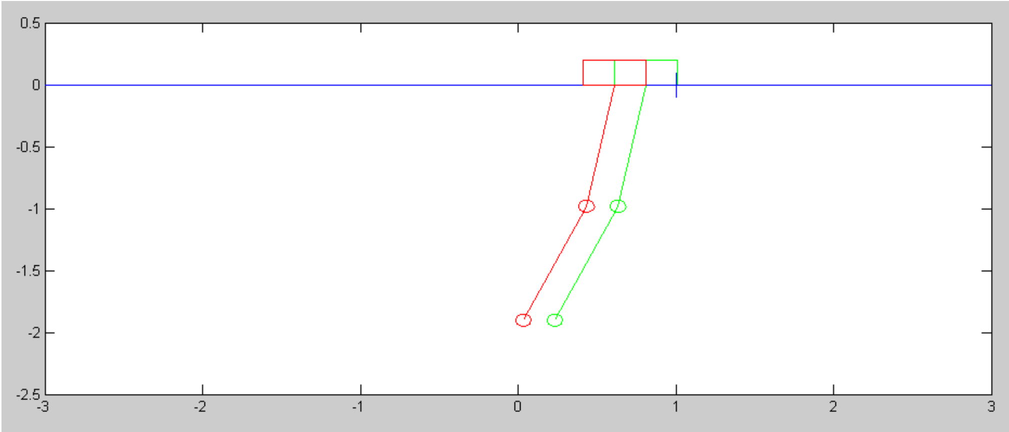
ECE 463/663 - Test #2: Name ________________

Due midnight Sunday, April 3rd. Individual Effort Only (no working in groups)



The linearized dynamics for a double gantry system are:

$$s\begin{bmatrix} x \\ \theta_1 \\ \theta_2 \\ \dot{x} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 2g & 0 & 0 & 0 & 0 \\ 0 & -3g & g & 0 & 0 & 0 \\ 0 & 3g & -3g & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta_1 \\ \theta_2 \\ \dot{x} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} F + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} d$$

C Level (max 80 points)

Design a feedback control law for the double pendulum assuming

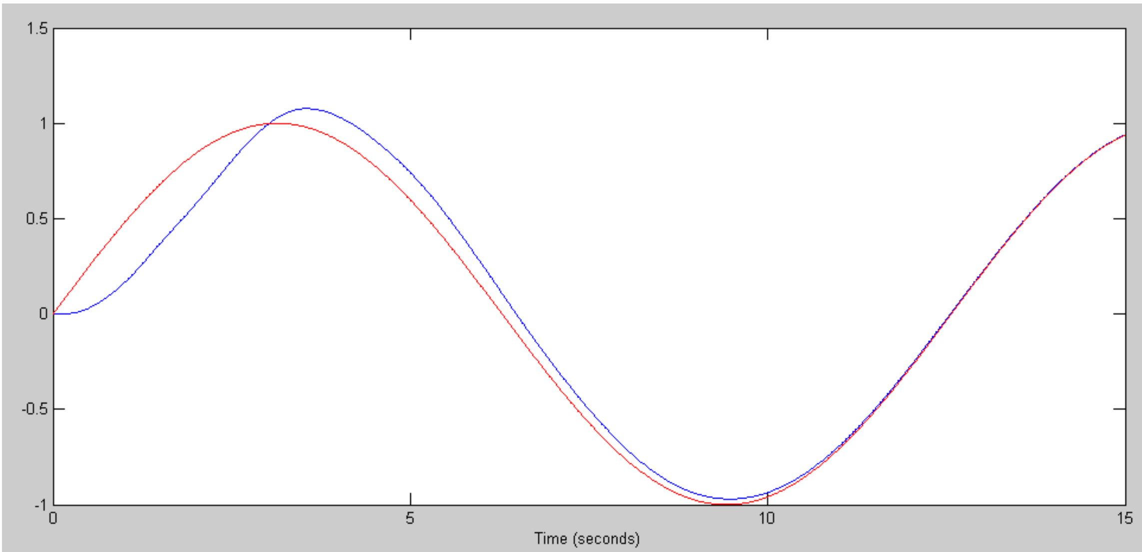
- All states are measured (no observer is needed)
- A sinusoidal set point (R(t) = sin(0.5t)), and
- A constant disturbance (d(t) = 5)

Validate your feedback control law on the linear system

Validate your feedback control law on the nonlinear system (Gantry2)

