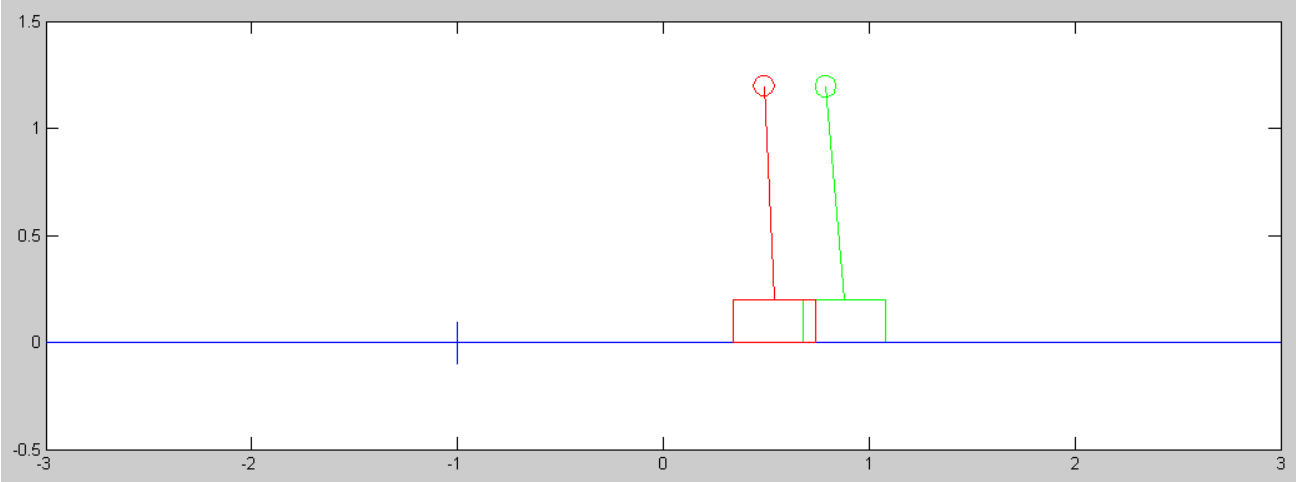
ECE 463: Homework #8

Linear Observers. Due Monday March 22nd, 10am (start of class)



Cart and Pendulum from homework #4 with a state estimator (green)

Use the dynamics for the cart and pendulum from homework set #4

1) Design a full-state feedback control law of the form

$$U = F = K_r R - K_x X$$

so that the closed-loop system has

- A 2% settling time of 6 seconds, and
- 10% overshoot for a step input.

Plot the step response of the linarized system in Matlab.

Translation: Place the dominant pole at

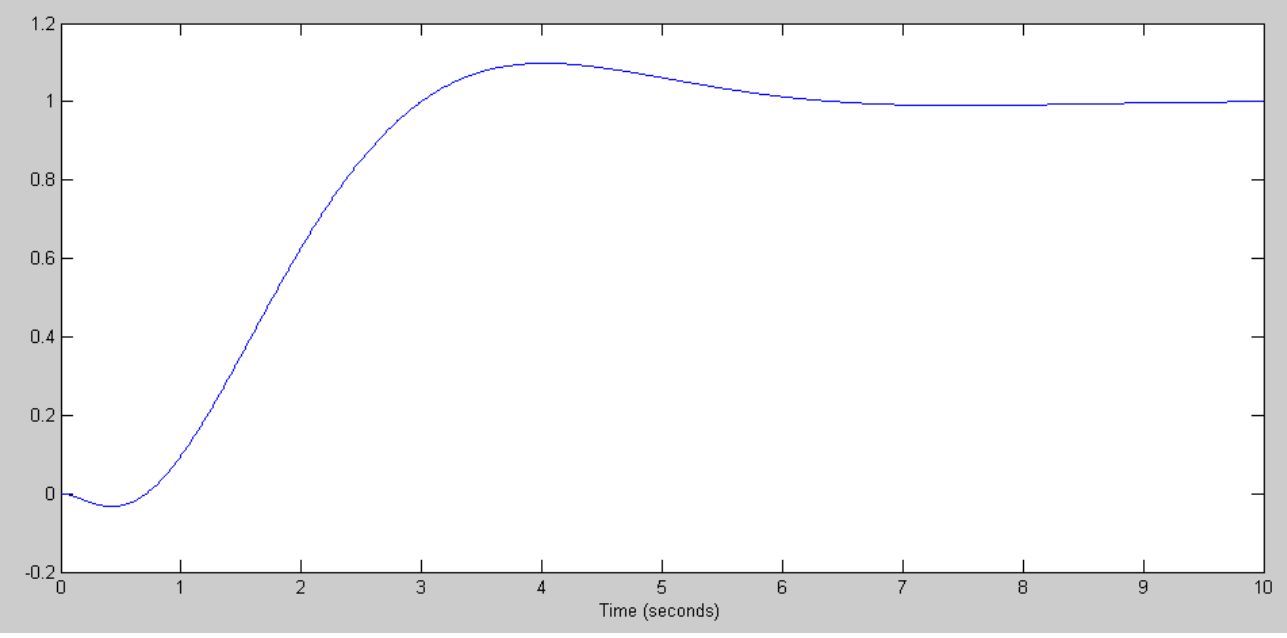
- s = -0.667 (Ts = 6 seconds)
- The damping ratio is 0.5910 (10% overshoot)
- s = -0.67 + j0.91

