ECE 463/663 - Homework #5

Full State Feedback. Due Monday, February 22nd

1) Write a Matlab m-file which is passed

- The system dynamics (A, B),
- The desired pole locations (P)

and then returns the feedback gains, Kx, so that roots(A - B Kx) = P

```
function [Kx] = ppl(A, B, P)
```

```
function [ Kx ] = ppl( A, B, P0)

N = length(A);

T1 = [];
for i=1:N
   T1 = [T1, (A^(i-1))*B];
end


P = poly(eig(A));
T2 = [];
for i=1:N
    T2 = [T2; zeros(1,i-1), P(1:N-i+1)];
end

T3 = zeros(N,N);
for i=1:N
    T3(i, N+1-i) = 1;
end

T = T1*T2*T3;

Pd = poly(P0);

dP = Pd - P;

Flip = [N+1:-1:2]';
Kz = dP(Flip);
Kx = Kz*inv(T);

end
```

Problem 2) Assume the following dynamic system:

$$sX = \begin{bmatrix} -10.5 & 5 & 0 & 0 & 0 \\ 5 & -10.5 & 5 & 0 & 0 \\ 0 & 5 & -10.5 & 5 & 0 \\ 0 & 0 & 5 & -10.5 & 5 \\ 0 & 0 & 0 & 5 & -5.5 \end{bmatrix} X + \begin{bmatrix} 30 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix} X$$

Find the feedback control law of the form

$$U = K_r R - K_x X$$

so that

- The DC gain is 1.000 and
- The closed-loop poles are at {-3, -4, -5, -6, -7}

Plot

- The resulting closed-loop step reponse, and
- The resulting input, U

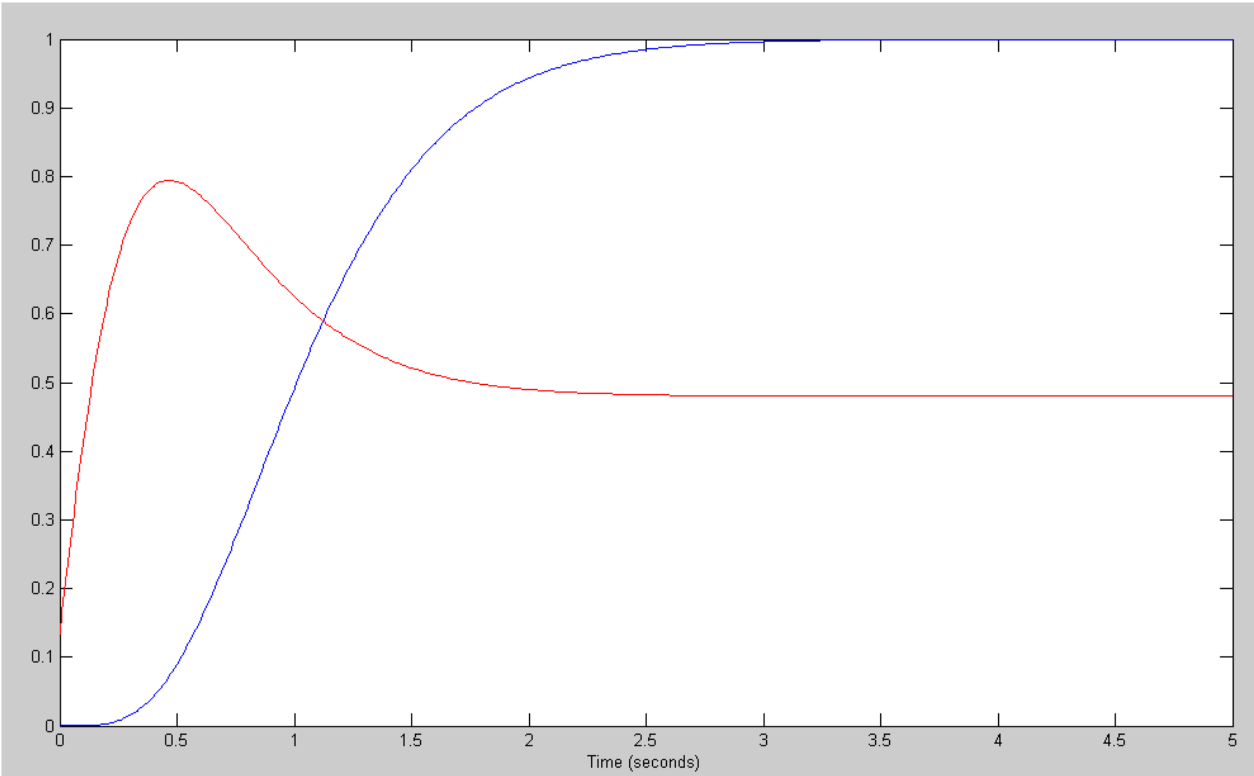
.

```
>> A = [-10.5,5,0,0,0;5,-10.5,5,0,0;0,5,-10.5,5,0;0,0,5,-10.5,5
0,0,0,5,-5.5]

A =

  -10.5000    5.0000         0         0         0
    5.0000  -10.5000    5.0000         0         0
         0    5.0000  -10.5000    5.0000         0
         0         0    5.0000  -10.5000    5.0000
         0         0         0    5.0000   -5.5000

>> B = [30;0;0;0;0];
>> C = [0,0,0,0,1];
>> D = 0;

>> Kx = ppl(A,B,[-3,-4,-5,-6,-7])

Kx =   -0.7500    1.9000   -2.9583    2.9418   -1.2251

>> DC = -C*inv(A-B*Kx)*B

DC =    7.4405

>> Kr = 1/DC

Kr =    0.1344
```

