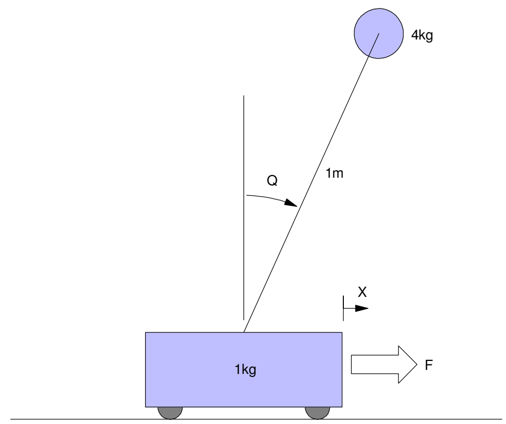
ECE 463/663 - Homework #6

Pole Placement. Due Monday, March 1st

Problem 1) (30pt) Use the dynamics of a Cart and Pendulum System from homework set #4:

$$s\begin{bmatrix} x \\ \theta \\ sx \\ s\theta \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -39.2 & 0 & 0 \\ 0 & 49 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta \\ sx \\ s\theta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}F$$



(10pt) Design a feedback control law of the form

U = Kr * R - Kx * X

so that the closed-loop system has

- A 2% settling time of 8 seconds, and
- 5% overshoot for a step input

(10pt) Check the step response of the linear system in Matlab

(10pt) Check the step response of the nonlinear system

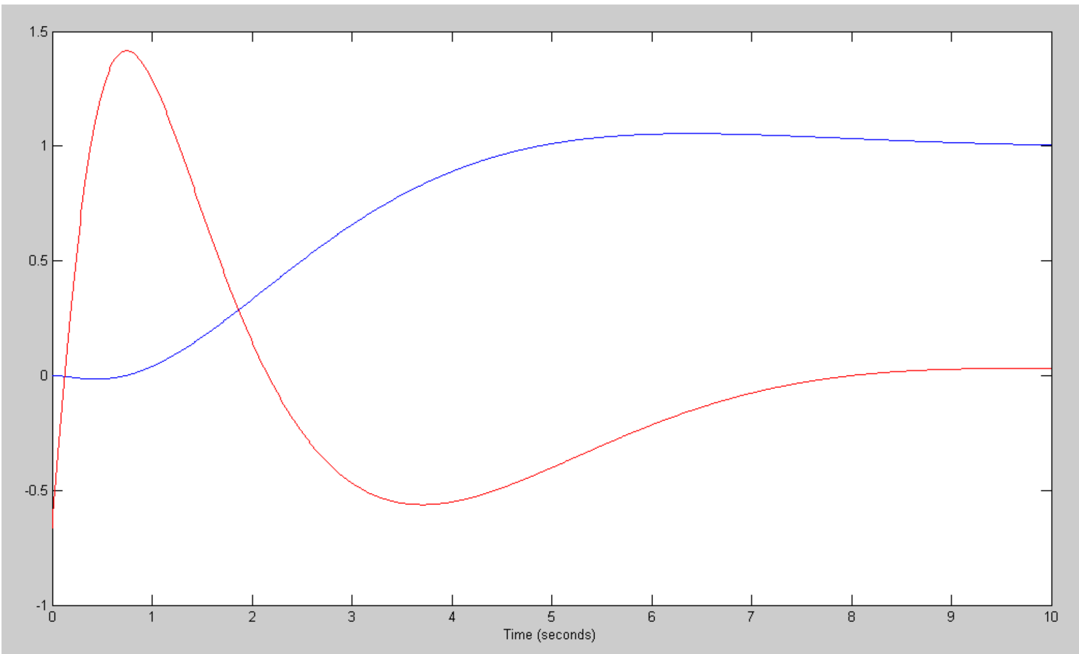
Step 1: Use pole placement to place the closed-loop dominant pole at s = -0.5 + j0.54

```
>> A = [0,0,1,0;0,0,0,1;0,-39.2,0,0;0,49,0,0]

         0         0    1.0000         0
         0         0         0    1.0000
         0  -39.2000         0         0
         0   49.0000         0         0

>> eig(A)

         0
         0
    7.0000
   -7.0000

>> B = [0;0;1;-1]

     0
     0
     1
    -1

>> C = [1,0,0,0]

     1     0     0     0

>> Kx = ppl(A, B, [-0.5+j*0.54,-0.5-j*0.54,-3,-4])

Kx =    -0.6632   -69.2048    -1.6113    -9.6113

>> DC = -C*inv(A-B*Kx)*B

DC =    -1.5079
```