In Matlab:

```
>> A = [0,0,1,0;0,0,0,1;0,-39.2,0,0;0,49,0,0]

          0         0    1.0000         0
          0         0         0    1.0000
          0  -39.2000         0         0
          0   49.0000         0         0

>> B = [0;0;1;-1]

     0
     0
     1
    -1

>> Kx = ppl(A,B,[-0.67+j*0.91,-0.67-j*0.91,-3,-4])

Kx =   -1.5637  -73.2207   -2.5530  -10.8930

>> C = [1,0,0,0];
>> DC = -C*inv(A-B*Kx)*B
>> Kr = 1/DC

Kr =   -1.5637
```

Plotting the step response

```
>> G = ss(A-B*Kx, B*Kr, C, 0);
>> t = [0:0.01:10]';
>> y = step(G,t);
>> plot(t,y)
```



2) Assume you can only measure the cart position and beam angle.

2a) Design a full-order observer to estimate all four states so that the observer is 2-5 times faster than the plant. You may use either cart position or beam angle (or both) as measurements.

Note: Pole placement doesn't work with two inputs. Ignore the beam angle and only use the cart position

```
>> C = [1,0,0,0];
>> H = ppl(A', C', [-3,-4,-5,-6])'

   18.0000
  -31.2245
  168.0000
 -219.1837
```

2b) Give the state-space model of the closde loop system using the actual states:

$$U = F = K_r R - K_x X$$

and plot the step response with initial conditions of

$X(0) = [0, 0, 0, 0]'$ $\quad$ $X_{observer}(0) = [0.1, 0.1, 0.1, 0.1]'$

(note: use the function step3)

