ECE 463/663 - Homework #9

Calculus of Variations. LQG Control. Due Wednesday, April 4th, 2022

Soap Film

1) Calculate the shape of a soap film connecting two rings around the X axis:
- $Y(0) = 10$
- $Y(4) = 9$

2) Calculate the shape of a soap film connecting two rings around the X axis:
- $Y(0) = 10$
- $Y(2)$ = free

Hanging Chain

3) Calculate the shape of a hanging chain subject to the following constraints
- Length of chain = 12 meters
- Left Endpoint: (0,0)
- Right Endpoint: (10,1)

Ricatti Equation

4) Find the function, x(t), which minimizes the following funcional

$$J = \int_0^{10} (x^2 + 4\dot{x}^2)dt$$

$x(0) = 6$

$x(10) = 4$

5) Find the function, x(t), which minimizes the following funcional

$$J = \int_0^{10} (x^2 + 9u^2)dt$$

$$\dot{x} = -0.1x + u$$

$x(0) = 6$

$x(10) = 4$

LQG Control

6) Cart & Pendulum (HW #4 & HW#6):

$$
s\begin{bmatrix} x \\ \theta \\ sx \\ s\theta \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -14.7 & 0 & 0 \\ 0 & 24.5 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ sx \\ s\theta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.5 \\ -0.5 \end{bmatrix} F
$$

Design a full-state feedback control law of the form

$$F = U = K_r R - K_x X$$

for the cart and pendulum system from homework #4 using LQG control so that

- The DC gain is 1.00
- The 2% settling time is 8 seconds, and
- There is less than 10% overshoot for a step input.

Compare your results with homework #6

- Where are the closed-loop poles with pole placement and with LQG control?
- Are the feedback gains larger or smaller with LQG control?
- Which one works better?

7) Ball and Beam (HW #4 & HW#6):

$$
s\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -7 & 0 & 0 \\ -4.2 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.143 \end{bmatrix} T
$$

Design a full-state feedback control law of the form

$$T = U = K_r R - K_x X$$

for the ball and beam system from homework #4 using LQG control so that

- The DC gain is 1.00
- The 2% settling time is 8 seconds, and
- There is less than 10% overshoot for a step input.

Compare your results with homework #6

- Where are the closed-loop poles with pole placement and with LQG control?
- Are the feedback gains larger or smaller with LQG control?
- Which one works better?