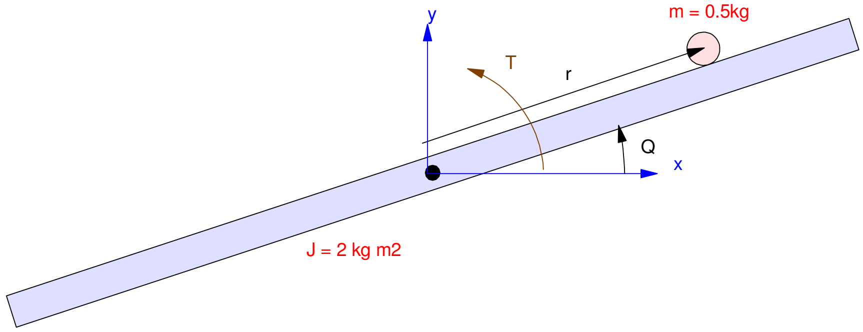
ECE 463/663 - Homework #7

Servo Compensators. Due Monday, March 8th



The dynamics of a Ball and Beam System (homework set #4) with a disturbance are

$$s\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -7 & 0 & 0 \\ -1.96 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.4 \end{bmatrix} T + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.4 \end{bmatrix} d$$

Full-State Feedback with Constant Disturbances

1) For the nonlinear simulation, use the feedback control law you computed in homework #6

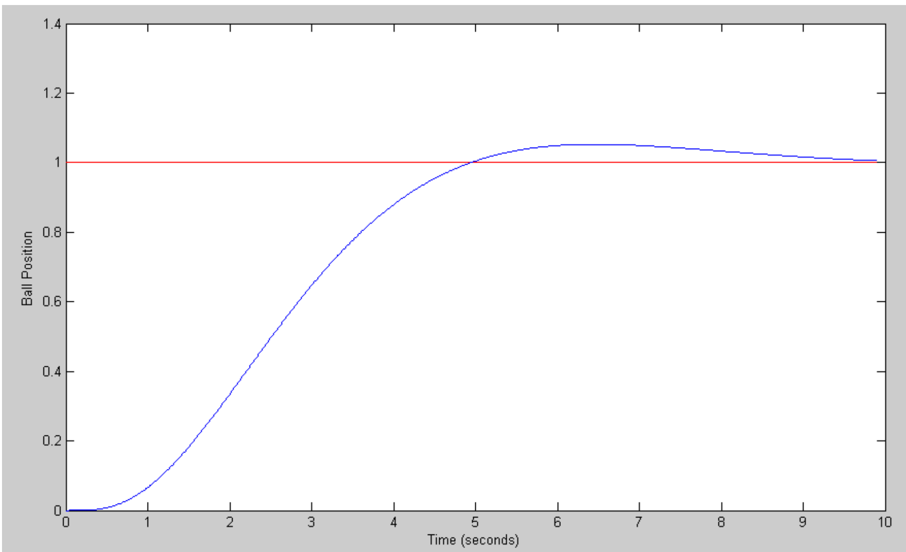
- With R = 1 and the mass of the ball = 0.5kg (same result you got for homework #6), and
- With R = 1 and the mass of the ball increased to 0.6kg

(i.e. a constant disturbance on the system due to the extra mass of the ball)

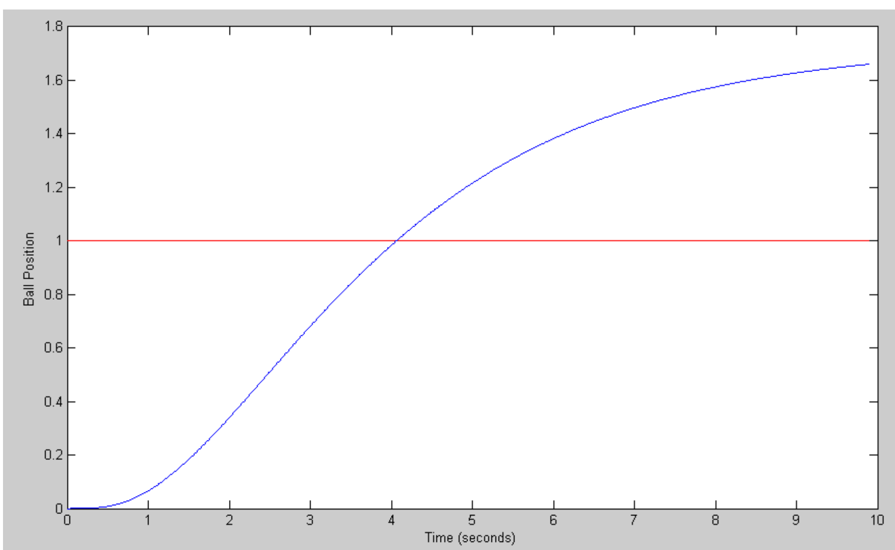
From homework #6

```
Kx =   -7.2211   48.8540   -5.6397   20.0000
Kr =   -2.3211
```