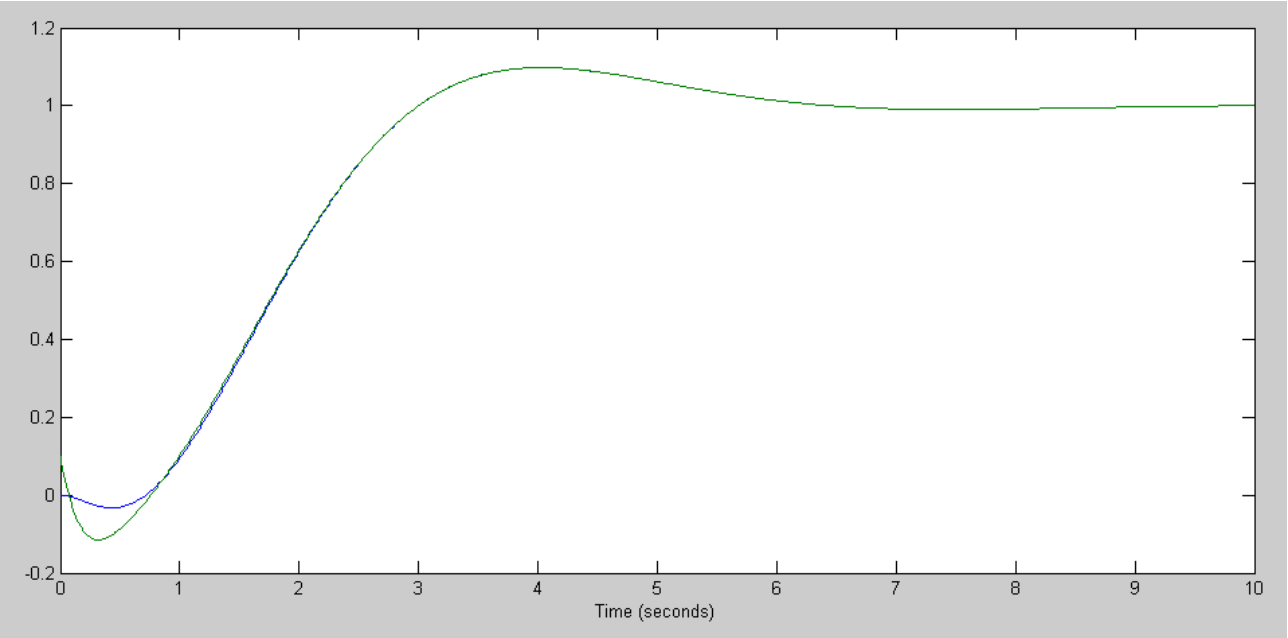
$$\begin{bmatrix} sX \\ sX_e \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix} \begin{bmatrix} X \\ X_e \end{bmatrix} + \begin{bmatrix} B \\ B \end{bmatrix} U + \begin{bmatrix} 0 \\ H \end{bmatrix} (Y - Y_e)$$

substituting for U

$$\begin{bmatrix} sX \\ sX_e \end{bmatrix} = \begin{bmatrix} A - BK_x & 0 \\ HC - BK_x & A - HC \end{bmatrix} \begin{bmatrix} X \\ X_e \end{bmatrix} + \begin{bmatrix} BK_r \\ BK_r \end{bmatrix} R$$

```
>> A8 = [A-B*Kx, zeros(4,4) ; H*C-B*Kx, A-H*C]

         0         0    1.0000         0         0         0         0         0
         0         0         0    1.0000         0         0         0         0
    1.5637   34.0207    2.5530   10.8930         0         0         0         0
   -1.5637  -24.2207   -2.5530  -10.8930         0         0         0         0
   18.0000         0         0         0  -18.0000         0    1.0000         0
  -31.2245         0         0         0   31.2245         0         0    1.0000
  169.5637   73.2207    2.5530   10.8930 -168.0000  -39.2000         0         0
 -220.7473  -73.2207   -2.5530  -10.8930  219.1837   49.0000         0         0

>> B8 = [B*Kr ; B*Kr]

         0
         0
   -1.5637
    1.5637
         0
         0
   -1.5637
    1.5637
```

```
>> C8 = [C,0*C ; 0*C, C]

     1     0     0     0     0     0     0     0
     0     0     0     0     1     0     0     0

>> D8 = zeros(2,1);
>> xlabel('Time (seconds)')>> X0 = [zeros(4,1) ; 0.1*ones(4,1)]

          0
          0
          0
          0
     0.1000
     0.1000
     0.1000
     0.1000

>> R = 0*t + 1;
>> y8 = step3(A8, B8, C8, D8, t, X0, R);
>> plot(t,y8)
>> xlabel('Time (seconds)')
```



Position (blue) & Observer Estimmate (green)

2c) Give the state-space model of the closed loop system using the state estimates:

$$U = K_r R - K_x X_{observer}$$

and plot the step response with initial conditions of

$X(0) = [0, 0, 0, 0]'$ $\quad$ $X_{observer}(0) = [0.1, 0.1, 0.1, 0.1]'$

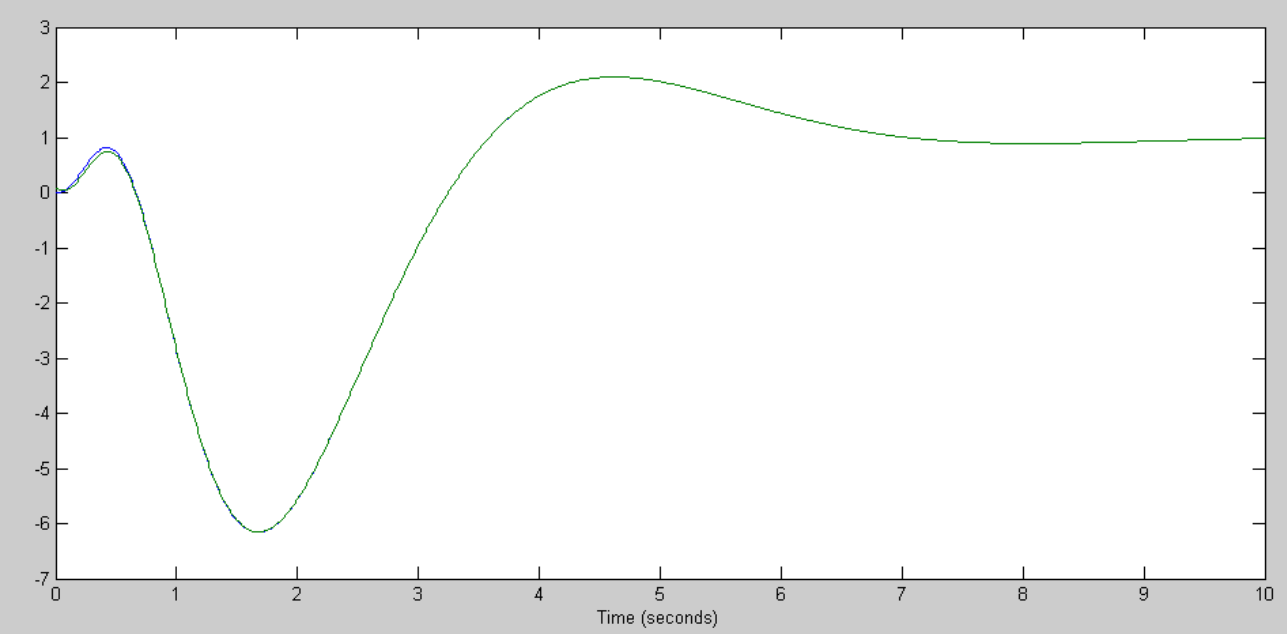
Feeding back the actual states

$$\begin{bmatrix} sX \\ sX_e \end{bmatrix} = \begin{bmatrix} A - BK_x & 0 \\ HC - BK_x & A - HC \end{bmatrix} \begin{bmatrix} X \\ X_e \end{bmatrix} + \begin{bmatrix} BK_r \\ BK_r \end{bmatrix} R$$

