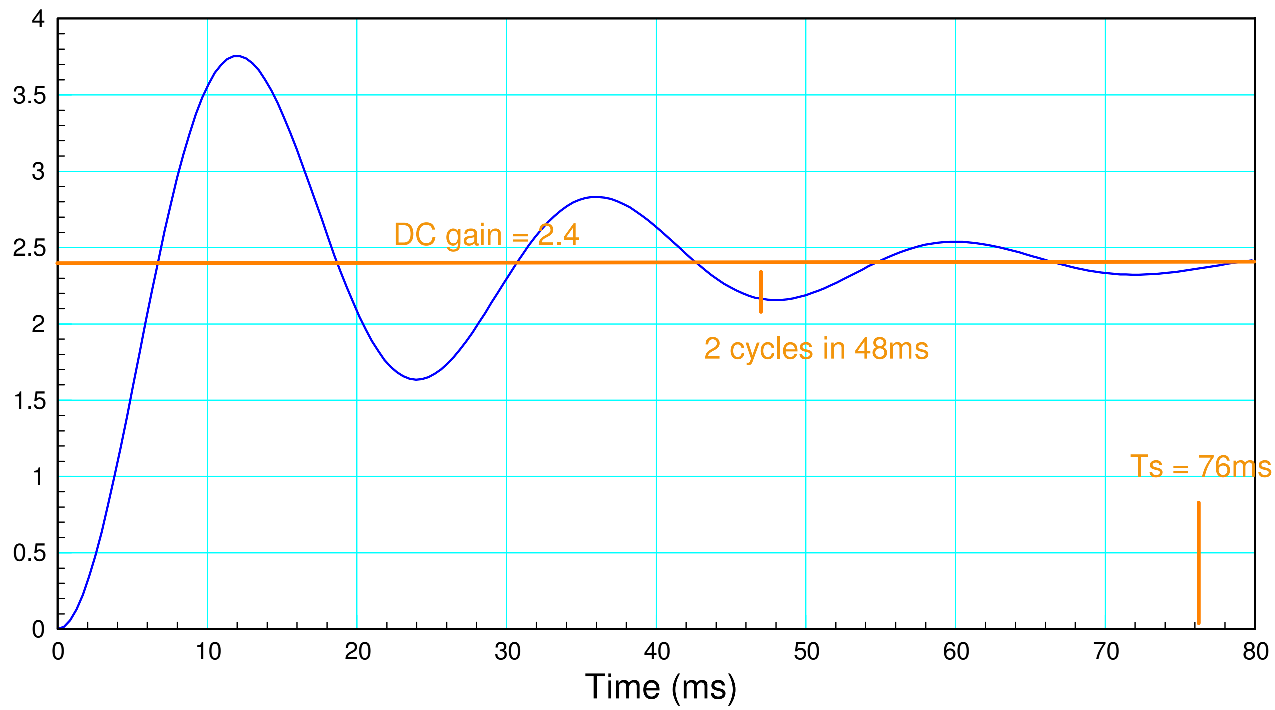
CE 463/663: Test #1.   Name ______________

Spring 2022. Open Book, Open Notes. Calculators & Matlab allowed. Individual Effort

1) Find the transfer funciton for a system with the following step response



The DC gain is about 2.4

The 2% settling time is about 76ms

$$real(s) \approx \frac{4}{76ms} = 52.6$$

The frequency of oscillation is

$$\omega_d \approx \left(\frac{2\text{ cycles}}{48ms}\right) 2\pi = 261 \frac{rad}{\text{sec}}$$

so

$$G(s) \approx \left(\frac{170{,}130}{(s+52.6+j261)(s+52.6-j261)}\right)$$

the numerator was whatever it took to make the DC gain equal to 2.40

2) Determine a 2nd-order system which has approximately the same step response as the following 7th-order system

