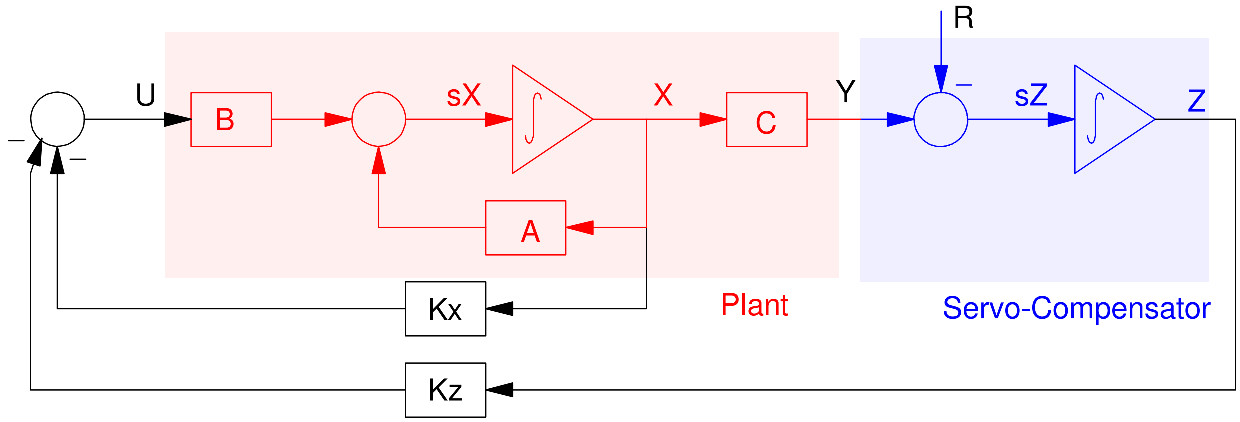
ECE 463/663 - Homework #10

LQG Control with Servo Compensators. Due Monday, April 12th



Cart and Pendulum (HW #7): Use LQG methods to design a full-state feedback control law of the form

$$U = -K_z Z - K_x X$$

$$\dot{Z} = (x - R)$$

for the cart and pendulum system from homework #5 using LQG control so that

- You track constant setpoints,
- You reject constant disturbances,
- The 2% settling time is 6 seconds, and
- There is less than 5% overshoot for a step input.

1) Give the control law (Kx and Kz) and explain how you chose Q and R

The desired response has poles at (5% overshoot, damping ratio = 0.6901 or more)

$$s = -0.67 + j0.7$$

$$G_d = \left(\frac{0.939}{(s+0.67\pm j0.7)}\right) = \left(\frac{0.939}{s^2+1.33s+0.939}\right)$$

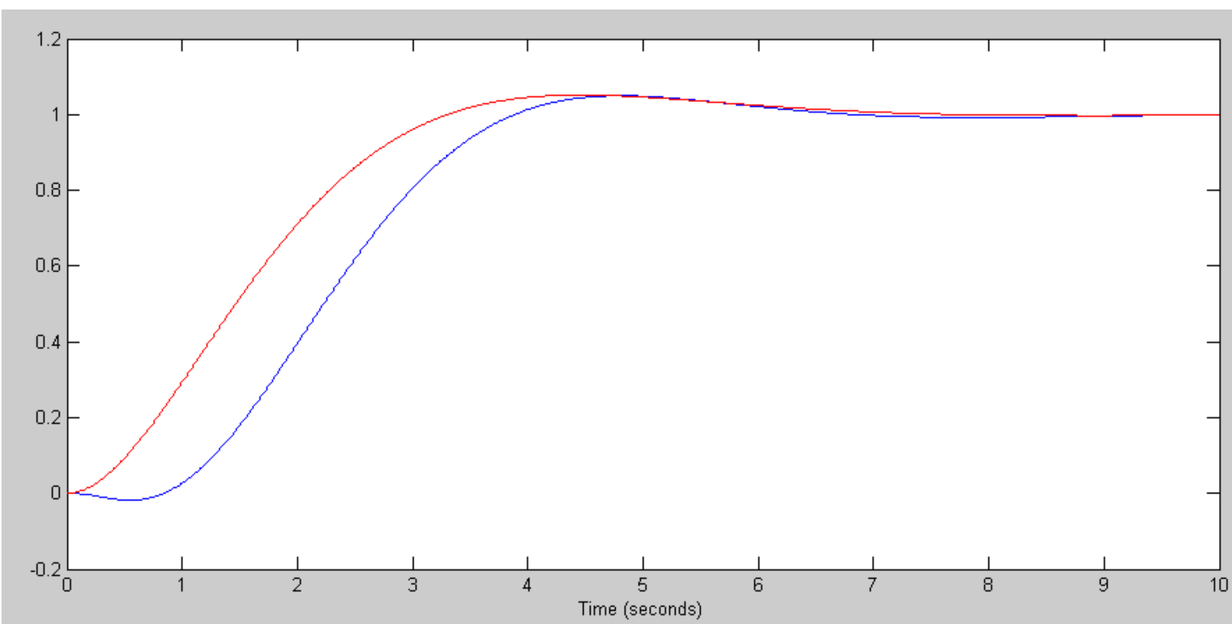
Q and R are adjusted until the plant & feedback has about the same step response as Gd