changes when you feed back the state estimates

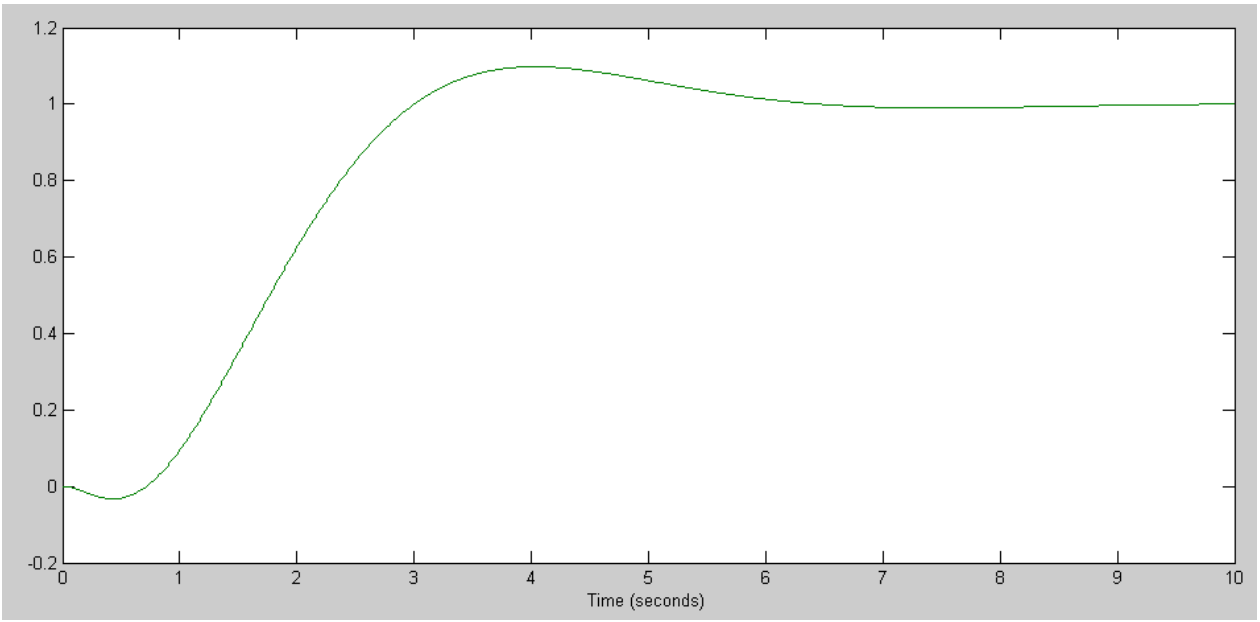
$$\begin{bmatrix} sX \\ sX_e \end{bmatrix} = \begin{bmatrix} A & -BK_x \\ HC & A - BK_x - HC \end{bmatrix} \begin{bmatrix} X \\ X_e \end{bmatrix} + \begin{bmatrix} BK_r \\ BK_r \end{bmatrix} R$$

```
>> A8 = [A, -B*Kx ; H*C, A-B*Kx-H*C]

         0         0    1.0000         0         0         0         0         0
         0         0         0    1.0000         0         0         0         0
         0  -39.2000         0         0    1.5637   73.2207    2.5530   10.8930
         0   49.0000         0         0   -1.5637  -73.2207   -2.5530  -10.8930
   18.0000         0         0         0  -18.0000         0    1.0000         0
  -31.2245         0         0         0   31.2245         0         0    1.0000
  168.0000         0         0         0 -166.4363   34.0207    2.5530   10.8930
 -219.1837         0         0         0  217.6200  -24.2207   -2.5530  -10.8930

>> B8 = [B*Kr ; B*Kr]

         0
         0
   -1.5637
    1.5637
         0
         0
   -1.5637
    1.5637

>> C8 = [C,0*C ; 0*C, C]

     1     0     0     0     0     0     0     0
     0     0     0     0     1     0     0     0

>> D8 = zeros(2,1);
>> y8 = step3(A8, B8, C8, D8, t, X0, R);
>> plot(t,y8)
>> xlabel('Time (seconds)')
>>
```



Step Response when Feeding back the state estimtes with different initial conditions

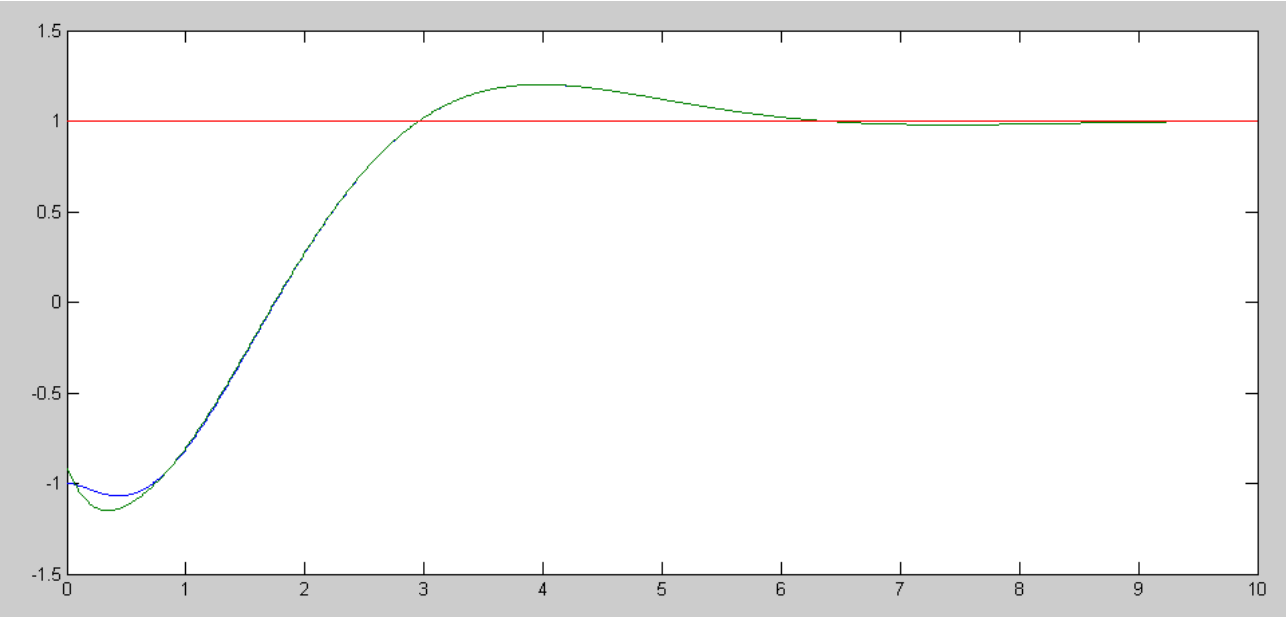
```
>> X0 = zeros(8,1);
>> y8 = step3(A8, B8, C8, D8, t, X0, R);
>> plot(t,y8)
>> xlabel('Time (seconds)')
```