$$Y = \left(\frac{10{,}000}{(s+0.2)(s+1)(s+3)(s+5)(s+8)(s+10)(s+12)} \right) X$$

Keep the two most dominant poles

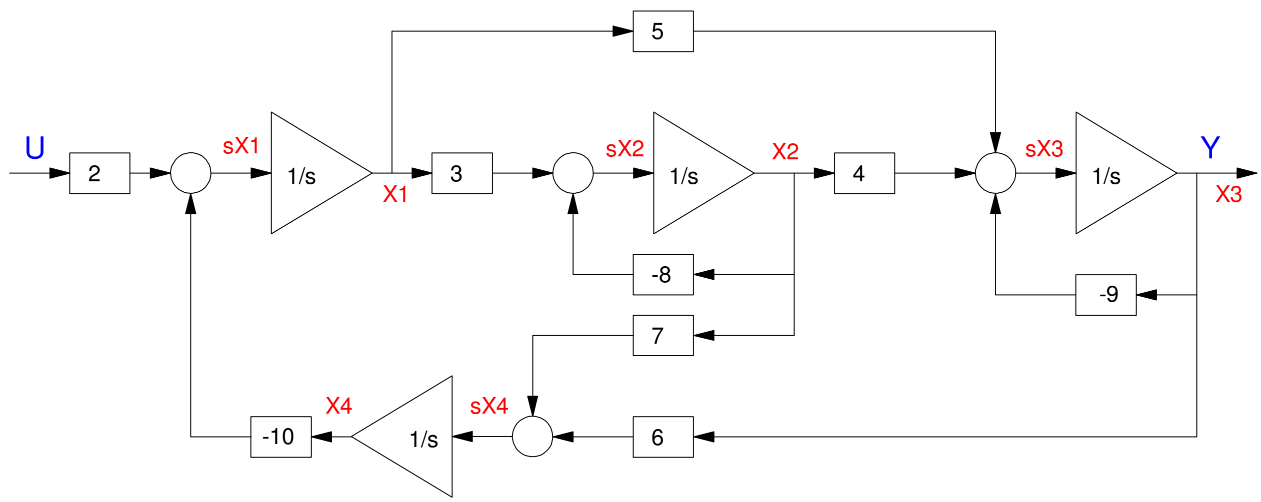
- (s + 0.2)
- (s + 1)

Match the DC gain

$$\left(\frac{10{,}000}{(s+0.2)(s+1)(s+3)(s+5)(s+8)(s+10)(s+12)} \right)_{s=0} = 3.4722$$

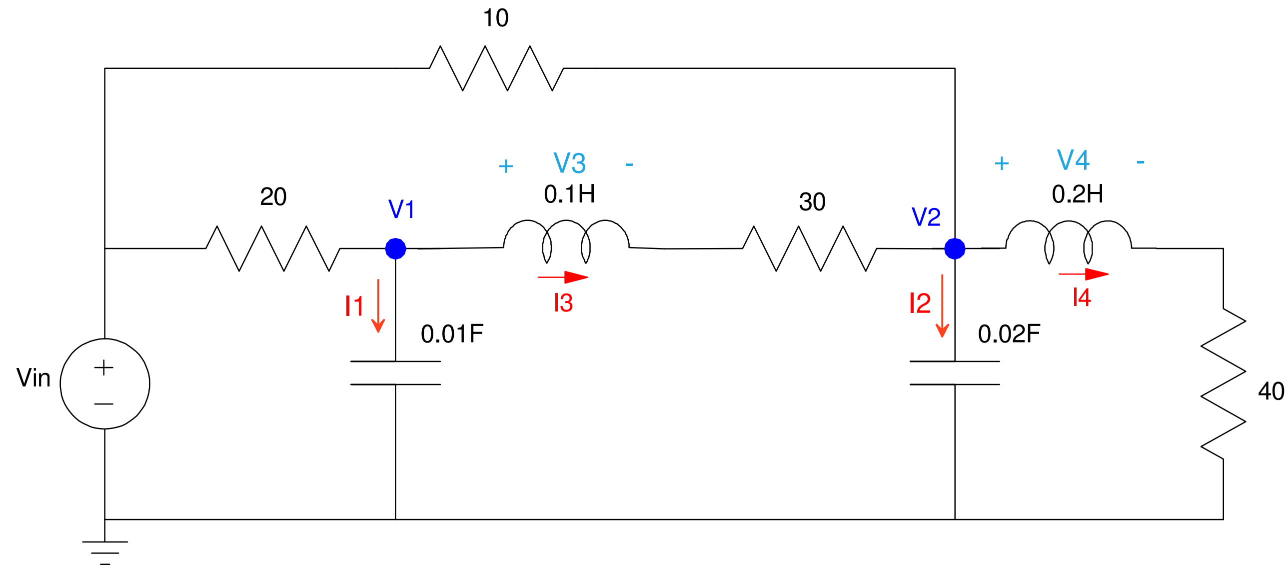
$$Y \approx \left(\frac{0.6944}{(s+0.2)(s+1)} \right) X$$