Plugging these in to the nonlinar simulation with m = 0.5kg and 0.6kg

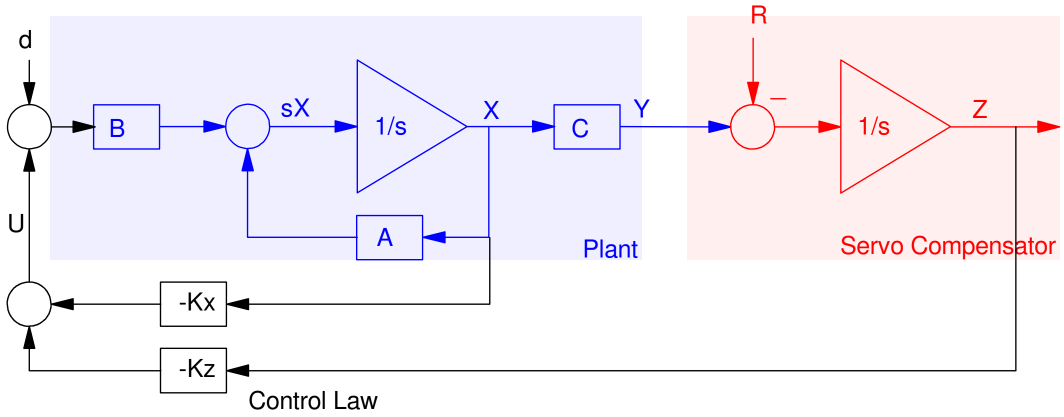


m = 0.5kg step response (nonlinear simulation)



m = 0.6kg step response (nonlinear simulation)

Servo Compensators with Constant Set-Points



2) Assume a constant disturbance and/or a constant set point. Design a feedback control law that results in

- The ability to track a constant set point (R = constant)
- The ability to reject a constant disturbance (d = constant),
- A 2% settling time of 6 seconds, and
- No overshoot for a step input.

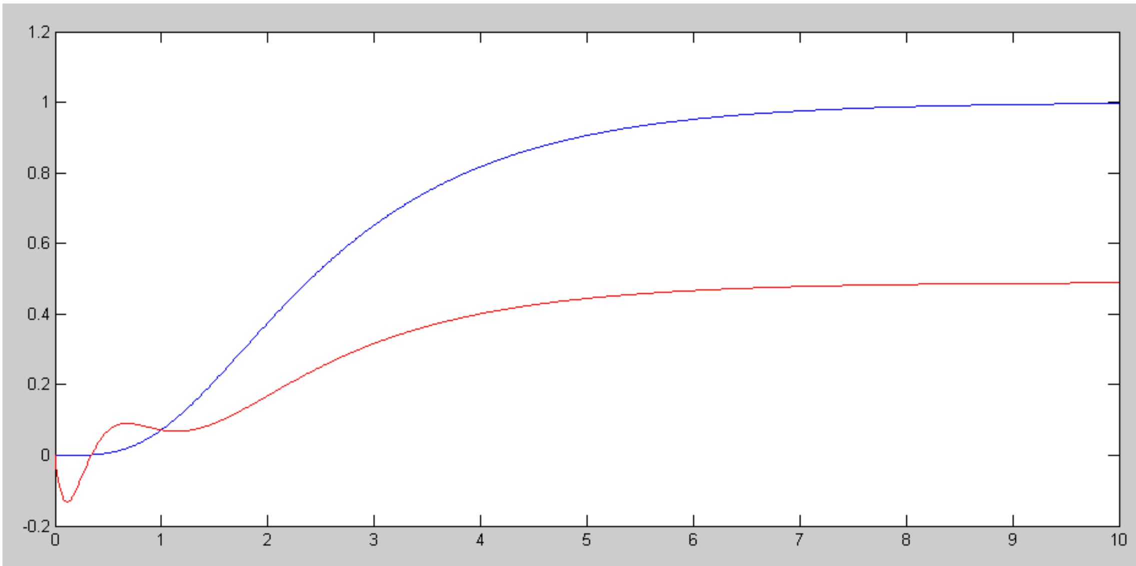
In matlab

```
>> A = [0,0,1,0;0,0,0,1;0,-7,0,0;-1.96,0,0,0];
>> B = [0;0;0;0.4];
>> C = [1,0,0,0];
>> A5 = [A,0*B ; C, 0]

         0         0    1.0000         0         0
         0         0         0    1.0000         0
         0   -7.0000         0         0         0
   -1.9600         0         0         0         0
    1.0000         0         0         0         0

>> B5 = [B;0]

         0
         0
         0
    0.4000
         0

>> B5r = [0*B;-1]

     0
     0
     0
     0
    -1

>> K5 = ppl(A5, B5u, [-0.67,-2, -3, -4, -5])

                       Kx                            Kz
K5 =   -84.6071   200.9500   -71.9893   36.6750   -28.7143
```