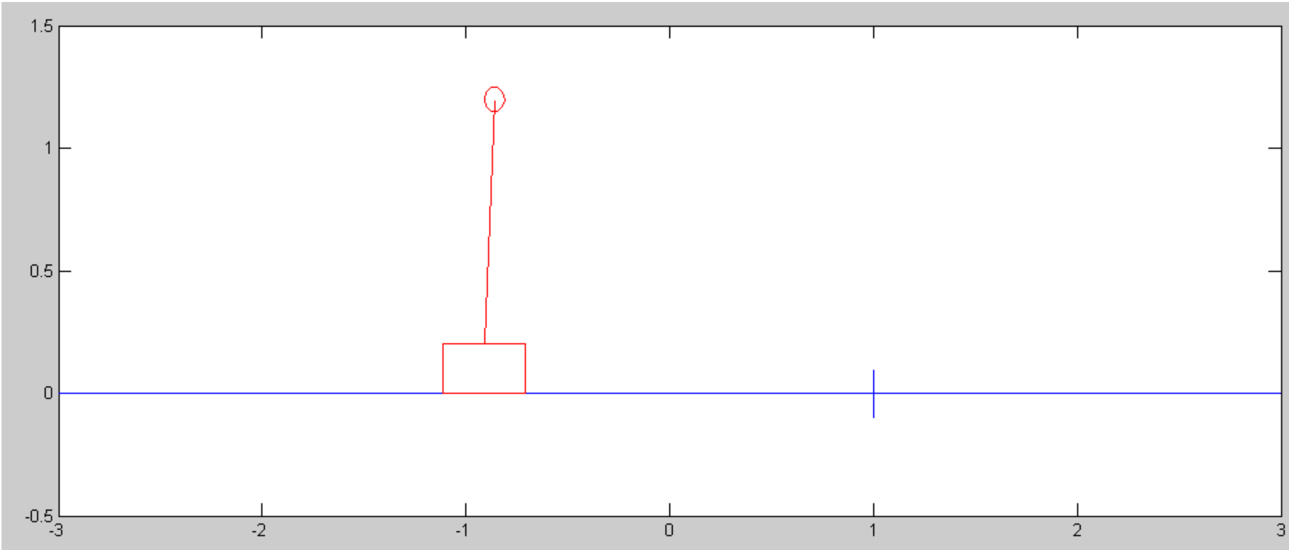
```
>> Kr = 1/DC

Kr =   -0.6632
```

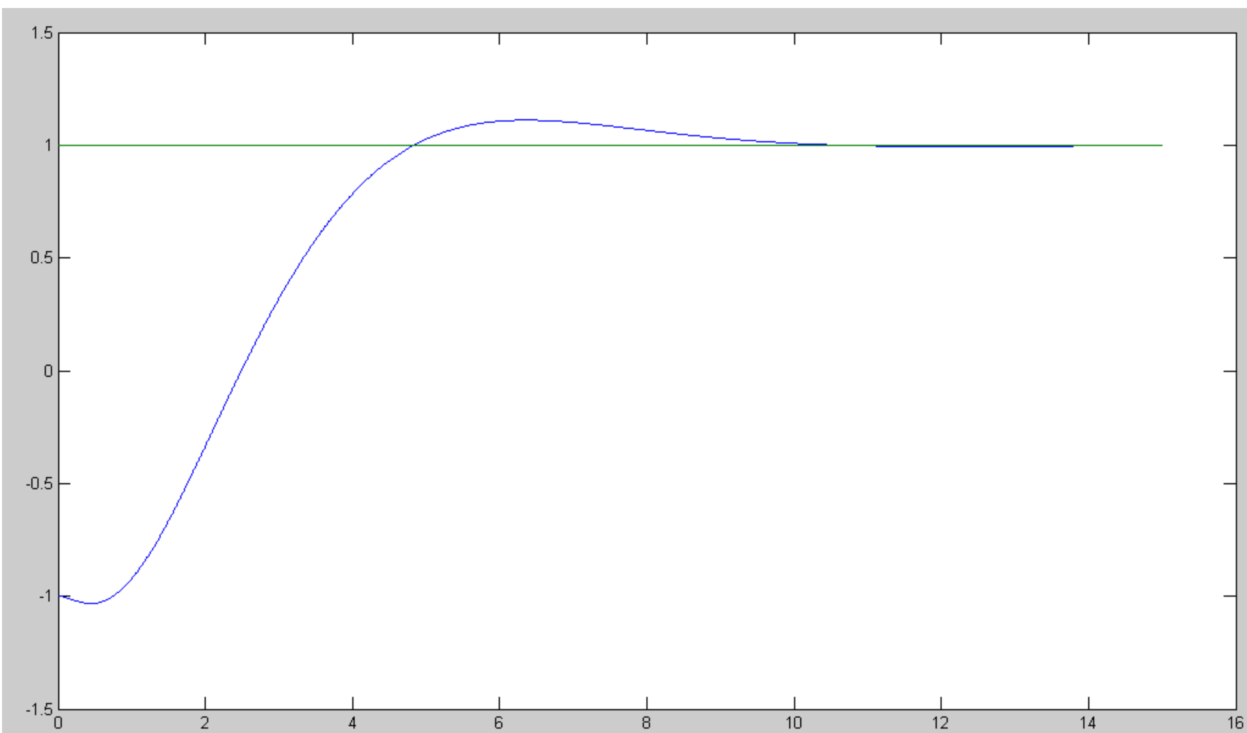
part 2: Check the step response of the linear system in Matlab

```
>> D = 0;
>> G = ss(A-B*Kx, B*Kr, [C ; -Kx], [D ; Kr]);
>> t = [0:0.01:10]';
>> y = step(G,t);
>> plot(t,y(:,1),'b',t,y(:,2),'r')
>> xlabel('Time (seconds)');
>>
```