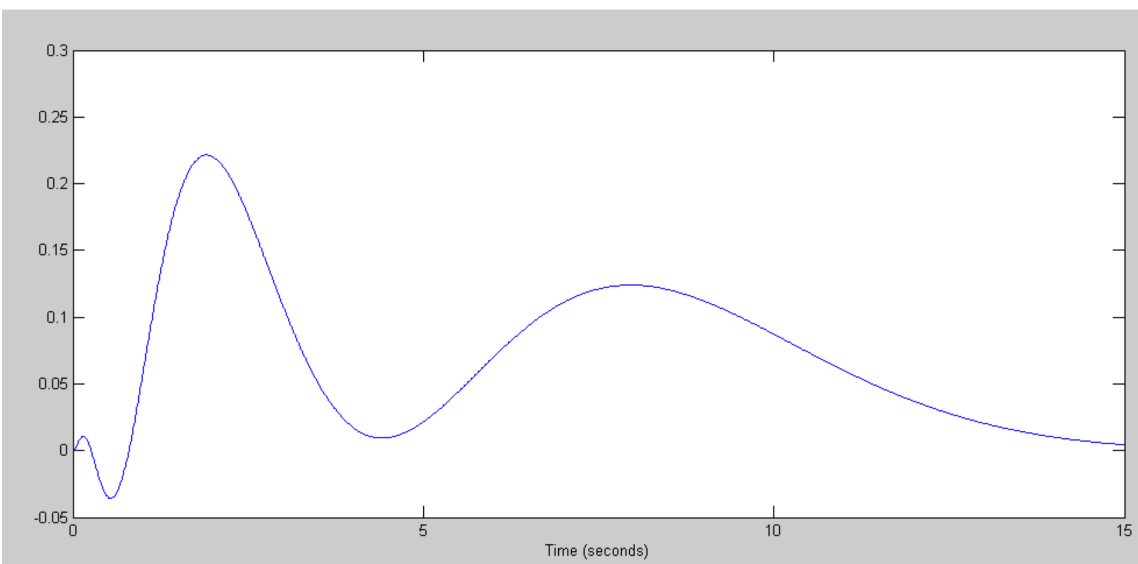
```
g = 9.8;
A = [0,0,0,1,0,0;0,0,0,0,1,0;0,0,0,0,0,1];
A = [A;0,-2*g,0,0,0,0;0,3*g,g,0,0,0;0,-3*g,3*g,0,0,0];
A

         0         0         0    1.0000         0         0
         0         0         0         0    1.0000         0
         0         0         0         0         0    1.0000
         0   19.6000         0         0         0         0
         0  -29.4000    9.8000         0         0         0
         0   29.4000  -29.4000         0         0         0

eig(A)
        0
        0
   0.0000 + 6.8099i
   0.0000 - 6.8099i
   0.0000 + 3.5250i
   0.0000 - 3.5250i

B = [0;0;0;1;-1;1];
Az = [0,0,0;0,0,0.5;0,-0.5,0]

         0         0         0
         0         0    0.5000
         0   -0.5000         0

Bz = [1;1;1];
C = [1,0,0,0,0,0];
D = 0;
A9 = [A, zeros(6,3) ; Bz*C, Az]

         0         0         0    1.0000         0         0         0         0         0
         0         0         0         0    1.0000         0         0         0         0
         0         0         0         0         0    1.0000         0         0         0
         0   19.6000         0         0         0         0         0         0         0
         0  -29.4000    9.8000         0         0         0         0         0         0
         0   29.4000  -29.4000         0         0         0         0         0         0
    1.0000         0         0         0         0         0         0         0         0
    1.0000         0         0         0         0         0         0         0    0.5000
    1.0000         0         0         0         0         0         0   -0.5000         0

B9u = [B;0*Bz];
B9r = [0*B;-Bz];
C9 = [1,0,0,0,0,0,0,0,0];
D9 = 0;

>> K9 = ppl(A9, B9u, [-0.5,-0.6,-0.7,-0.5+j*0.5,-0.5-j*0.5,-0.5+j*3.5,-0.5-j*3.5, -0.5+j*7,-0.5-j*7])

K9 =

   10.5028    0.0263    2.1935    9.4793    6.6959    2.0166    1.3461    1.5339    1.9423
```

```
t = [0:0.01:15]';
X0 = zeros(9,1);
R = sin(0.5*t);
d = 0*t + 5;

y = step3(A9-B9u*K9,[B9r,B9u],C9,[0,0],t,X0,[R,0*d]);
plot(t,y)
plot(t,y,t,R)
```