3) For the linear system, plot the step response

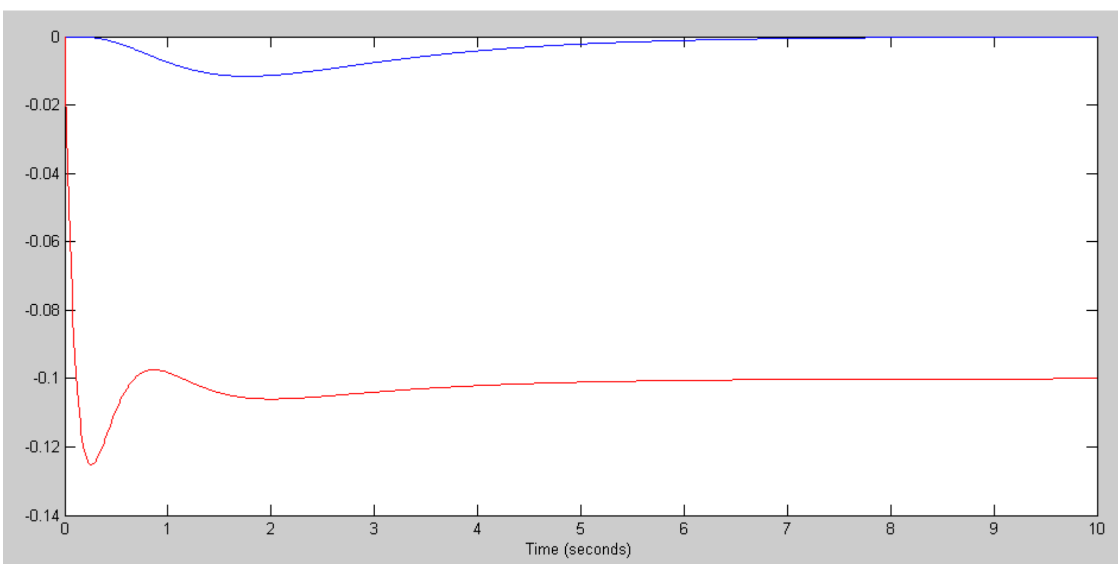
Step response with respect to R: (Just for fun, plot both position and input (U))

```
>> C5 = [C,0 ; -K5];
>> D5 = [0;0];
>> G5 = ss(A5 - B5*K5, B5r, C5, D5);
>> y = step(G5,t);
>> plot(t,y(:,1),'b',t,y(:,2)/10,'r');
```



Step Resposne with respect to R: Position (blue) & U/10 (red)
The system can track a constant set point