3) Give {A and B} for the the state-space model for the following system



$$\begin{bmatrix} sX1 \\ sX2 \\ sX3 \\ sX4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -10 \\ 3 & -8 & 0 & 0 \\ 5 & 4 & -9 & 0 \\ 0 & 7 & 6 & 0 \end{bmatrix} \begin{bmatrix} X1 \\ X2 \\ X3 \\ X4 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} U$$

4a) Write four coupled differential equations to describe the following circuit



$$I_1 = 0.01sV_1 = \left(\frac{V_{in}-V_1}{20}\right) - I_3$$

$$I_2 = 0.02sV_2 = I_3 + \left(\frac{V_{in}-V_2}{10}\right) - I_4$$

$$V_3 = 0.1sI_3 = V_1 - 30I_3 - V_2$$

$$V_4 = 0.2sI_4 = V_2 - 40I_4$$

4b) Express the A and B matricies for the dynamics in state-space form

Simplifying

$$sV_1 = 5V_{in} - 5V_1 - 100I_3$$

$$sV_2 = 50I_3 + 5V_{in} - 5V_2 - 50I_4$$

$$sI_3 = 10V_1 - 300I_3 - 10V_2$$

$$sI_4 = 5V_2 - 200I_4$$

$$\begin{bmatrix} sV1 \\ sV2 \\ sI3 \\ sI4 \end{bmatrix} = \begin{bmatrix} -5 & 0 & -100 & 0 \\ 0 & -5 & 50 & -50 \\ 10 & -10 & -300 & 0 \\ 0 & 5 & 0 & -200 \end{bmatrix} \begin{bmatrix} V1 \\ V2 \\ I3 \\ I4 \end{bmatrix} + \begin{bmatrix} 5 \\ 5 \\ 0 \\ 0 \end{bmatrix} \text{Vin}$$

5) Assume the LaGrangian is:

$$L = 4x^2\dot{x}^3\dot{\theta}^2 + 5x\dot{x}\cos(\theta) - 2g\sin(\theta)$$

Determine

$$F = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \left(\frac{\partial L}{\partial x}\right)$$

Taking partial derivatives

$$F = \frac{d}{dt}\left(12x^2\dot{x}^2\dot{\theta}^2 + 5x\cos\theta\right) - \left(8x\dot{x}^3\dot{\theta}^2 + 5\dot{x}\cos\theta\right)$$

$$\begin{aligned} F = {} & 24x\dot{x}^3\dot{\theta}^2 + 24x^2\dot{x}\ddot{x}\dot{\theta}^2 + 24x^2\dot{x}^2\dot{\theta}\ddot{\theta} \\ & +5\dot{x}\cos\theta - 5x\dot{\theta}\sin\theta \\ & -8x\dot{x}^3\dot{\theta}^2 - 5\dot{x}\cos\theta \end{aligned}$$

Simplifying (not necessary)

$$F = 16x\dot{x}^3\dot{\theta}^2 + 24x^2\dot{x}\ddot{x}\dot{\theta}^2 + 24x^2\dot{x}^2\dot{\theta}\ddot{\theta} - 5x\dot{\theta}\sin\theta$$