comment: This is actually a decent design:

- *The gains are reasonable.*
- *The input (u) is reasonable - a slight hump to speed up the system*
- *Negative gains in Kx are worrying: they provide positive feedback*

```
>> G2 = ss(A-B*Kx, B*Kr, [C ; -Kx], [D ; Kr]);
>> t = [0:0.01:5]';
>> y2 = step(G2,t);
>> plot(t,y2(:,1),'b',t,y2(:,2),'r')
>> xlabel('Time (seconds)');
>
```



Open-Loop Step Response (blue) and contrl input U (red)

Note: This is kind of what you're looking for in a step response:

- The output meets the requirements (2 second settling time, no overshoot)
- The input has a reasonable peak which tries to speed up the response (similar to hitting the gas pedal when the light turns green and then backing off to maintain speed)

3) Repeat problem #2 but find Kx and Kr so that

- The DC gain is 1.000 and
- The closed-loop dominant pole is at s = -3 and the other four poles don't move

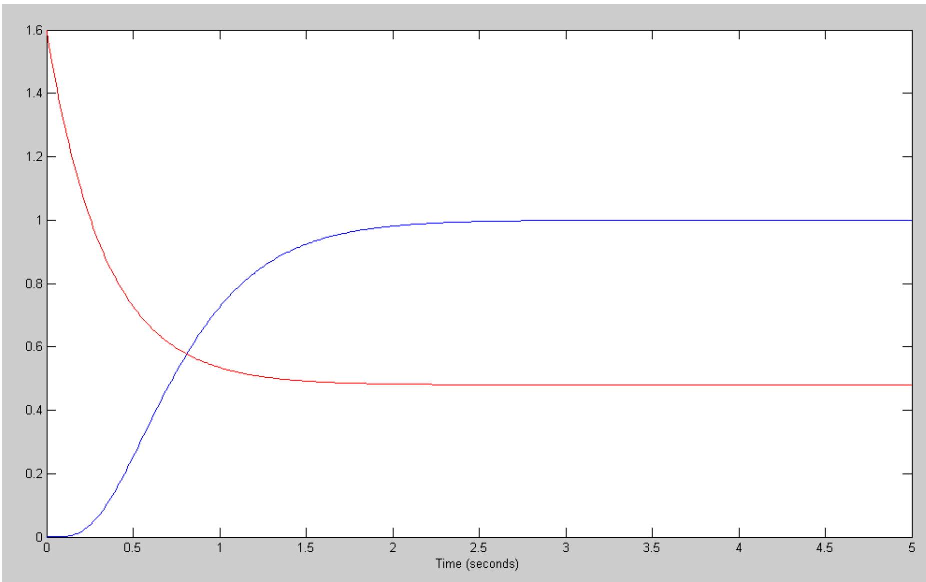
Plot the resulting closed-loop step reponse.

```
>> eig(A)

  -18.9125
  -14.6542
   -9.0769
   -3.9514
   -0.9051

>> Kx = ppl(A,B,[-3,-4,-9,-14.6,-19])

Kx =     0.0700     0.1320     0.1961     0.2226     0.2474

>> DC = -C*inv(A-B*Kx)*B

DC =     0.6259

>> Kr = 1/DC

Kr =     1.5978

>> G3 = ss(A-B*Kx, B*Kr, [C ; -Kx], [D ; Kr]);
>> y3 = step(G3,t);
>> plot(t,y2(:,1),'b',t,y2(:,2),'r')
>> plot(t,y3(:,1),'b',t,y3(:,2),'r')
>> xlabel('Time (seconds)');
>> >> plot(t,y0,'r',t,y3,'b')
```