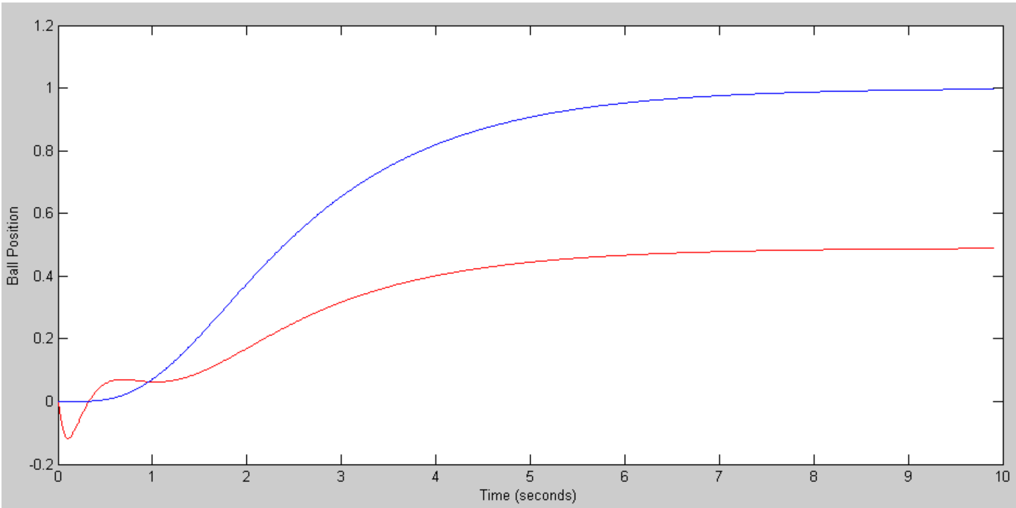
```
>> G5 = ss(A5 - B5*K5, B5, C5, D5);
>> y = step(G5,t);
>> plot(t,y(:,1),'b',t,y(:,2)/10,'r');
>> xlabel('Time (seconds)');
```



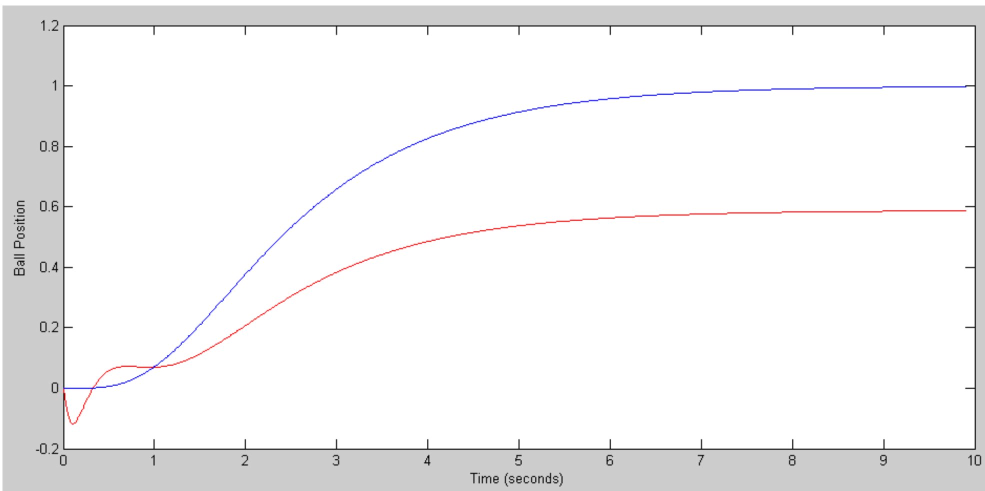
Step Resposne with respect to d: Position (blue) & U/10 (red)
The system can reject a constant disturbance

4) Implement your control law on the nonlinear ball and beam system

- With R = 1 and the mass of the ball being 0.5kg, and
- With R = 1 and the mass of the ball being 0.6kg



Step response when m = 0.5kg. Position (blue) and U/10 (red)