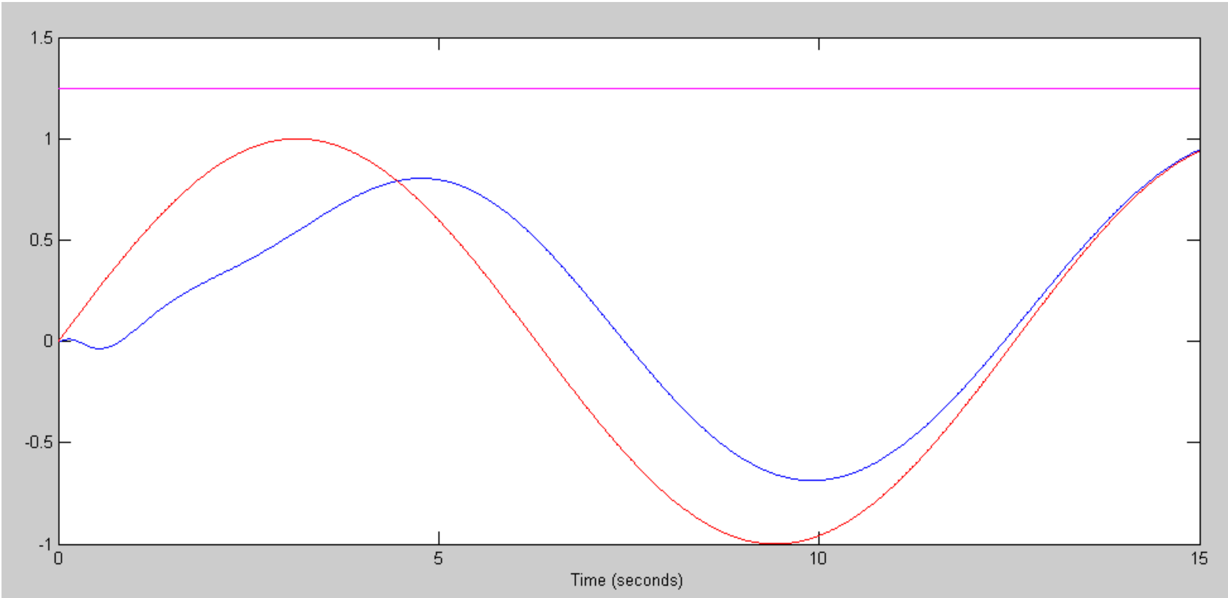
Step response with respect to R = sin(0.5t)

```
>> y = step3(A9-B9u*K9,[B9r,B9u],C9,[0,0],t,X0,[0*R,d]);
>> plot(t,y,'b')
>> xlabel('Time (seconds)');
```

