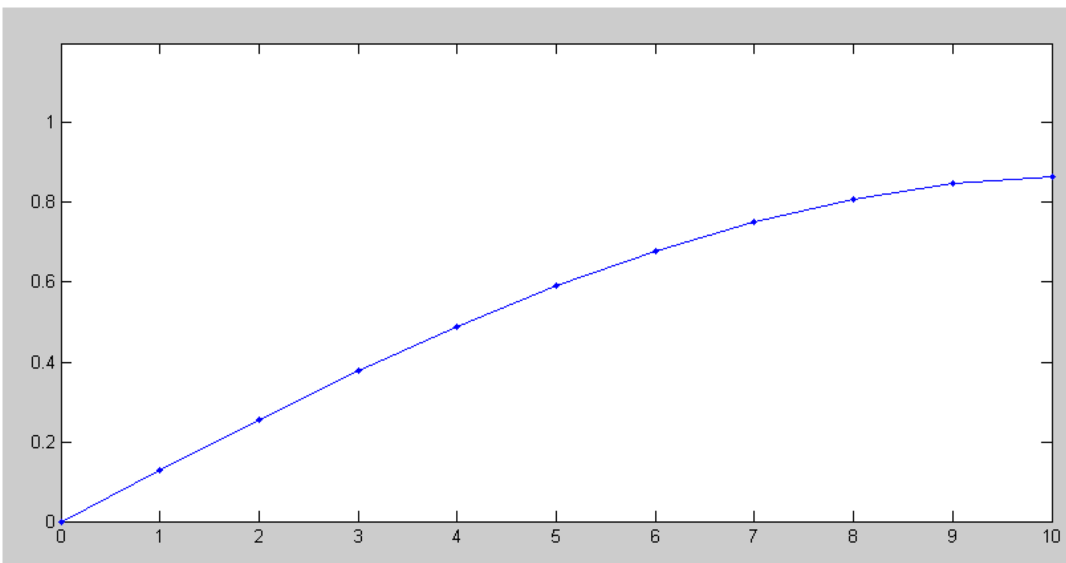
If the initial condition is proportional to the slow eigenvector, the transient decays as exp(-0.6117t)

7) Modify the program *heat.m* to match the dynamics you calculated for this problem.

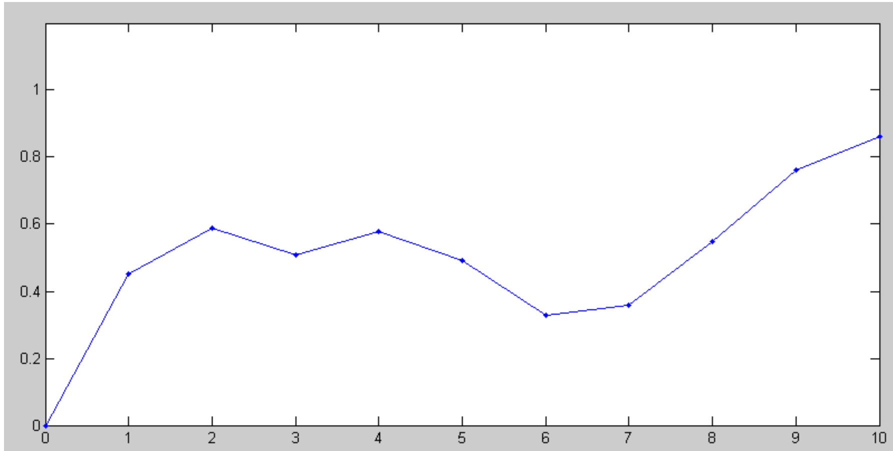
- Give the program listing
- Give the response for Vin = 0 and the initial conditions being
  - The slowest eigenvector
  - The fastest eigenvector
  - A random set of voltages



Fast Mode: Quickly decays to zero



Slow Mode: Slowly decays



Random initial condition: quickly converges to the slow mode

Code

```
% 10-stage RC Filter

V = rand(10,1);

V0 = 0;
DATA = [V0;V];
dV = zeros(10,1);

dt = 0.001;
t = 0;

while(t < 10)

   dV(1)  = 5*V0   - 10.5*V(1) + 5*V(2);
   dV(2)  = 5*V(1) - 10.5*V(2) + 5*V(3);
   dV(3)  = 5*V(2) - 10.5*V(3) + 5*V(4);
   dV(4)  = 5*V(3) - 10.5*V(4) + 5*V(5);
   dV(5)  = 5*V(4) - 10.5*V(5) + 5*V(6);
   dV(6)  = 5*V(5) - 10.5*V(6) + 5*V(7);
   dV(7)  = 5*V(6) - 10.5*V(7) + 5*V(8);
   dV(8)  = 5*V(7) - 10.5*V(8) + 5*V(9);
   dV(9)  = 5*V(8) - 10.5*V(9) + 5*V(10);
   dV(10) = 5*V(9) - 5.5*V(10);

   V = V +  dV*dt;
   t = t + dt;

   plot([0:10], [V0;V], '.-');
   ylim([0,1.2]);
   pause(0.01);
   end
```