- Start with increasing the weighting on Z until you're fast enough (Q(5,5) = 50))
- Increase the weighting on Y to reduce the overshoot (Q(1,1) = 10)

Matlab Code

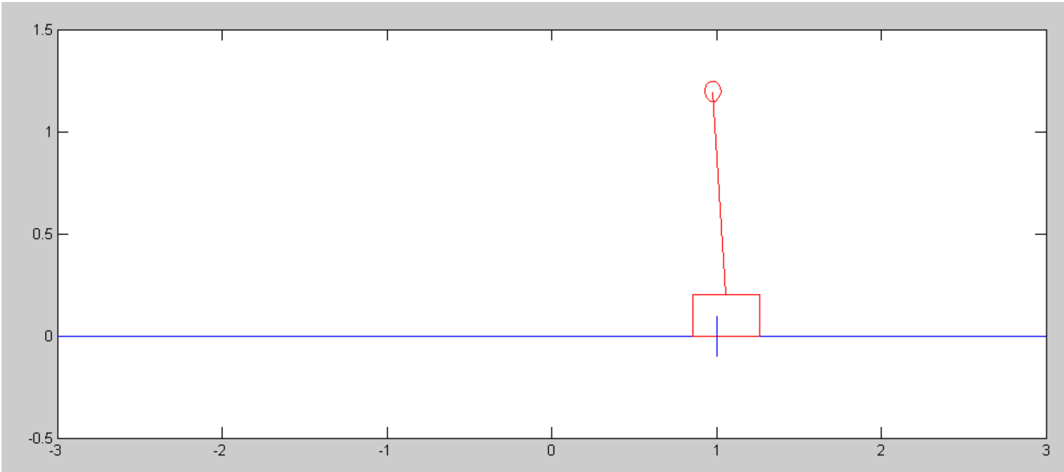
```
A = [0,0,1,0;0,0,0,1;0,-39.2,0,0;0,49,0,0];
B = [0;0;1;-1];
C = [1,0,0,0];

A5 = [A, 0*B ; C, 0];
B5 = [B; 0];
B5r = [0*B; -1];
C5 = [C, 0];

Gd = tf(0.939,[1,1.33,0.939]);

K5 = lqr(A5, B5, diag([10,0,0,0,50]), 1);

t = [0:0.01:10]';
G = ss(A5-B5*K5, B5r, C5, 0);
y = step(G,t);
yd = step(Gd, t);

plot(t,y,'b',t,yd,'r');
xlabel('Time (seconds)');
eig(A - B*Kx)

>> K5

                            Kx                            Kz
K5 =   -15.5841 -149.1354  -16.4659   -32.8092    -7.0711
```

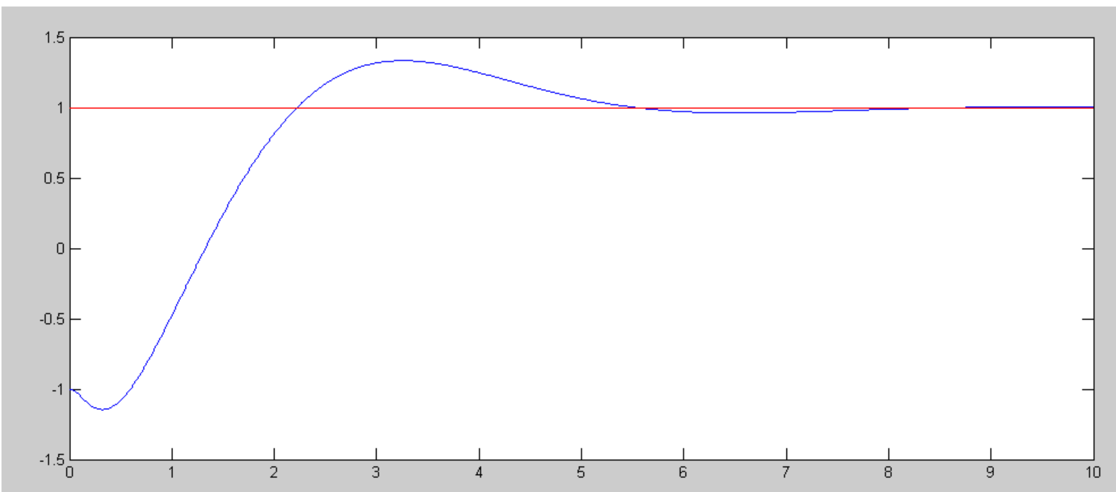
2) Plot the step response of the linear system



