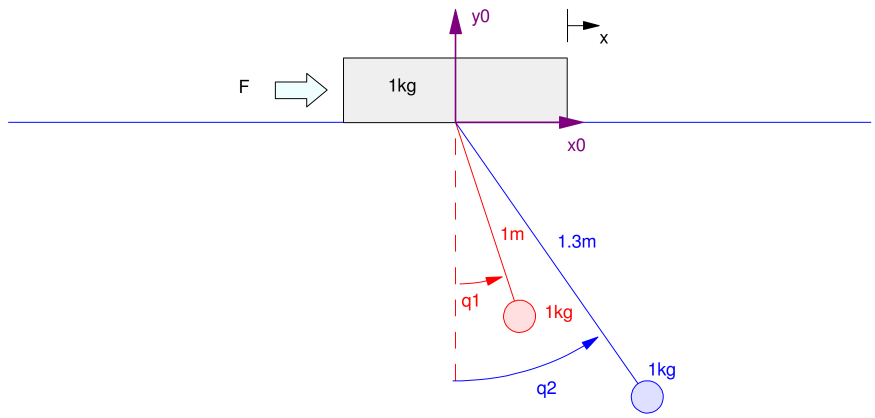
ECE 463/663 - Test #3: Name ________________

Due midnight Sunday, May 8th. Individual Effort Only (no working in groups)



The linearized dynamics for a cart with two pendulums are:

$$s\begin{bmatrix} x \\ \theta_1 \\ \theta_2 \\ \dot{x} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & g & g & 0 & 0 & 0 \\ 0 & -2g & -g & 0 & 0 & 0 \\ 0 & -0.7692g & -1.5385g & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta_1 \\ \theta_2 \\ \dot{x} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ -0.7692 \end{bmatrix}(F+d)$$

Design a feedback control law using LQR or LQG/LTR or VSS techniques (your pick) which results in

- A 2% settling time between 6 to 12 seconds
- Less than 10% overshoot for a step input, and
- An ability to track a constant set point

Turn in for your exam

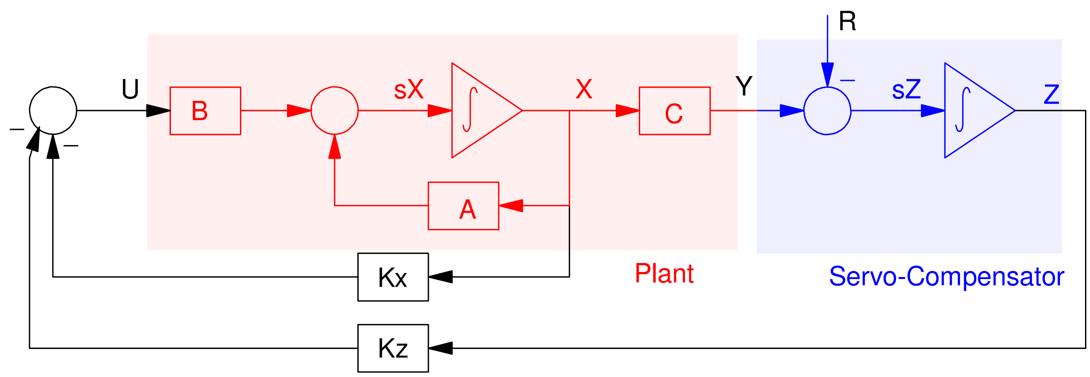
- A block diagram of your plant and controller
- Matlab code used to determine your control law,
- The resulting control law
- A step response with respect Ref = 1, d = 1 for the linear model (above),
- A step response for the nonlinear simulation (Cart2 / Cart2Display / Cart2Dynamics) with your control law, and
- The main calling routine (Cart2.m) you used to generate this step response.

C Level (max 80 points)

Assume

- No noise
- All states are measured
- A constant set point, and
- A constant disturbance (d = 1)

Use a servo-compensator so that you can track a constant set-point



The augmented system is then

$$s\begin{bmatrix} X \\ Z \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}\begin{bmatrix} X \\ Z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} U + \begin{bmatrix} 0 \\ -1 \end{bmatrix} R$$