Part 3) Check with the nonlinear simulation. Note: Make sure the dynamics are for Sp21





Code:

```
% Cart and Pendulum ( Sp21 version)
% m1 = 1.0kg
% m2 = 4.0kg
% L = 1.0m

X = [-1; 0 ; 0 ; 0];
Ref = 1;
dt = 0.01;
t = 0;
Kx = [-0.6632  -69.2048   -1.6113   -9.6113];
Kr = -0.6632;

y = [];
while(t < 15)
 Ref = 1;
 U = Kr * Ref - Kx*X;
 dX = CartDynamics(X, U);
 X = X + dX * dt;
 t = t + dt;

 CartDisplay(X, Ref);
 y = [y ; X(1), Ref];
end

t = [1:length(y)]' * dt;
plot(t,y);
```

Problem 2) (30pt) Use the dynamics for the Ball and Beam system from homework set #4.

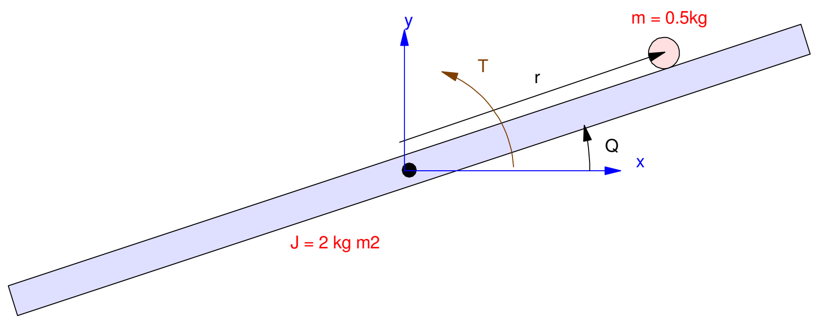
$$s\begin{bmatrix} r \\ \theta \\ sr \\ s\theta \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -7 & 0 & 0 \\ -1.96 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} r \\ \theta \\ sr \\ s\theta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.4 \end{bmatrix} T$$