comment: By allowing the system to behave the way it wants,

- *The feedback gains are all positive (good), and*
- *Their amplitude is a low lower.*

Lower gains tend to work better (less sensitive to modeling errors, sensor dynamics, delays, etc)



Open-Loop Response (red) and closed-loop response (blue)

4) Repeat problem #2 but find Kx and Kr so that

- The DC gain is 1.000
- The 2% settling time is 2 seconds, and
- There is 5% overshoot for a step input.

Plot the resulting closed-loop step response

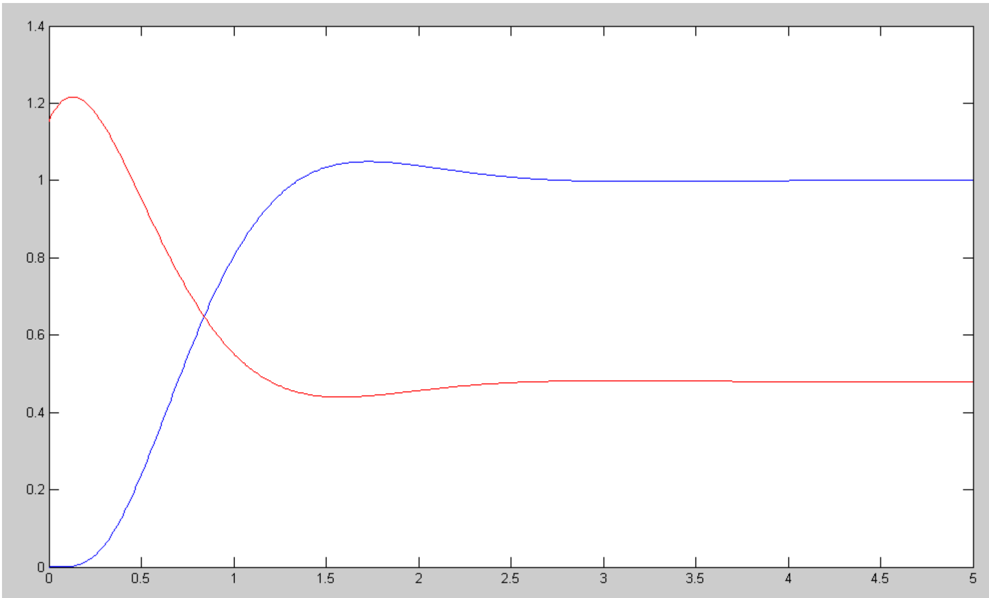
Translating the requirements to pole location

- The real part of the dominant pole is at s = -2
- The damping ratio associated with 5% overshoot is 0.68 (approx)
- The angle of the dominant pole is 47.2 degrees
- s = -2 + j2.16

Place the closed-loop poles at s = {-2+j2.16, -2-j2.16, -9, -14.6, -19}

```
>> Kx = ppl(A,B,[-2+j*2.16,-2-j*2.16,-9,-14.6,-19])

Kx =    -0.0300    -0.0022     0.1149     0.2346     0.3339

>> DC = -C*inv(A-B*Kx)*B

DC =     0.8667

>> Kr = 1/DC

Kr =     1.1538

>> G4 = ss(A-B*Kx, B*Kr, [C ; -Kx], [D ; Kr]);
>> y4 = step(G4,t);
>> plot(t,y4(:,1),'b',t,y4(:,2),'r')
```

note: by allowing the dominant pole to be complex at 45 degrees, you can get the same speed (approx) with lower gains and lower input. Pole placement can place the poles anywhere - some locations work better than others though.



Open-Loop Response (red) and closed-loop response (blue)