Find Kx and Kz using LQR techniques

```
>> Z = zeros(3,3);
>> I = eye(3,3);
>> g = 9.8;
>> K = [0,1,1;0,-2,-1;0,-0.7692,-1.5385]*g;
>> A = [Z,I;K,Z];
>> eig(A)

         0
         0
   0.0000 + 5.1211i
   0.0000 - 5.1211i
  -0.0000 + 2.9071i
  -0.0000 - 2.9071i
```

all poles are on the jw axis: gravity is in the correct direction

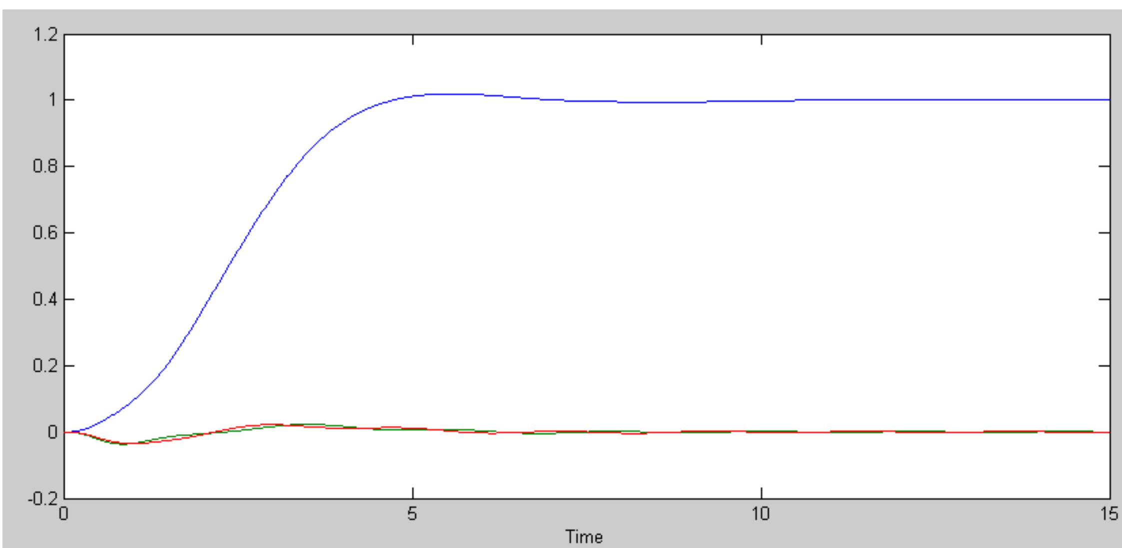
```
>> B = [0;0;0;1;-1;-0.7692];
>> Cx = [1,0,0,0,0,0];
```

now form the augemented system

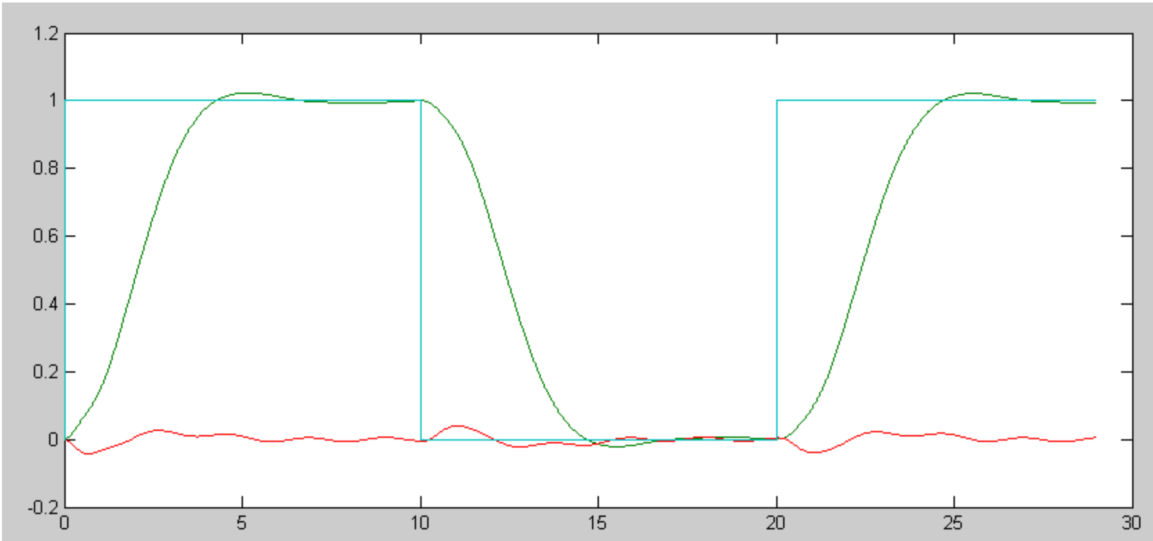
```
>> A7 = [A, zeros(6,1) ; Cx, 0]

         0         0         0    1.0000         0         0         0
         0         0         0         0    1.0000         0         0
         0         0         0         0         0    1.0000         0
         0    9.8000    9.8000         0         0         0         0
         0  -19.6000   -9.8000         0         0         0         0
         0   -7.5382  -15.0773         0         0         0         0
    1.0000         0         0         0         0         0         0

>> B7 = [B;0];
>> B7r = [zeros(6,1);-1];

>> C7 = [1,0,0,0,0,0,0;0,1,0,0,0,0,0;0,0,1,0,0,0,0];
>> D7 = [0;0;0];

>> t = [0:0.01:15]';
```