Desired Step Response (red) & Linear System's Step Response (blue)

3) Check your design with the nonlinear simulation of the cart and pendulum system.





Ball and Beam (HW #7): Use LQG methods to design a full-state feedback control law of the form

$$U = -K_z Z - K_x X$$

$$\dot{Z} = (x - R)$$

for the ball and beam system from homework #5 using LQG control so that

- You track constant setpoints,
- You reject constant disturbances,
- The 2% settling time is 6 seconds, and
- There is less than 5% overshoot for a step input.

4) Give the control law (Kx and Kx) and explain how you chose Q and R

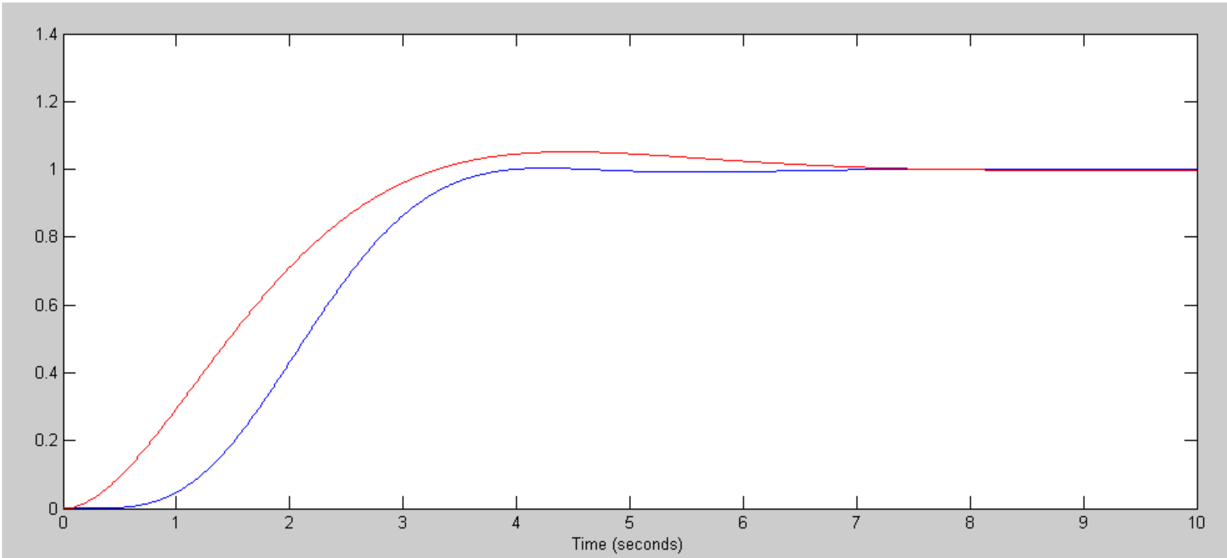
Same as problem #1 but using the (A,B,C,D) matrix for the ball and beam system

- Start with Q weighting Z (Q(5,5) = 1). Adjust until it's fast enough.
- The resulting response is pretty close, so you're done

Code:

```
A = [0,0,1,0;0,0,0,1;0,-7,0,0;-1.96,0,0,0];
B = [0;0;0;0.4];
C = [1,0,0,0];

A5 = [A, 0*B ; C, 0];
B5 = [B; 0];
B5r = [0*B; -1];
C5 = [C, 0];

Gd = tf(0.939,[1,1.33,0.939]);

K5 = lqr(A5, B5, diag([0,300,0,0,40]), 1);

t = [0:0.01:10]';
G = ss(A5-B5*K5, B5r, C5, 0);
y = step(G,t);
yd = step(Gd, t);

plot(t,y,'b',t,yd,'r');
xlabel('Time (seconds)');
eig(A - B*Kx)



>> K5

                        Kx                          Kz
K5 =   -18.6196   50.3087  -12.9825   15.8601   -6.3246
```

5) Plot the step response of the linear system



Desired Step Response (red) & Linear System's Step Response (blue)

6) Check your design with the nonlinear simulation of the cart and pendulum system.