ECE 463/663 - Homework #11

Kalman Filters. Due Monday, April 19th

Kalman Filters

Cart and Pendulum (HW #6): The dynamics for a cart and pendulum system with sensor and input noise is as follows

$$s\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -39.2 & 0 & 0 \\ 0 & 49 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}(F + n_u)$$

$$y = x + n_y$$

where there is Gaussian noise at the input and output

$n_u \sim N(0, 0.02^2)$ *mean zero, standard deviation 0.02*

$n_y \sim N(0, 0.01^2)$ *mean zero, standard deviation 0.01*

1) Use a servo-compensator to force the DC gain to one (i.e. use the servo compensator from homework set #10.

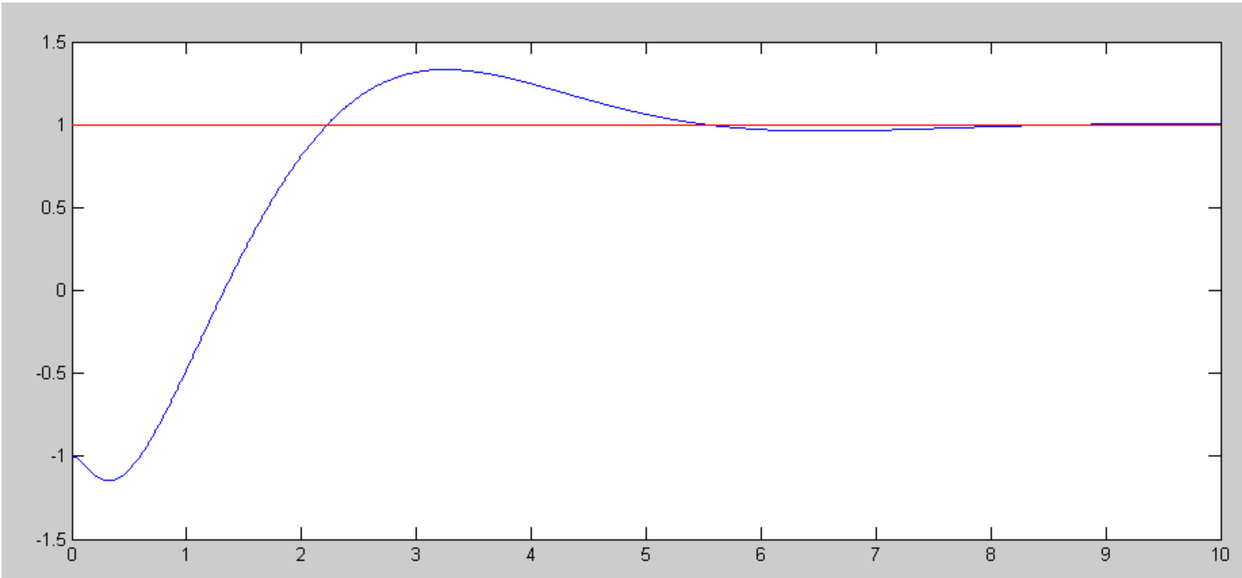
Plot the step response

- Without noise (same as homework set #10)
- With noise

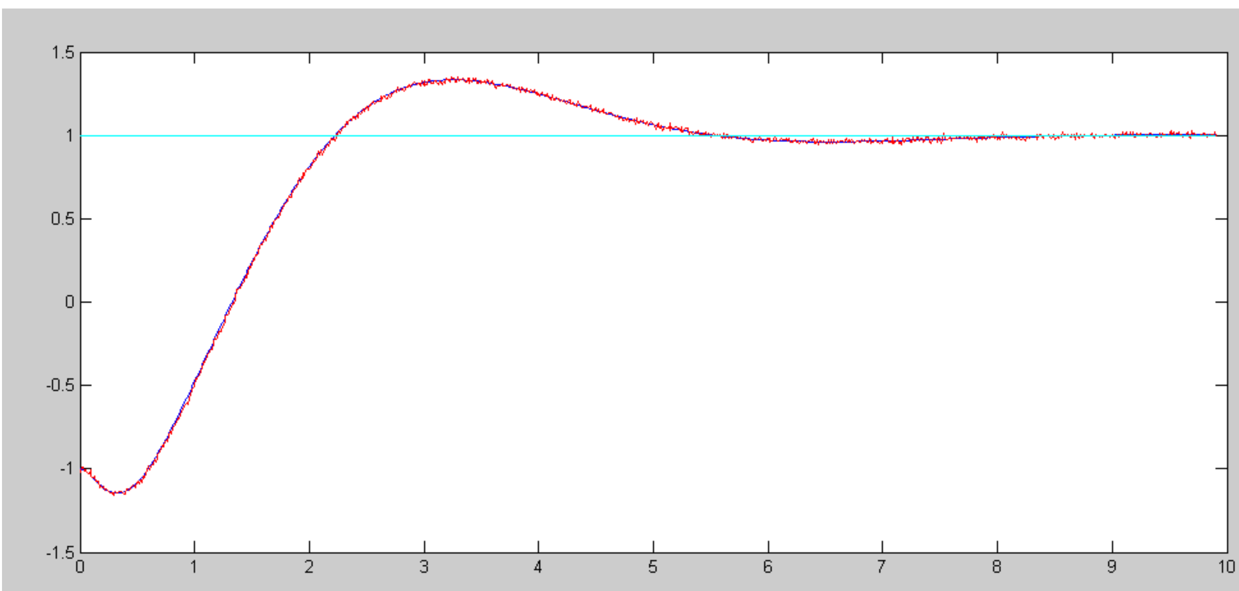
From homework set #10

```
                               Kx                        Kz
K5 =  -15.5841 -149.1354  -16.4659   -32.8092   -7.0711
```

Without noise (same as homework #10)



With noise, feeding back the actual states



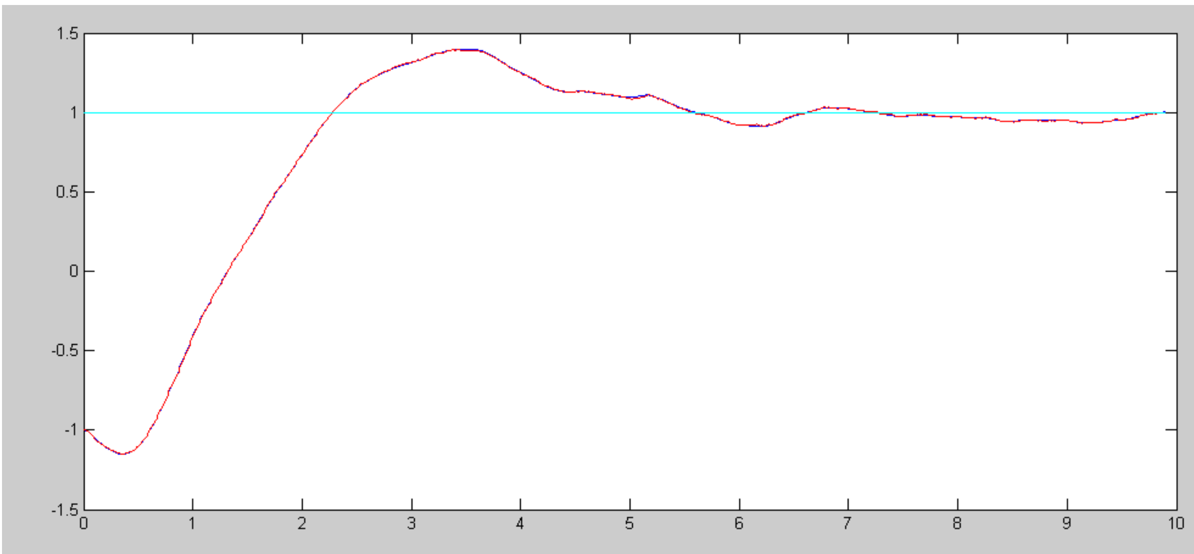
2) Design a full-order observer using pole-placement to place the observer poles at {-3, -4, -5, -6}

- Simulate the response of the cart with noise added at the input and output.
- Plot the states of the plant and the observer with noise,.