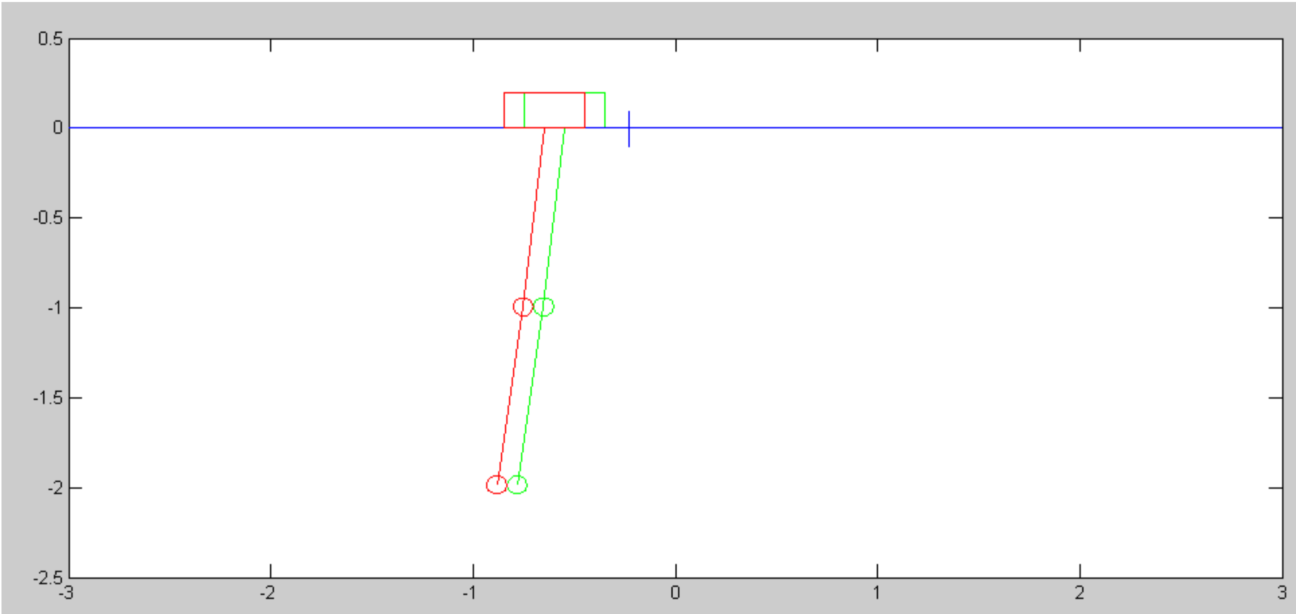


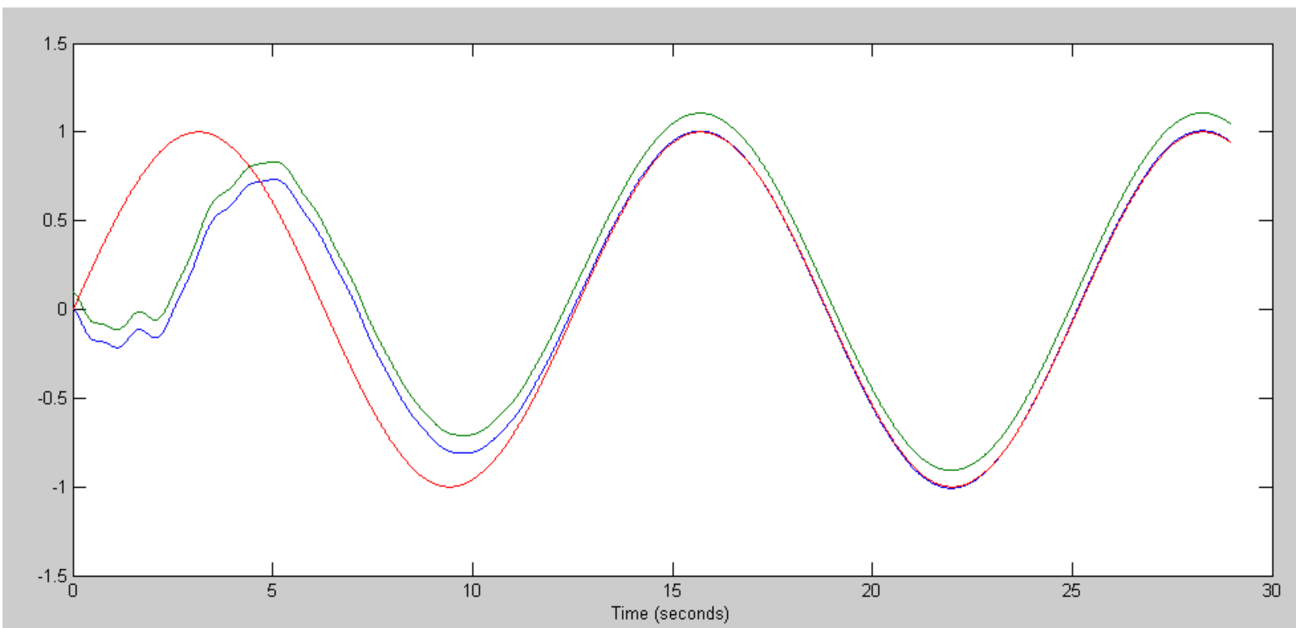
Response to d = 5

```
>> y = step3(A9-B9u*K9,[B9r,B9u],C9,[0,0],t,X0,[R,d]);
>> plot(t,y,'b',t,d,'r')
>> xlabel('Time (seconds)');
```



Response of linear system to R = sin(0.5t), d = 5





Nonlinear Simulation Result

```
% Double Pendulum system - main routine
% ECE 463 Test #2
% cart = 1kg
% m1 = m2 = 1kg
% L1 = L2 = 1m
% X = [x, q1, q2, dx, dq1, dq2]

X = [0, 0, 0, 0, 0, 0]';
Xe = X;

Az = [0,0,0;0,0,0.5;0,-0.5,0];
Bz = [1;1;1];
Z = [1;1;1];

Kx = [10.5028    0.0263    2.1935    9.4793    6.6959    2.0166];
Kz = [1.3461    1.5339    1.9423];

dt = 0.01;
U = 0;
t = 0;
n = 0;

y = [];

while(t < 29)
 Ref = sin(0.5*t);

 U = -Kz*Z - Kx*X;
 dX = Gantry2Dynamics(X, U);
 dZ = Az*Z + Bz*(X(1) - Ref);

 X = X + dX * dt;
 Xe = X + [0.1,0,0,0,0,0]';
 Z = Z + dZ * dt;

 t = t + dt;
 n = mod(n+1,5);
 if(n == 0)
    Gantry2Display(X, Xe, Ref);
    plot([Ref, Ref],[-0.1,0.1],'b');
 end
 y = [y ; X(1), Xe(1), Ref];

end

hold off
t = [1:length(y)]' * dt;
plot(t,y)
xlabel('Time (seconds)');
```

B Level (max 90 points)