(10pt) Design a feedback control law so that the closed-loop system has

- A 2% settling time of 8 seconds, and
- 5% overshoot for a step input

(10pt) Check the step response of the linear system in Matlab

(10pt) Check the step response of the nonlinear system

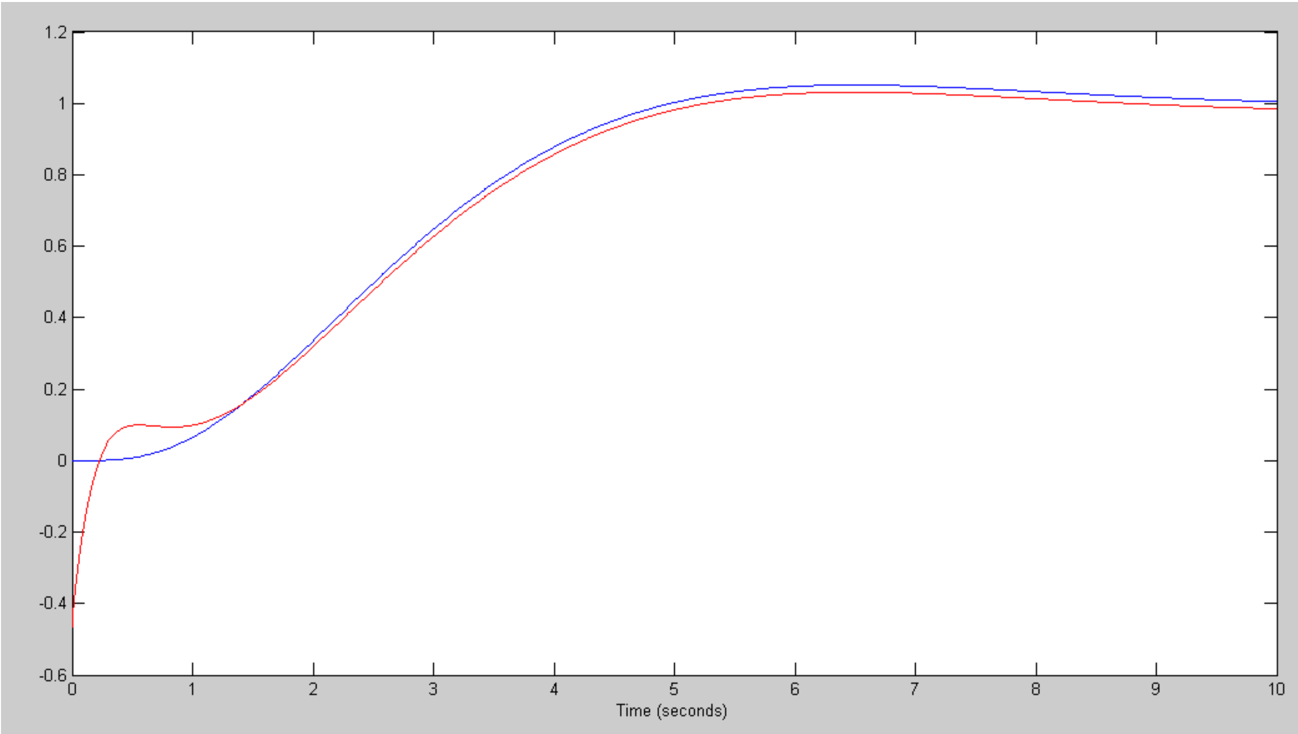


Part 1)

```
>> A = [0,0,1,0;0,0,0,1;0,-7,0,0;-1.96,0,0,0]

         0         0    1.0000         0
         0         0         0    1.0000
         0   -7.0000         0         0
   -1.9600         0         0         0

>> B = [0;0;0;0.4]

         0
         0
         0
    0.4000

>> C = [1,0,0,0]

C =      1     0     0     0

>> D = 0;
>> Kx = ppl(A, B, [-0.5+j*0.54,-0.5-j*0.54,-3,-4])

Kx =    -7.2211    48.8540    -5.6397    20.0000

>> DC = -C*inv(A-B*Kx)*B

DC =    -0.4308

>> Kr = 1/DC

Kr =    -2.3211
```

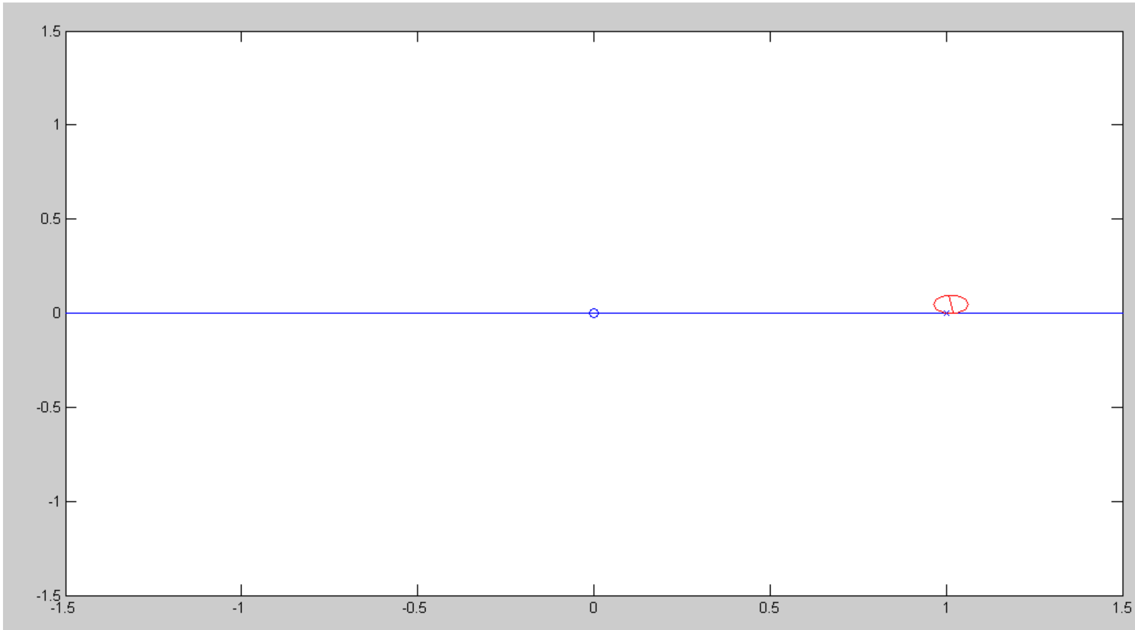
Part 2)