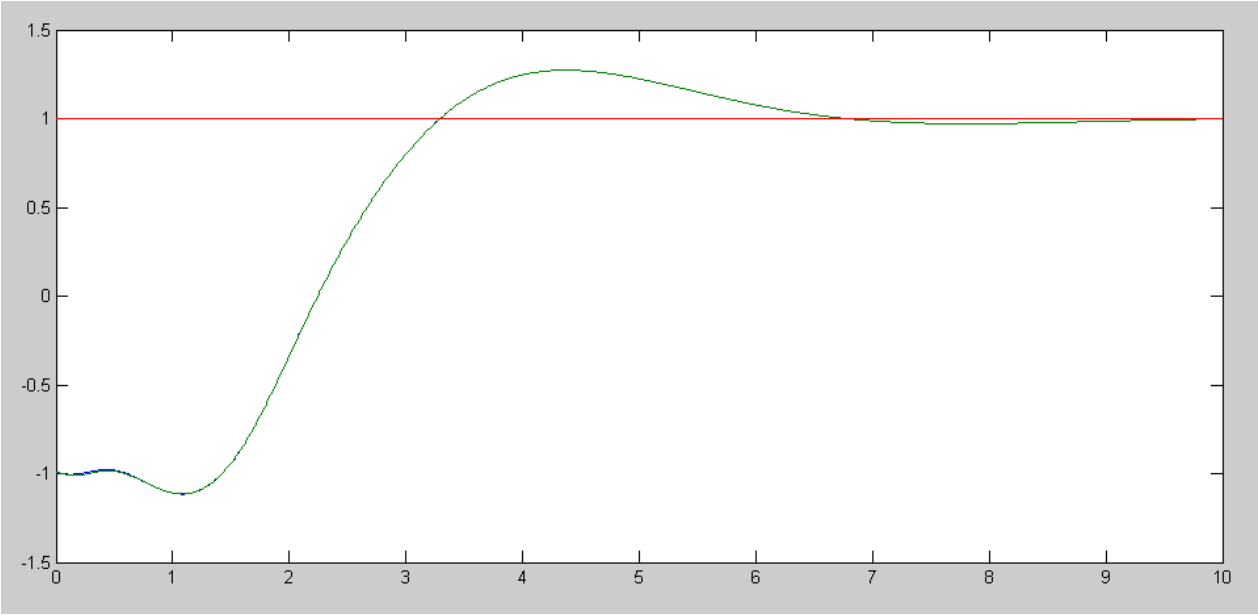
Step Response when Feeding back the state estimtes with same initial conditions