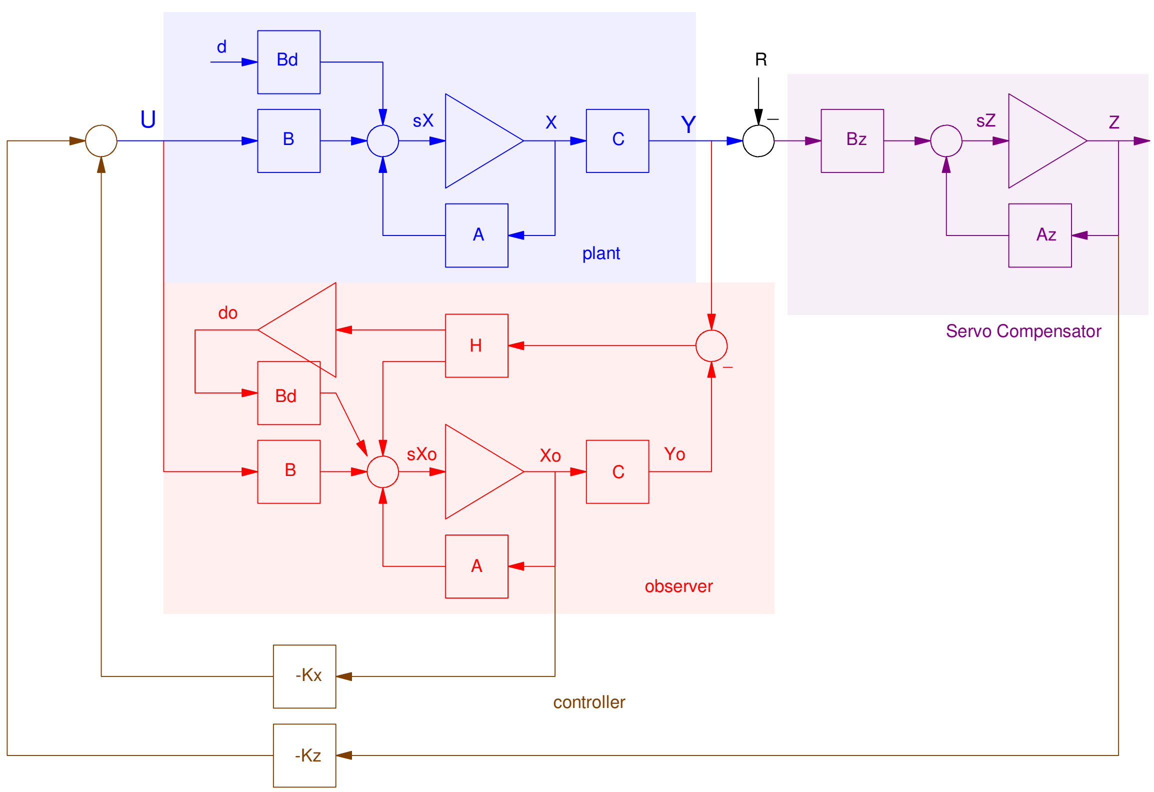
Design a feedback control law for the double pendulum assuming

- Only position and angles are measured, (observer is required)
- A sinusoidal set point (R(t) = sin(0.5t)), and
- No disturbance (d(t) = 0)

Step 1: Design a full-order observer

```
>> g = 9.8;
Ae = [0,0,0,1,0,0;0,0,0,0,1,0;0,0,0,0,0,1];
Ae = [Ae;0,2*g,0,0,0,0;0,-3*g,g,0,0,0;0,3*g,-3*g,0,0,0];
Be = [0;0;0;1;-1;1];
Xe = X + 0.1*randn(6,1);
Ce = [1,0,0,0,0,0];
H = ppl(Ae', Ce', [-1,-1,-1+j*3,-1-j*3,-1+j*6,-1-j*6])'

H =

    6.0000
   -7.7959
    9.7297
    1.2000
   -1.9286
    4.1120
```

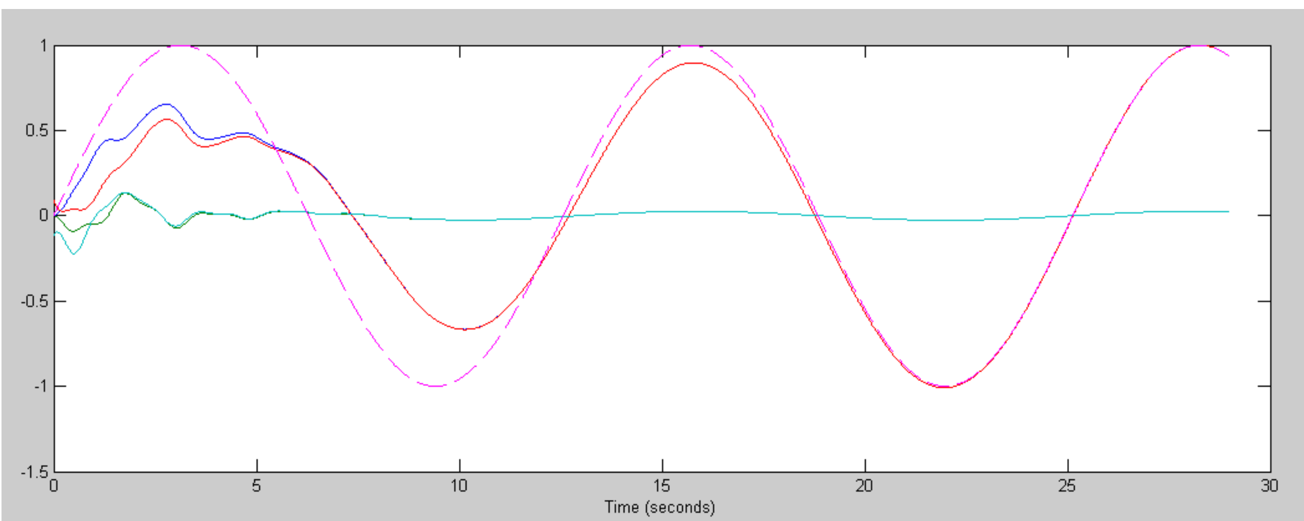
Validate your feedback control law on the linear system