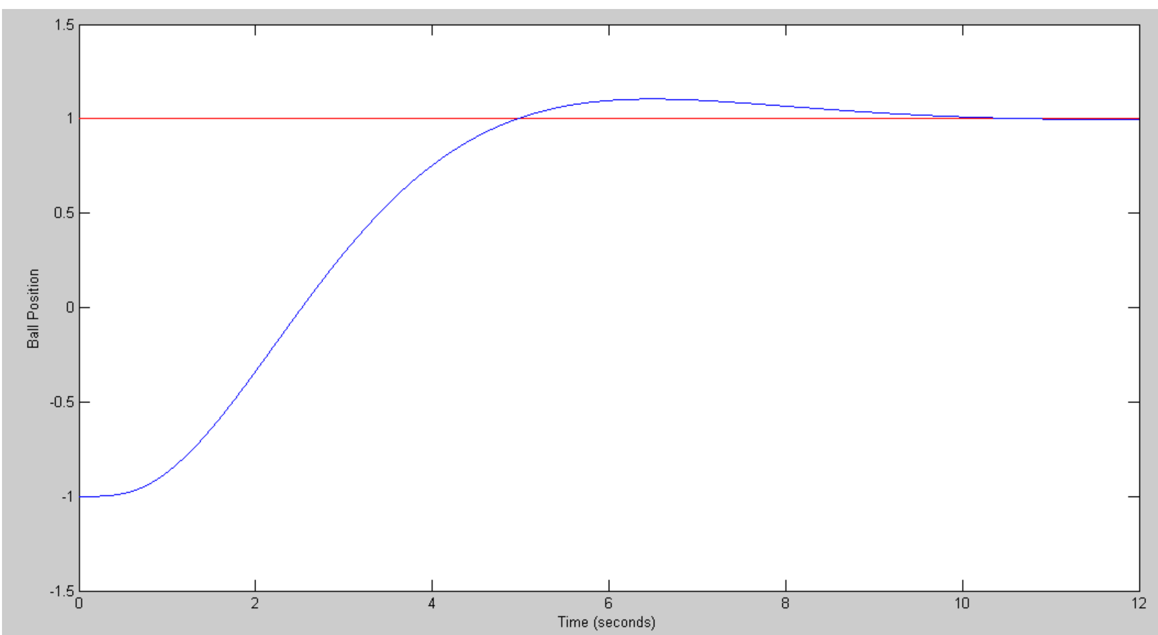
```
>> G = ss(A-B*Kx, B*Kr, [C ; -Kx], [D ; Kr]);
>> t = [0:0.01:10]';
>> y = step(G,t);
>> plot(t,y(:,1),'b',t,y(:,2),'r')
>> xlabel('Time (seconds)');
>> plot(t,y(:,1),'b',t,y(:,2)*0.2,'r')
>> xlabel('Time (seconds)');
```



Ball Position (blue) and Control Input * 0.2 (red)