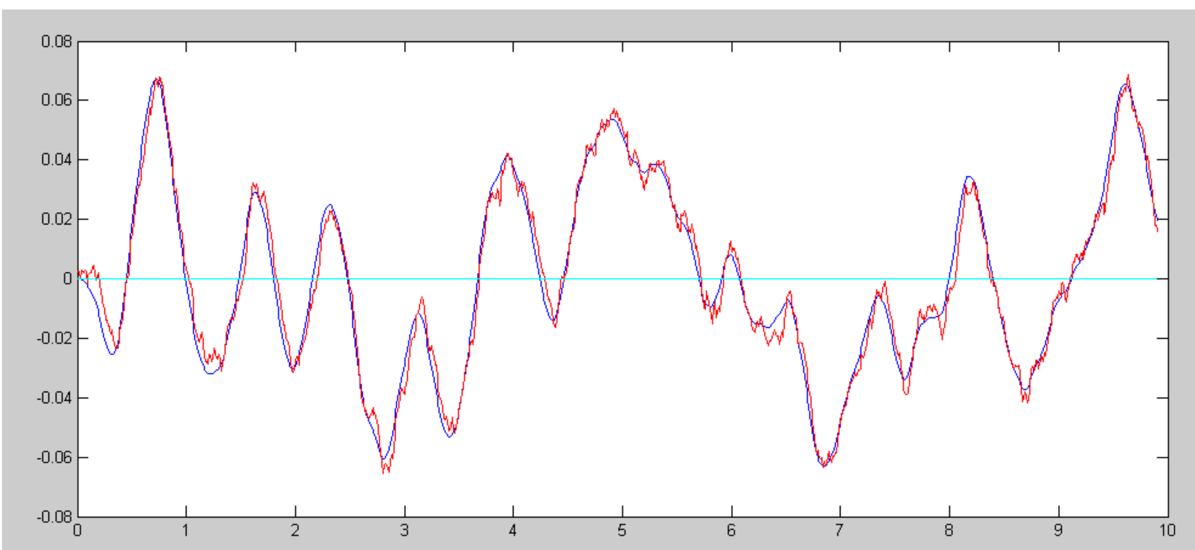
```
>> H = ppl(A', C', [-3,-4,-5,-6])'

   18.0000
  -48.8571
  119.0000
  -53.3886
```

