ECE 463/663 - Homework #12

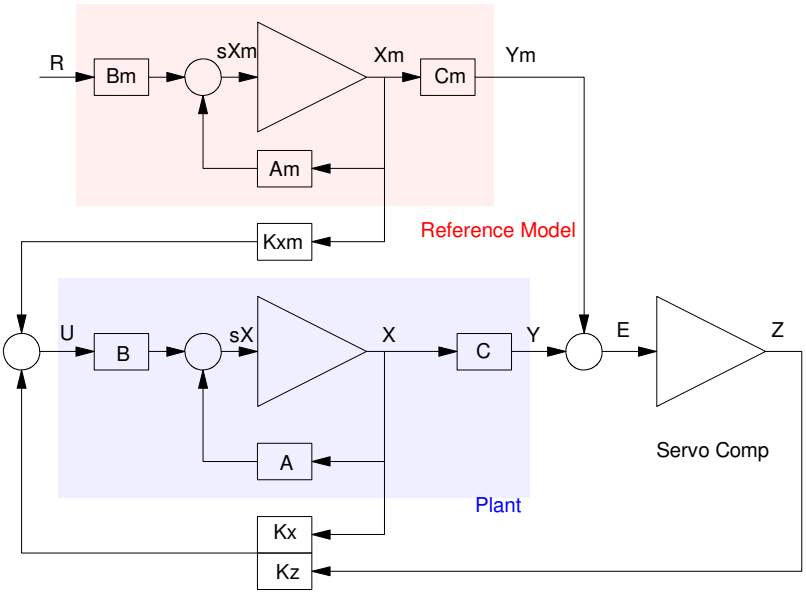
LQG/LTR. Due Monday, April 26th

LQG / LTR

1) Design a control law so that the ball and beam system behaves like the following reference model:

$$y_m = \left(\frac{0.5}{s^2+s+0.5}\right) R$$

1) Give a block diagram for your controller



$$s\begin{bmatrix} X \\ Z \\ X_m \end{bmatrix} = \begin{bmatrix} A & 0 & 0 \\ C & 0 & -C_m \\ 0 & 0 & A_m \end{bmatrix}\begin{bmatrix} X \\ Z \\ X_m \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} U + \begin{bmatrix} 0 \\ 0 \\ B_m \end{bmatrix} R$$

$$U = -\begin{bmatrix} K_x & K_z & K_{xm} \end{bmatrix}\begin{bmatrix} X \\ Z \\ X_m \end{bmatrix}$$

The system output is

$$Y = Z = \begin{bmatrix} 0 & I & 0 \end{bmatrix}\begin{bmatrix} X \\ Z \\ X_m \end{bmatrix}$$

2) Plot the step response of the model and the linearlized plant for your control law for