$$s\begin{bmatrix} X \\ X_e \\ Z \end{bmatrix} = \begin{bmatrix} A & -BK_x & -BK_z \\ HC & A-HC-BK_x & -BK_z \\ B_zC & 0 & A_z \end{bmatrix}\begin{bmatrix} X \\ X_e \\ Z \end{bmatrix} + \begin{bmatrix} B \\ B \\ 0 \end{bmatrix} d + \begin{bmatrix} 0 \\ 0 \\ -B_z \end{bmatrix} R$$

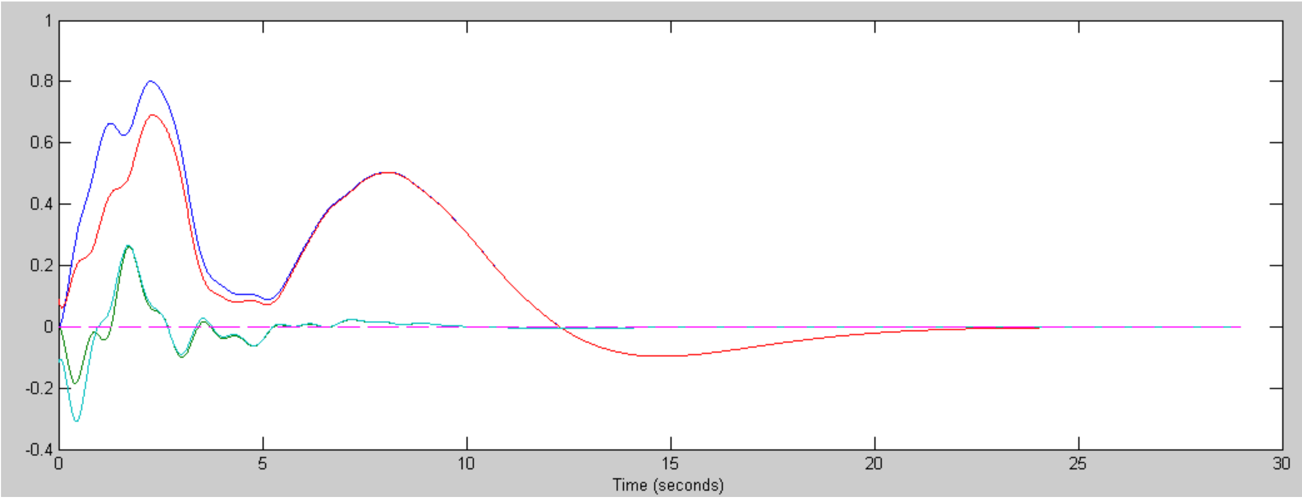
Putting everything together in Matlab

```
>> A15 = [A, -B*Kx,-B*Kz ; H*C, A-H*C-B*Kx,-B*Kz ; Bz*C,zeros(3,6),Az];

>> B15d = [B;B;0*Bz];
>> B15r = [0*B;0*B;-Bz];
>> X0 = zeros(15,1);
>> X0(7:12) = 0.1*randn(6,1);

>> t = [0:0.01:29]';
>> R = sin(0.5*t);
>> d = 0*t + 5;

>> C15 = zeros(4,15);
>> D15 = zeros(4,1);
>> C15(1,1) = 1;          position
>> C15(2,2) = 1;          angle
>> C15(3,7) = 1;          position estimate
>> C15(4,8) = 1;          angle estimate
>> y = step3(A15, B15r, C15, D15, t, X0, R);
>> plot(t,y,t,R,'m--');
>> xlabel('Time (seconds)');
>>
```