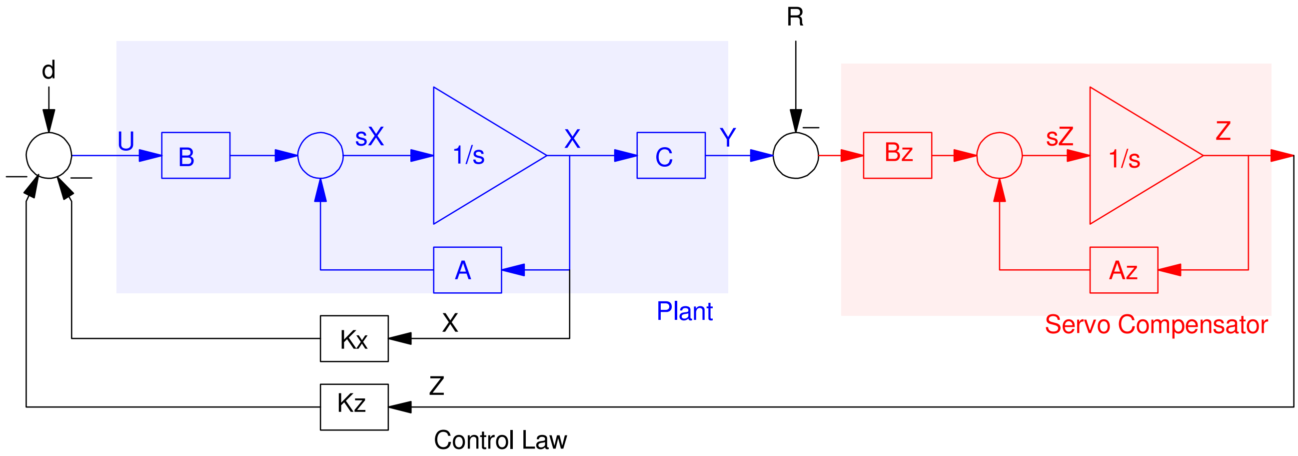


Step response when m = 0.6kg. Position (blue) and U/10 (red)

Code:

```
% Ball & Beam System
% Lecture #16
% Servo Compensators at DC

X = [0, 0, 0, 0]';
Z = 0;
dt = 0.01;
t = 0;
% Full-State Feedback Gains
Kx = [-7.2211   48.8540   -5.6397   20.0000];
Kr = -2.3211;

% Servo Compensator Gains
Kx = [  -84.6071  200.9500  -71.9893   36.6750];
Kz = -28.7143;
n = 0;
y = [];

while(t < 9.9)
   Ref = 1;
   U = -Kz*Z - Kx*X;
   %U = Kr*Ref - Kx*X;

   dX = BeamDynamics(X, U);
   dZ = X(1) - Ref;

   X = X + dX * dt;
   Z = Z + dZ * dt;
   t = t + dt;

   y = [y ; U/10, X(1)];
   n = mod(n+1,5);
   if(n == 0)
      BeamDisplay(X, Ref);
      end
   end

t = [1:length(y)]' * dt;

plot(t,y(:,1),'r',t,y(:,2),'b');
xlabel('Time (seconds)');
ylabel('Ball Position');
```

Servo Compensators with Sinulsoidal Set-Points



5) Assume a 1 rad/sec disturbance and/or set point (R). Design a feedback control law that results in

- The ability to track a constant set point (R = sin(t))
- The ability to reject a constant disturbance (d = sin(t)), and
- A 2% settling time of 6 seconds

First, input the plant and servo compensator

```
>> A = [0,0,1,0;0,0,0,1;0,-7,0,0;-1.96,0,0,0];
>> B = [0;0;0;0.4];
>> C = [1,0,0,0];
>> Az = [0,1;-1,0];
>> Bz = [1;1];
```

Create the augmented system

```
>> A6 = [A, zeros(4,2) ; Bz*C, Az]

        0         0    1.0000         0         0         0
        0         0         0    1.0000         0         0
        0   -7.0000         0         0         0         0
  -1.9600         0         0         0         0         0
   1.0000         0         0         0         0    1.0000
   1.0000         0         0         0   -1.0000         0

>> B6 = [B ; 0*Bz]

        0
        0
        0
   0.4000
        0
        0

>> B6r = [0*B; -Bz]

    0
    0
    0
    0
   -1
   -1
```