Step response: Feedback uses state estimates



Ref = 0 response

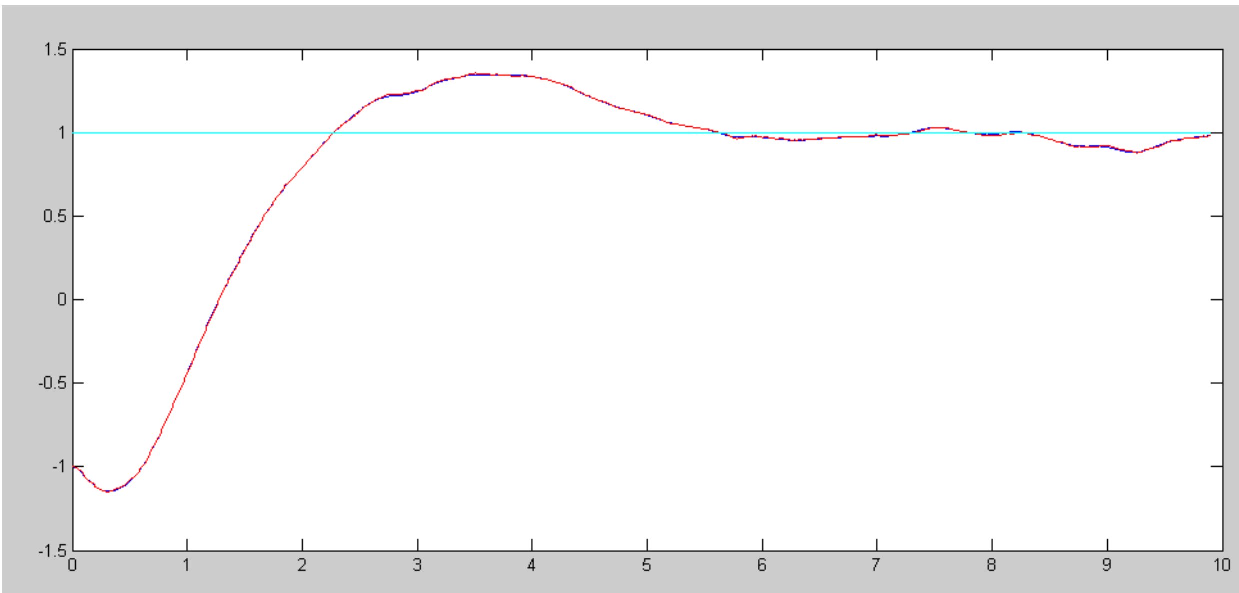


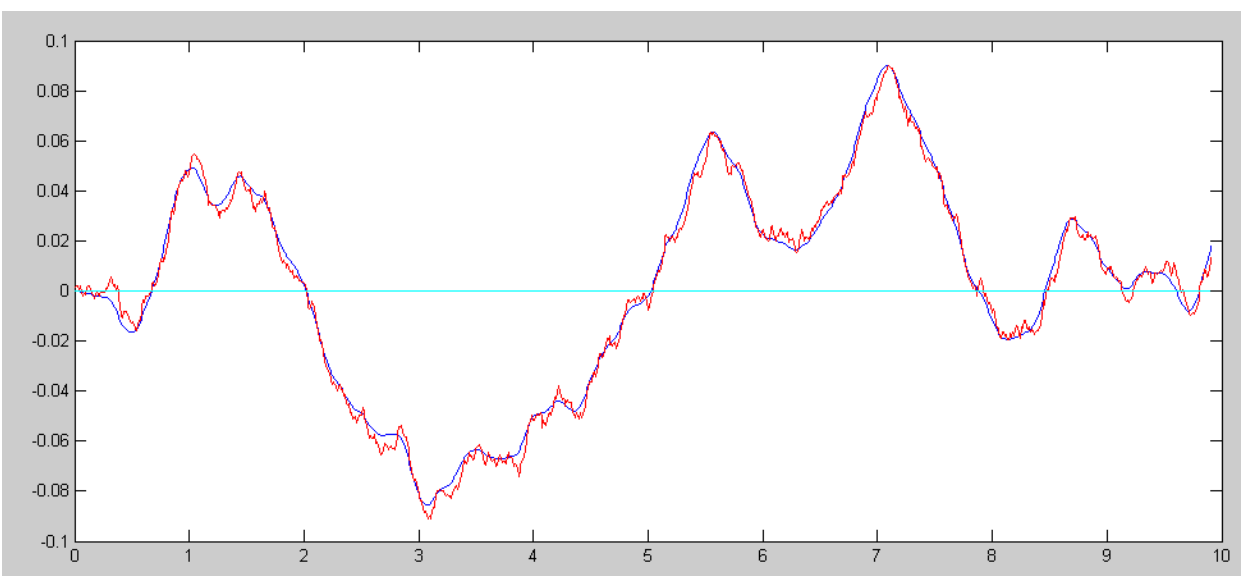
3) Design a Kalman filter (i.e. a full-order observer with a specific Q and R)

- Simulate the response of the cart with noise added at the input and output.
- Plot the states of the plant and the observer with noise,.

```
Q = B*B' * 0.02;
R = 0.01;
H = lqr(A', C', Q, R)';

   14.7607
  -19.5029
  108.9392
 -136.5275
```





Note: The Kalman filter is slightly better. The standard deviations for the actual vs. estimated position and angle and

	Pole Placement	Kalman Filter
std(x - xe)	0.0042	0.0036
std(q - qe)	0.0073	0.0048