Part 3) Nonlinear Simulation

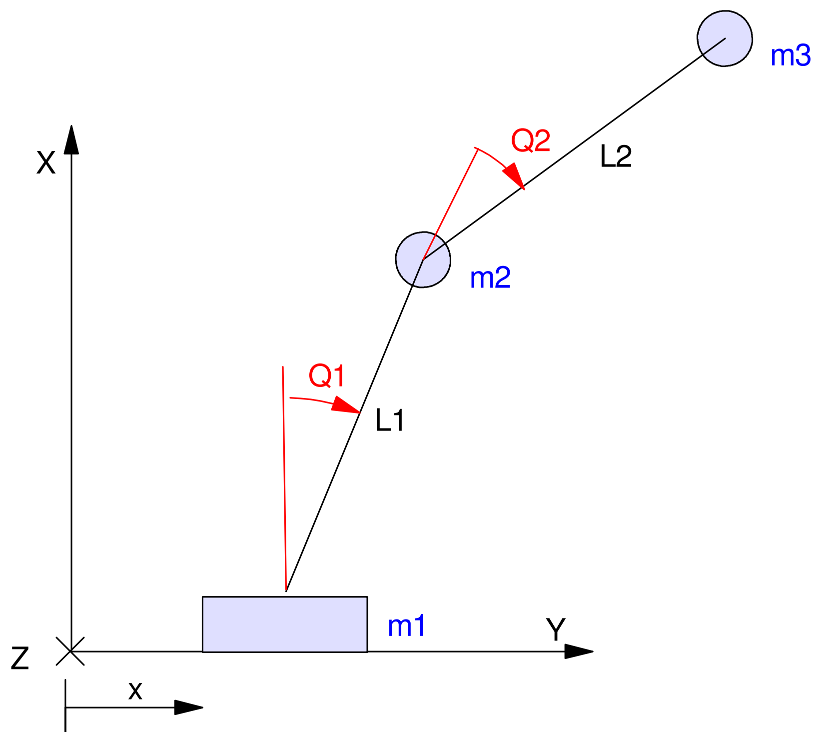




Code:

```
% Ball & Beam System
% Sp21 Version
% m = 0.5kg
% J = 2.0 kg m^2

X = [-1, 0, 0, 0]';
dt = 0.002;
t = 0;
Kx = [ -7.2211   48.8540   -5.6397   20.0000];
Kr =   -2.3211;
n = 0;
y = [];

while(t < 12)
 Ref = 1;
 U = Kr*Ref - Kx*X;
 dX = BeamDynamics(X, U);

 X = X + dX * dt;
 t = t + dt;

 y = [y ; Ref, X(1)];
 n = mod(n+1,5);
 if(n == 0)
    BeamDisplay(X, Ref);
 end
 end

t = [1:length(y)]' * dt;


plot(t,y(:,1),'r',t,y(:,2),'b');
xlabel('Time (seconds)');
ylabel('Ball Position');

>>
```

Problem #3 (30pt): The dynamics of a double pendulum are



$$s\begin{bmatrix} x \\ \theta_1 \\ \theta_2 \\ \dot{x} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -2g & 0 & 0 & 0 & 0 \\ 0 & 3g & -g & 0 & 0 & 0 \\ 0 & -3g & 3g & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta_1 \\ \theta_2 \\ \dot{x} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} F$$

(10pt) Design a feedback control law of the form

U = Kr * R - Kx * X

so that the closed-loop system has

- A 2% settling time of 8 seconds, and
- 5% overshoot for a step input

(10pt) Determine the step response of the linear system in Matlab

(10pt) Determine the step response of the nonlinear system

part 1)

```
>> g = 9.8

g =    9.8000

>>  A = [0,0,0,1,0,0;
0,0,0,0,1,0;
0,0,0,0,0,1;
0,-2*g,0,0,0,0;
0,3*g,-g,0,0,0;
0,-3*g,3*g,0,0,0]

A =

         0         0         0    1.0000         0         0
         0         0         0         0    1.0000         0
         0         0         0         0         0    1.0000
         0  -19.6000         0         0         0         0
         0   29.4000   -9.8000         0         0         0
         0  -29.4000   29.4000         0         0         0


>> eig(A)

         0
         0
   -6.8099
   -3.5250
    6.8099
    3.5250

>> B = [0,0,0,1,-1,1]'

     0
     0
     0
     1
    -1
     1

>> C = [1,0,0,0,0,0];

>> Kx = ppl(A, B, [-0.5+j*0.54,-0.5-j*0.54,-3,-4,-5,-6])
```