Find the full-state feedback gains

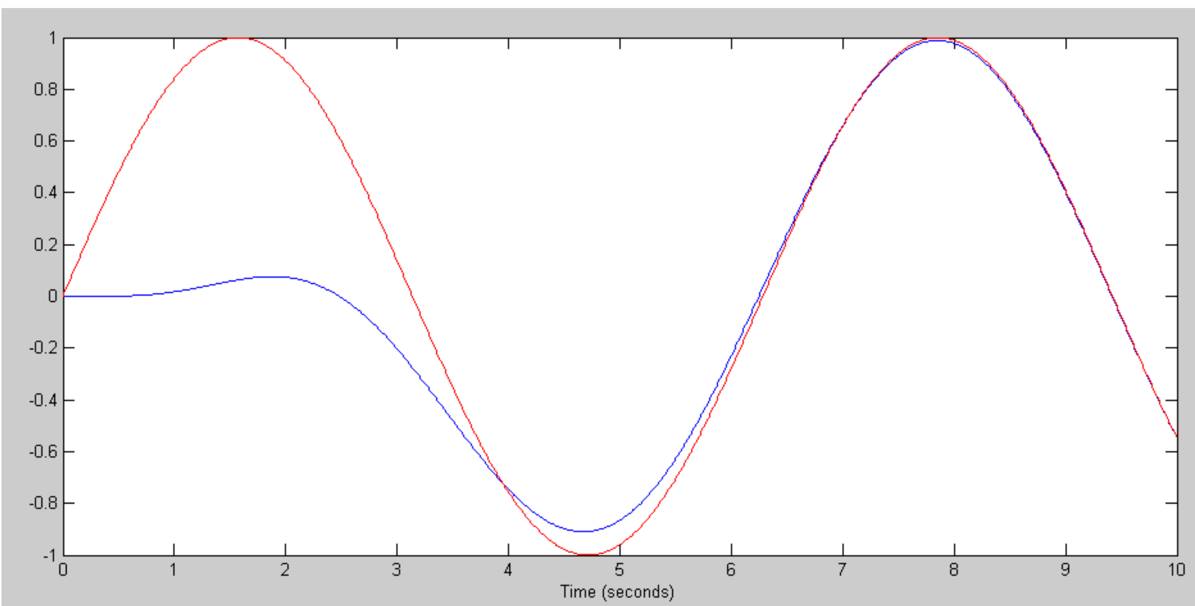
```
>> K6 = ppl(A6, B6, [-0.67+j, -0.67-j, -2, -3, -4, -5])

                          Kx                                Kz
K6 = -125.9796  225.5223  -90.7445   38.3500    6.3052  -52.6787
```

6) For the linear system, plot the response

- With R(t) = sin(t), and
- With d(t) = sin(t)

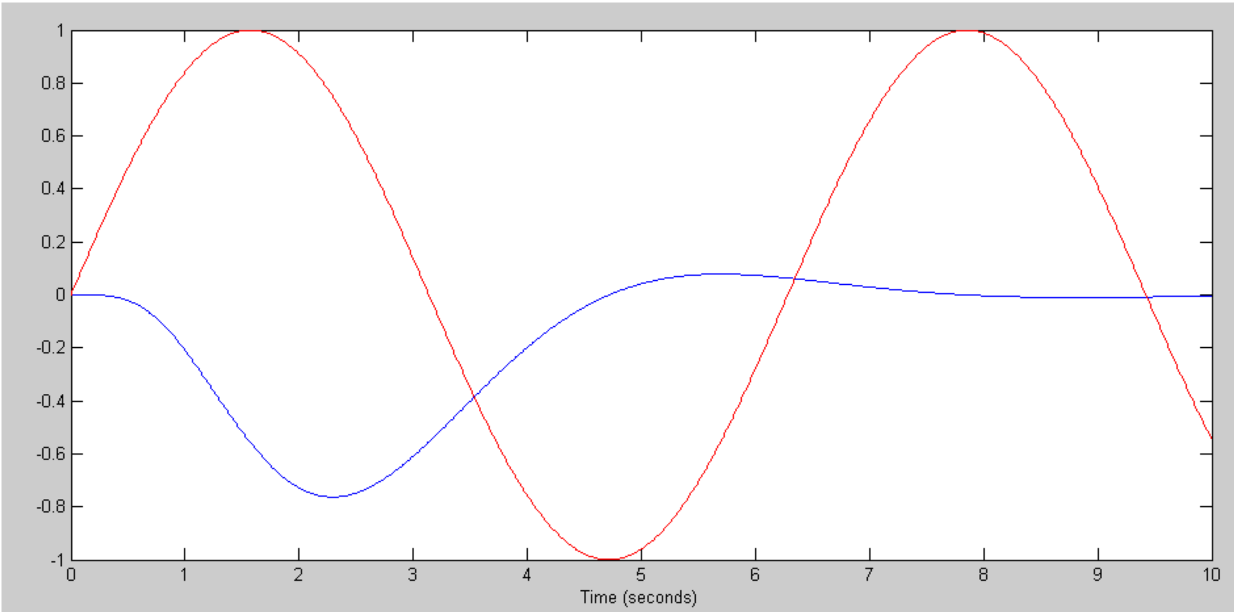
Take the response to a sinusoidal input using the step3() command (user created function in Matlab)

```
function [ y ] = step3( A, B, C, D, t, X0, U )

>> C6 = [C, 0,0 ; -K6];
>> D6 = [0;0];

>> X0 = zeros(6,1);
>> t = [0:0.01:10]';

>> R = sin(t);
>> y = step3(A6-B6*K6, B6r, C6, D6, t, X0, R);
>> plot(t,y(:,1),'b',t,R,'r');
>> xlabel('Time (seconds)');
```