3) Modify the cart and pendulum system to include

- your control law, and
- A full-order observer



Step Response with U = Kr*R - Kx * X

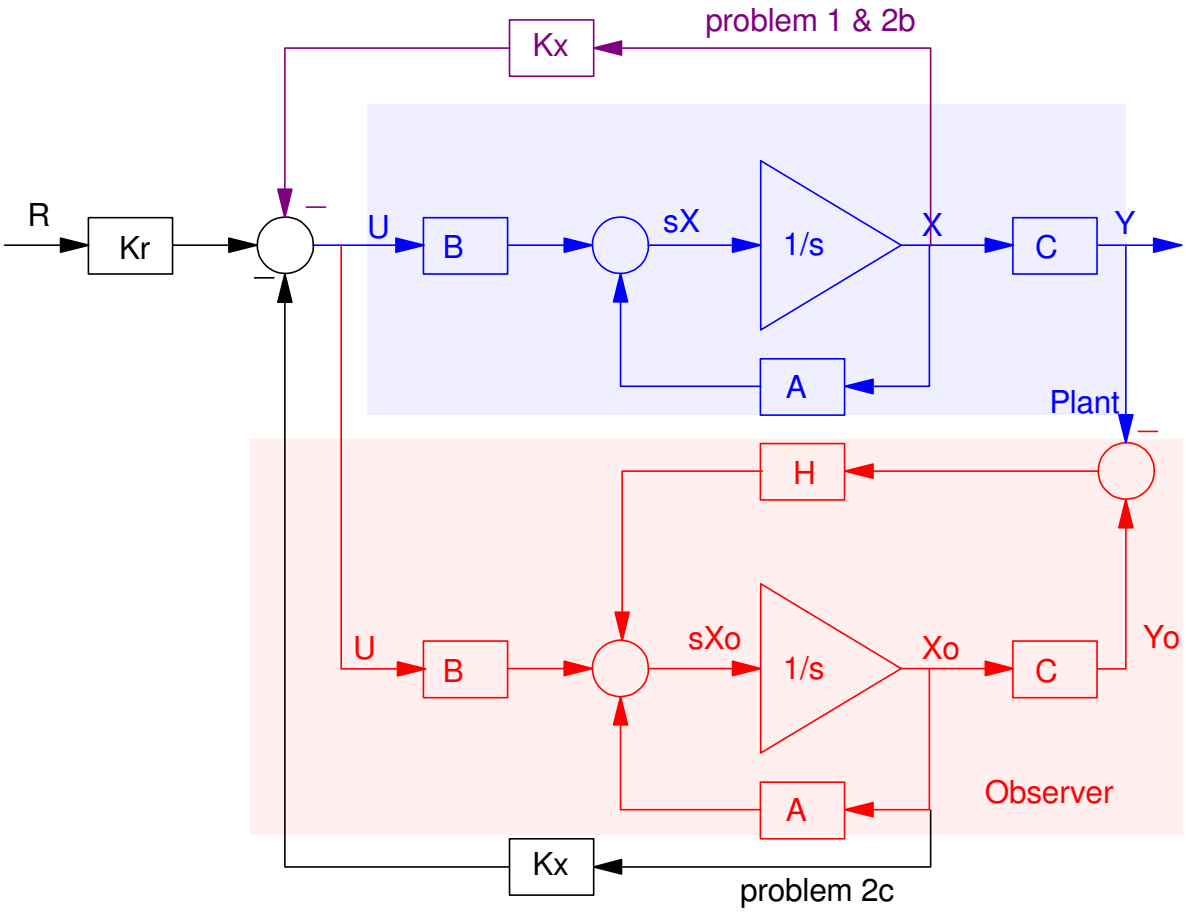


Step Response with U = Kr*R - Kx*Xe

Code:

```
% Cart and Pendulum ( Sp21 version)
% m1 = 1.0kg
% m2 = 4.0kg
% L = 1.0m

X = [-1; 0 ; 0 ; 0];
Ref = 1;
dt = 0.01;
t = 0;
Kx = [-1.5637  -73.2207   -2.5530  -10.8930];
Kr = -1.5637;

% Full-order observer
A = [0,0,1,0;0,0,0,1;0,-39.2,0,0;0,49,0,0];
B = [0;0;1;-1]
C = [1,0,0,0];
H = [18.0000  -31.2245  168.0000 -219.1837]'
Xe = X + 0.01*ones(4,1);

y = [];
while(t < 10)
 Ref = 1;
 U = Kr * Ref - Kx*Xe;
 dX = CartDynamics(X, U);
 dXe = A*Xe + B*U + H*(C*X - C*Xe);

 X = X + dX * dt;
 Xe = Xe + dXe * dt;
 t = t + dt;

 CartDisplay(X, Xe, Ref);
 y = [y ; X(1), Xe(1), Ref];
end

clf
t = [1:length(y)]' * dt;
plot(t,y);
```

Block Diagram


problem 1 & 2b
Kx
R
Kr
U
B
sX
1/s
X
C
Y
A
Plant
H
U
B
sXo
1/s
Xo
C
Yo
Observer
A
Kx
problem 2c