Kx =   1.0151   11.7348 207.0613   2.8385   29.6949   45.8563

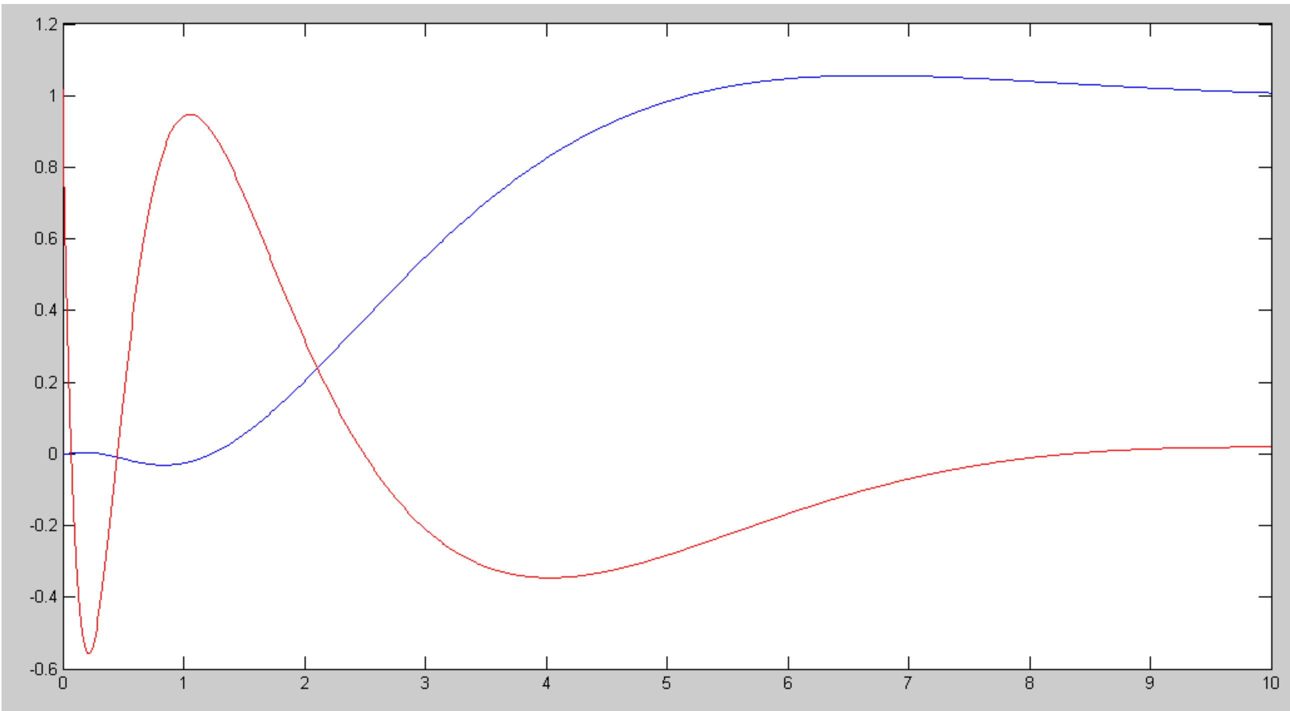
```
>> DC = -C*inv(A-B*Kx)*B

DC =    0.9851

>> Kr = 1/DC
```