Response to a sinusoidal set point. Output (blue) and set point (red)

```
>> y = step3(A6-B6*K6, B6, C6, D6, t, X0, R);
>> plot(t,y(:,1)*100,'b',t,R,'r');
>> xlabel('Time (seconds)');
```



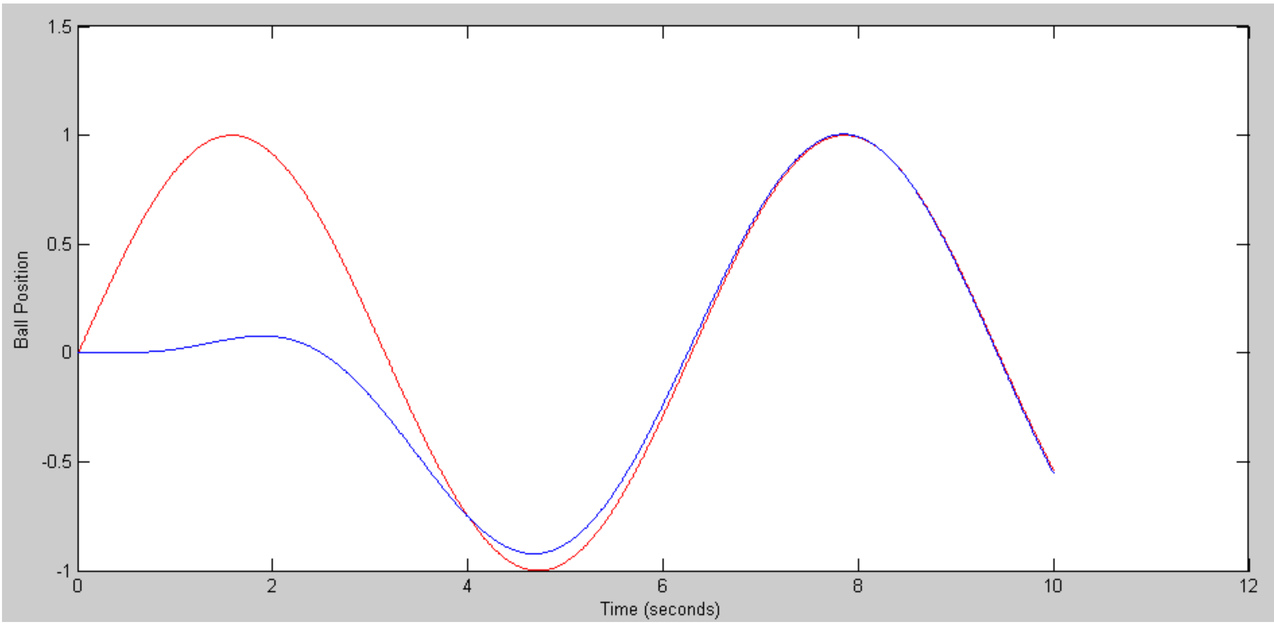
Response to a sinusoidal disturbance. Disturbance, D (red) and Position*100 (blue)