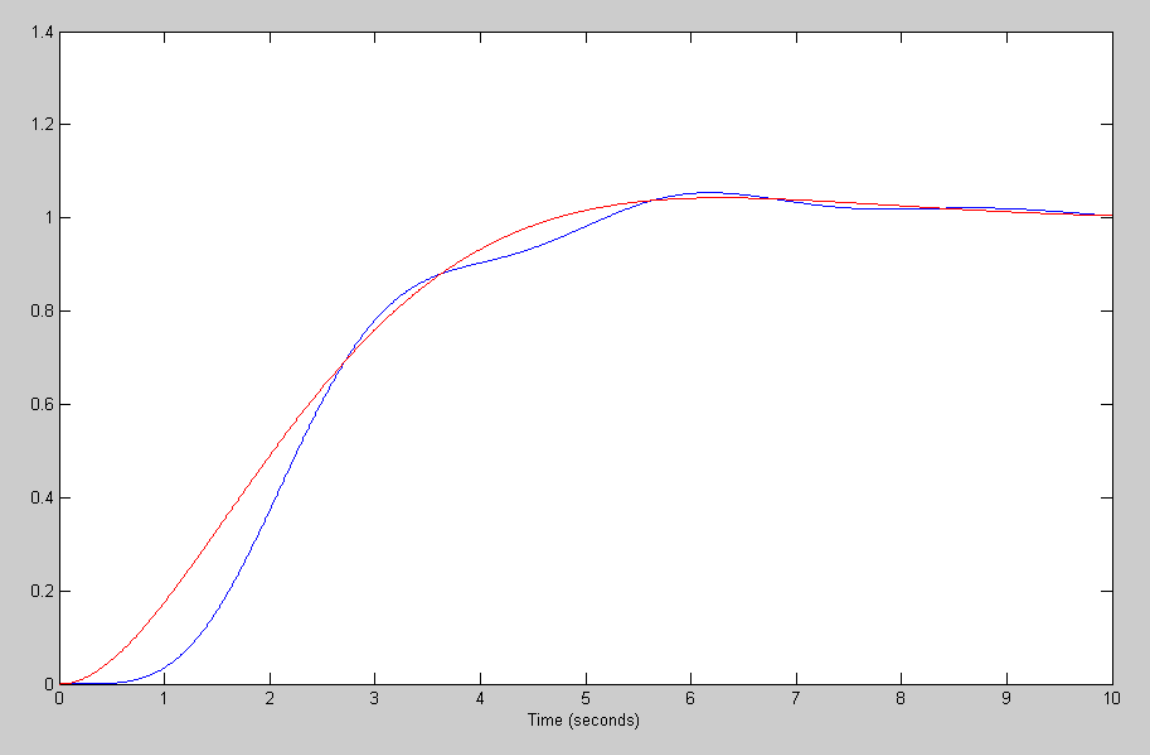
- Q = 100
- Q = 1000
- Q = 10,000

LQG/LTR with Q = 100 z^2

```
>> K7 = lqr(A7, B7, Q7*1e2, R7)

                        Kx                                  Kz              Kxm
K7 =   -21.8367   44.4987  -13.1421   14.9162  -10.0000    6.9855    3.9188

>> y = step3(A7-B7*K7, Br, C7, zeros(2,1), t, X0, 0*t+1);
>> plot(t,y(:,1),'b',t,y(:,2), 'r')
>> xlabel('Time (seconds)');
```

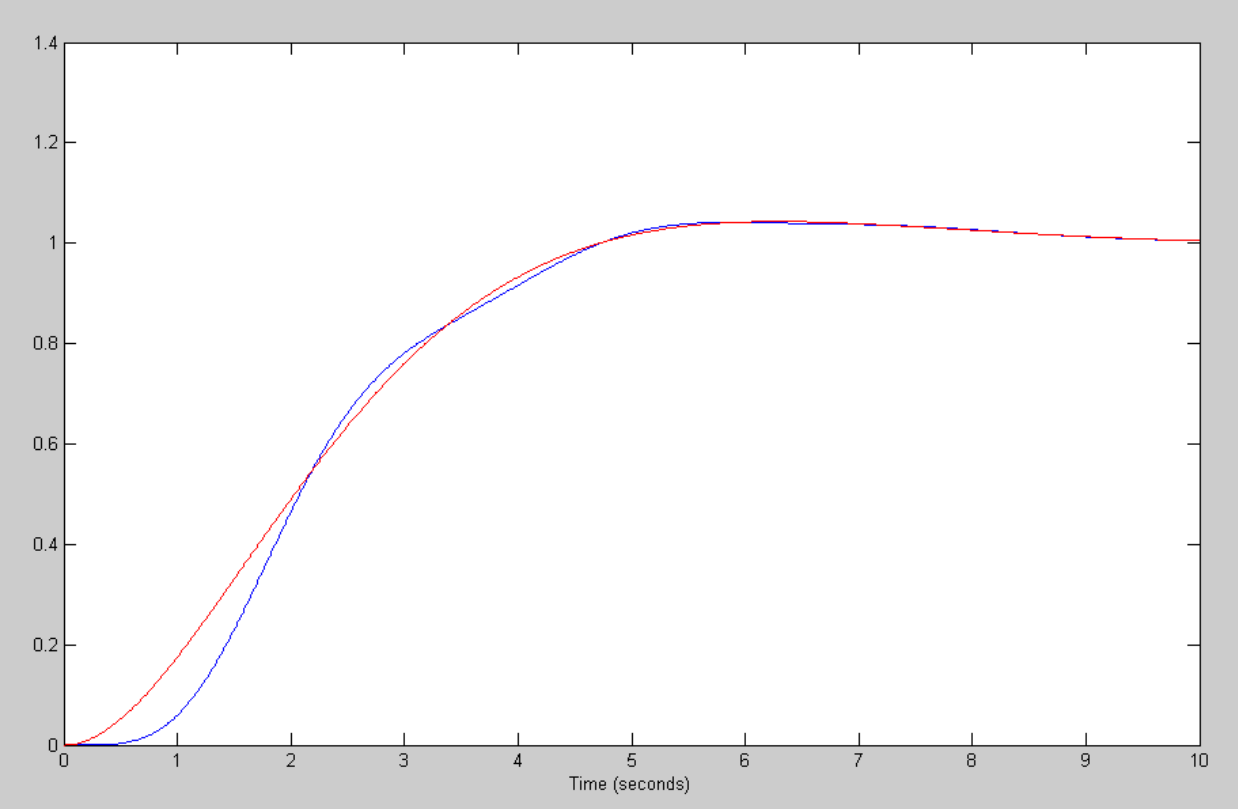


LQG/LTR with Q = 1000 z^2

```
>> K7 = lqr(A7, B7, Q7*1e3, R7)

                        Kx                                  Kz                  Kxm
K7 =  -46.1684   74.5729  -26.5485   19.3097  -31.6228   18.4937    8.6785

>> y = step3(A7-B7*K7, Br, C7, zeros(2,1), t, X0, 0*t+1);
>> plot(t,y(:,1),'b',t,y(:,2), 'r')
>> xlabel('Time (seconds)');
```



LQG/LTR with Q = 10,000 z^2

```
>> K7 = lqr(A7, B7, Q7*1e4, R7)

                      Kx                                  Kz              Kxm
K7 = -108.9703  121.9121  -54.0330   24.6893 -100.0000   48.4818   19.2180

>> y = step3(A7-B7*K7, Br, C7, zeros(2,1), t, X0, 0*t+1);
>> plot(t,y(:,1),'b',t,y(:,2), 'r')
>> xlabel('Time (seconds)');
```