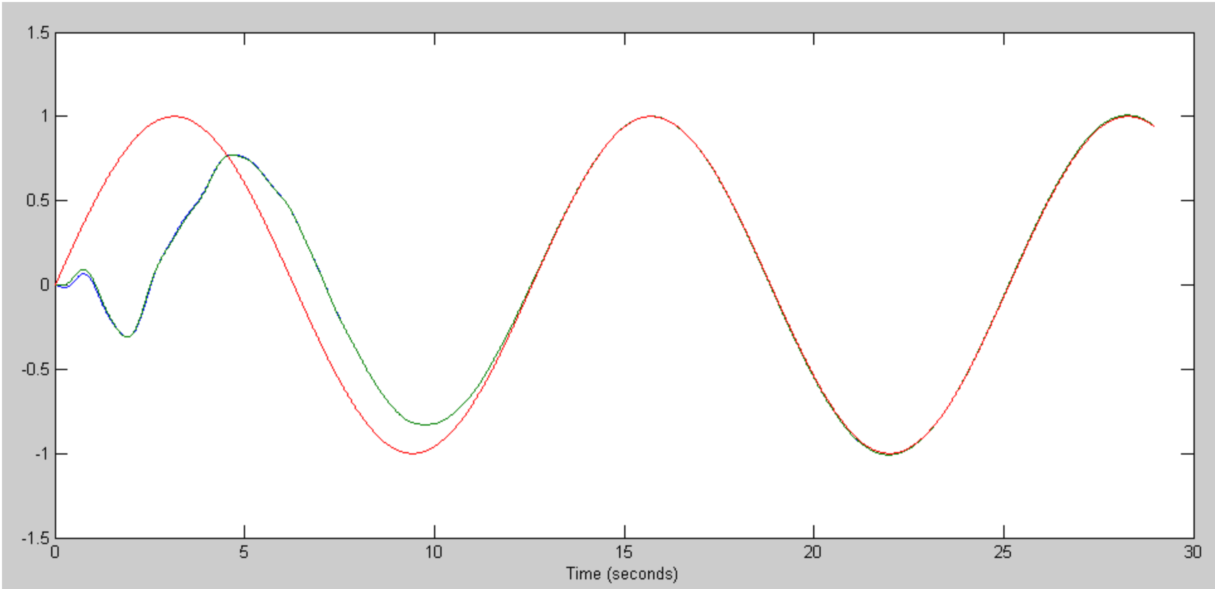
Response to tracking a sinusoidal setpoint (magenta dashed line)

```
>> d = 0*t + 5;
>> y = step3(A15, B15d, C15, D15, t, X0, d);
>> plot(t,y,t,0*R,'m--');
>> xlabel('Time (seconds)');
```

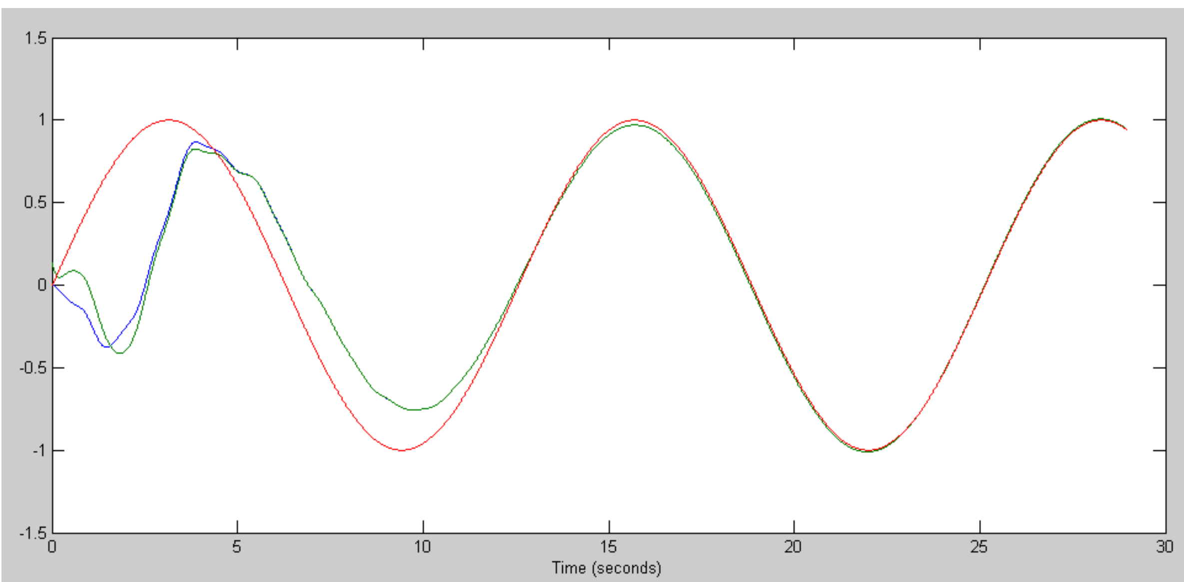


Response to a disturabance (rejects)

Validate your feedback control law on the nonlinear system (Gantry2)