Kr =   1.0151

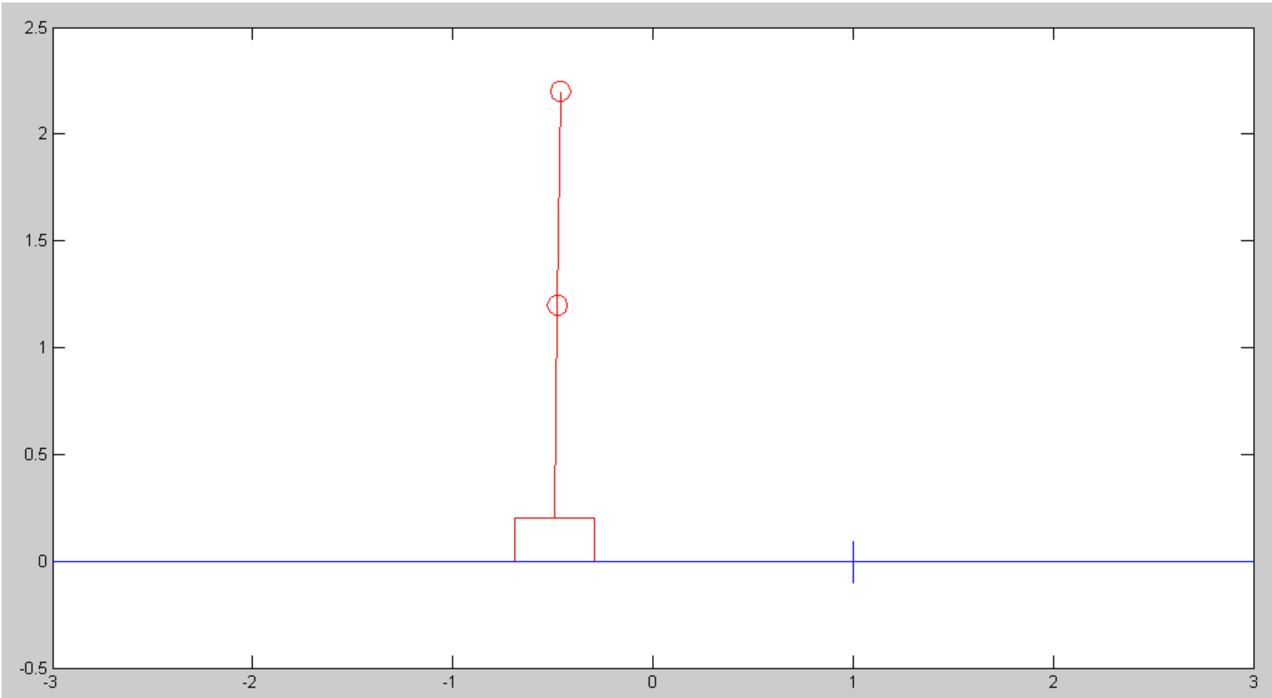
```
>>
```

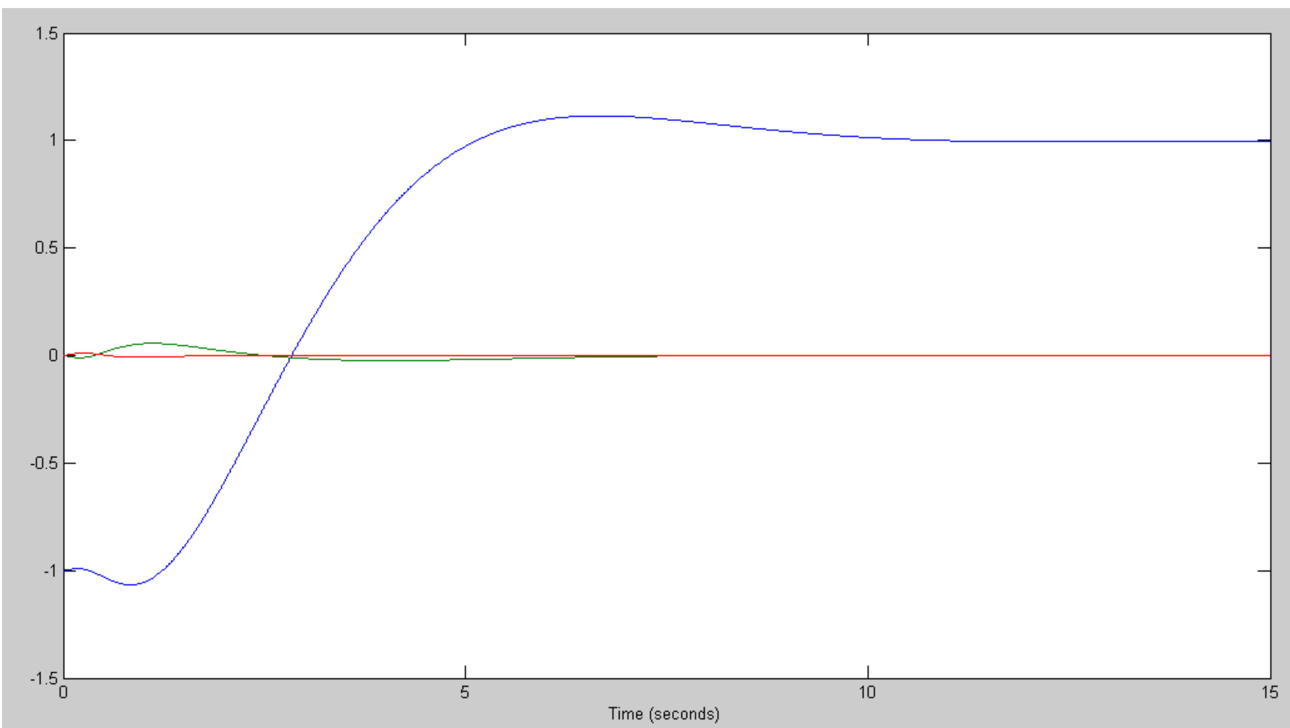
part 2)

```
>> G = ss(A-B*Kx, B*Kr, [C ; -Kx], [D ; Kr]);
>> t = [0:0.01:10]';
>> y = step(G,t);
>> plot(t,y(:,1),'b',t,y(:,2),'r')
```



Step Response: Position (b lue) & control input (red)


2.5
2
1.5
1
0.5
0
-0.5
-3
-2
-1
0
1
2
3


1.5
1
0.5
0
-0.5
-1
-1.5
0
5
10
15
Time (seconds)

Code:

```
X = [-1, 0, 0, 0, 0, 0]';
Ref = 1;
dt = 0.01;
U = 0;
t = 0;
Kx = [1.0151 11.7348 207.0613  2.8385  29.6949  45.8563];
Kr = 1.0151;
n = 0;
y = [];
while(t < 15)
 Ref = 1;
 U = Kr*Ref - Kx*X;
 dX = Cart2Dynamics(X, U);
 X = X + dX * dt;
 t = t + dt;
 y = [y ; X(1),X(2),X(3)];
 n = mod(n+1,5);
 if(n == 0)
    Cart2Display(X, Ref);
    plot([Ref, Ref],[-0.1,0.1],'b');
 end
end

t = [0:length(y)-1]' * dt;
plot(t,y);
xlabel('Time (seconds)');
```