As expected, the sevo compensator

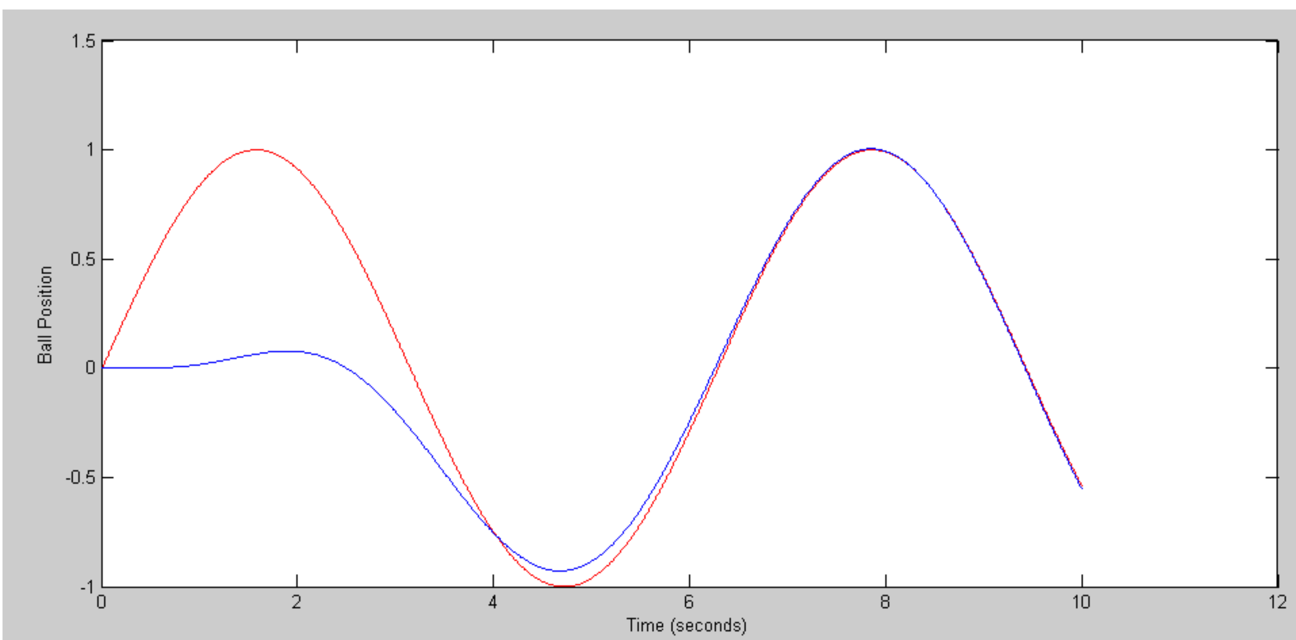
- Tracks a 1 rad/sec set point, and
- Rejects a 1 rad/sec disturbance.

7) Implement your control law on the nonlinear ball and beam system

- With R = sin(t) and the mass of the ball being 0.5kg (nominal), and
- With R = sin(t) and the mass of the ball being 0.6kg (ball has an extra 0.1kg)



Tracking a 1 rad/sec set point when m = 0.5kg



Tracking a 1 rad/sec set point when m = 0.6kg

Code:

```
% Ball & Beam System
% Lecture #17
% Servo Compensators at AC

X = [0,0,0,0]';
Z = zeros(2,1);
dt = 0.01;
t = 0;

% Servo Compensator Gains
Kx = [-125.9796  225.5223  -90.7445   38.3500];
Kz = [6.3052  -52.6787];
Az = [0,1 ; -1,0];
Bz = [1;1];

n = 0;
y = [];

while(t < 10)
   Ref = 1*sin(t);
   U = -Kz*Z - Kx*X;

   dX = BeamDynamics(X, U);
   dZ = Az*Z + Bz*(X(1) - Ref);

   X = X + dX * dt;
   Z = Z + dZ * dt;
   t = t + dt;

   y = [y ; Ref, X(1)];
   n = mod(n+1,5);
   if(n == 0)
      BeamDisplay(X, Ref);
      end
   end

t = [1:length(y)]' * dt;

plot(t,y(:,1),'r',t,y(:,2),'b');
xlabel('Time (seconds)');
ylabel('Ball Position');
```