Nonlinear Response with U = -Kz*Z - Kx*X



Respnse with U = -Kz*Z - Kx*Xe

```
% Double Pendulum system - main routine
% ECE 463 Test #2
% cart = 1kg
% m1 = m2 = 1kg
% L1 = L2 = 1m
% X = [x, q1, q2, dx, dq1, dq2]

X = [0, 0.2, 0.2, 0, 0, 0]';
Xe = X;

Az = [0,0,0;0,0,0.5;0,-0.5,0];
Bz = [1;1;1];
Z = [1;1;1];

Kx = [10.5028    0.0263    2.1935    9.4793    6.6959    2.0166];
Kz = [1.3461    1.5339    1.9423];


% Observer
g = 9.8;
Ae = [0,0,0,1,0,0;0,0,0,0,1,0;0,0,0,0,0,1];
Ae = [Ae;0,2*g,0,0,0,0;0,-3*g,g,0,0,0;0,3*g,-3*g,0,0,0];
Be = [0;0;0;1;-1;1];
Xe = X + 0.1*randn(6,1);
Ce = [1,0,0,0,0,0];
H = ppl(Ae', Ce', [-1,-1,-1+j*3,-1-j*3,-1+j*6,-1-j*6])'

dt = 0.01;
U = 0;
t = 0;
n = 0;

y = [];

while(t < 29)
 Ref = sin(0.5*t);

 U = -Kz*Z - Kx*Xe;
 dX = Gantry2Dynamics(X, U);
 dXe = Ae*Xe + Be*U + H*(X(1) - Xe(1));
 dZ = Az*Z + Bz*(X(1) - Ref);

 X = X + dX * dt;
 Xe = Xe + dXe * dt;
 Z = Z + dZ * dt;

 t = t + dt;
 n = mod(n+1,5);
 if(n == 0)
    Gantry2Display(X, Xe, Ref);
    plot([Ref, Ref],[-0.1,0.1],'b');
 end
 y = [y ; X(1), Xe(1), Ref];

end

hold off
t = [1:length(y)]' * dt;
plot(t,y)
xlabel('Time (seconds)');
```

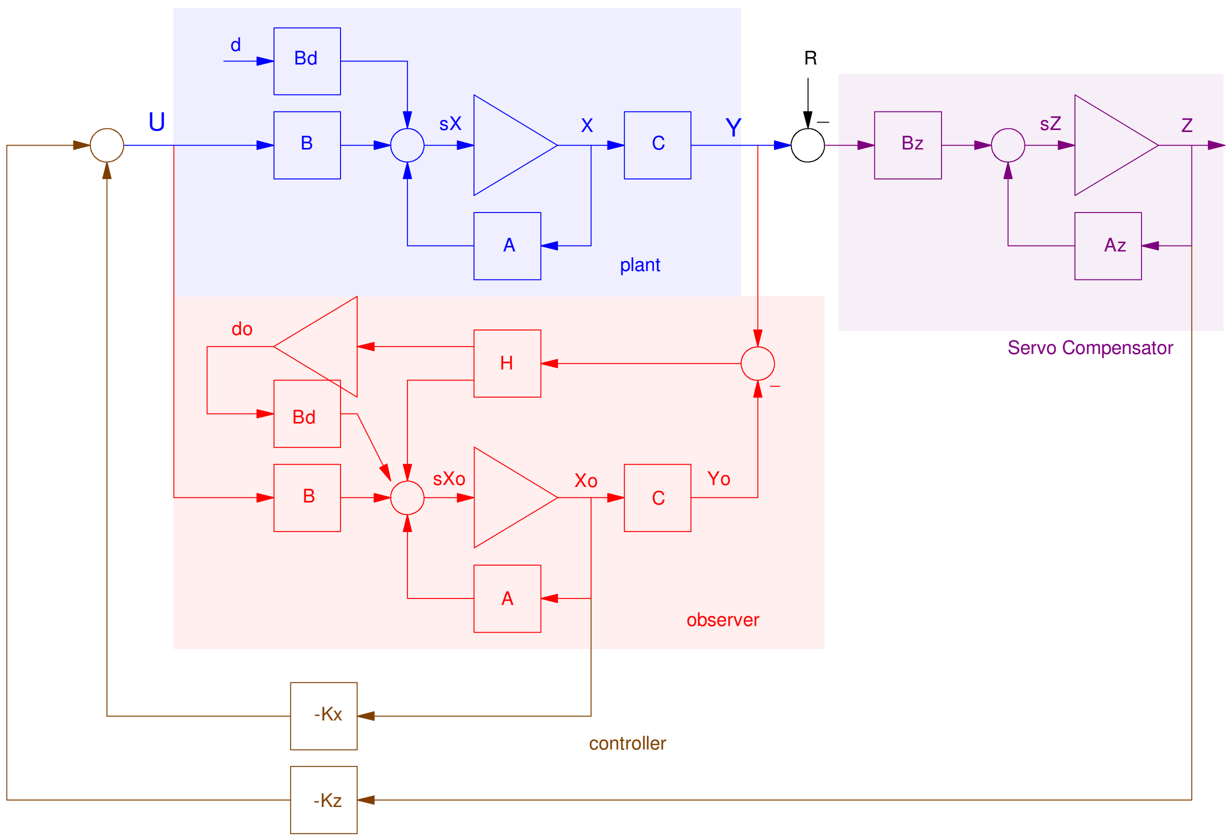
A Level (max 100 points)

Design a feedback control law for the double pendulum assuming

- Only position and angles are measured, (observer is requried)
- A sinusoidal set point (R(t) = sin(0.5t)), and
- A constant disturbance (d(t) = 5)

Validate your feedback control law on the linear system

Validate your feedback control law on the nonlinear system (Gantry2)



Block diagram for the Plant, Servo Compensator, Disturbance, Observer, and Full-State Feedback