eventually after some trial and error on Q

```
>> K7 = lqr(A7, B7, diag([10,0,0,0,10,10,10]),1);
>> y = step3(A7-B7*K7, B7r, C7, D7, t, X0, R);
>> plot(t,y);
>> K7

K7 =    7.5004   -2.5600   -2.7131    7.3137    2.0521   -1.4036    3.1623
```



Checking with the nonlinear system:



Code:

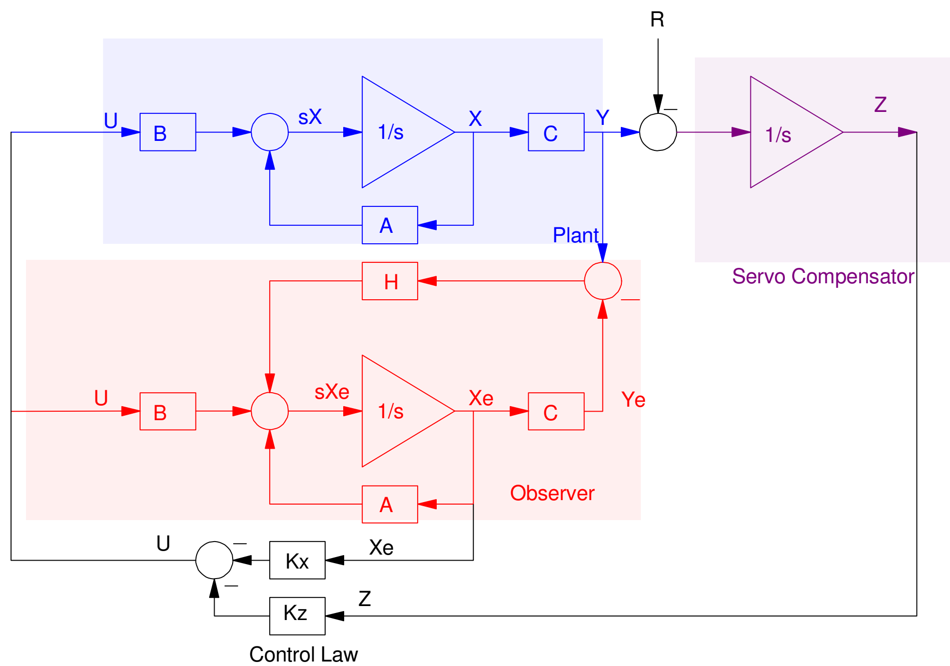
```
% ECE 463/663 Final Exam
% Cart with two pendulums

Ref = 1;
dt = 0.01;
t = 0;
n = 0;
y = [];
X = [0 0 0 0 0 0]';
Z = 0;

Kx = [7.5004   -2.5600   -2.7131    7.3137    2.0521   -1.4036];
Kz = 3.1623;

while(t < 29)
 Ref = 1*(sin(0.314*t) > 0);
 U = -Kz*Z - Kx*X;

 dX = Cart2Dynamics(X, U + 1);
 dZ = X(1) - Ref;

 X = X + dX * dt;
 Z = Z + dZ * dt;

 t = t + dt;
 n = mod(n+1, 5);
 if(n == 0)
    Cart2Display(X, Ref);
 end
 y = [y ; X(1), X(1), X(3), Ref];
end

hold off;
t = [1:length(y)]' * dt;
plot(t,y);
```

B Level (max 90 points)

Assume

- No noise
- Only positions and angles are measured $\{x, \theta_1, \theta_2\}$
- A constant set point, and
- No disturbance (d = 0)

Add a full-order observer



Use LQR techniques to find H

```
>> C = [1,0,0,0,0,0;0,1,0,0,0,0;0,0,1,0,0,0]

     1     0     0     0     0     0
     0     1     0     0     0     0
     0     0     1     0     0     0

>> D = zeros(3,1)

     0
     0
     0

>> H = lqr(A', C', diag([10,10,10,10,10,10]),diag([1,1,1]))'

    4.7503   -0.4871   -0.3095
   -0.4871    3.2428   -0.2064
   -0.3095   -0.2064    3.0833
    6.4494    0.3264    1.3896
   -4.1559    0.3980   -1.2554
   -3.7136    0.1005   -0.1773
```

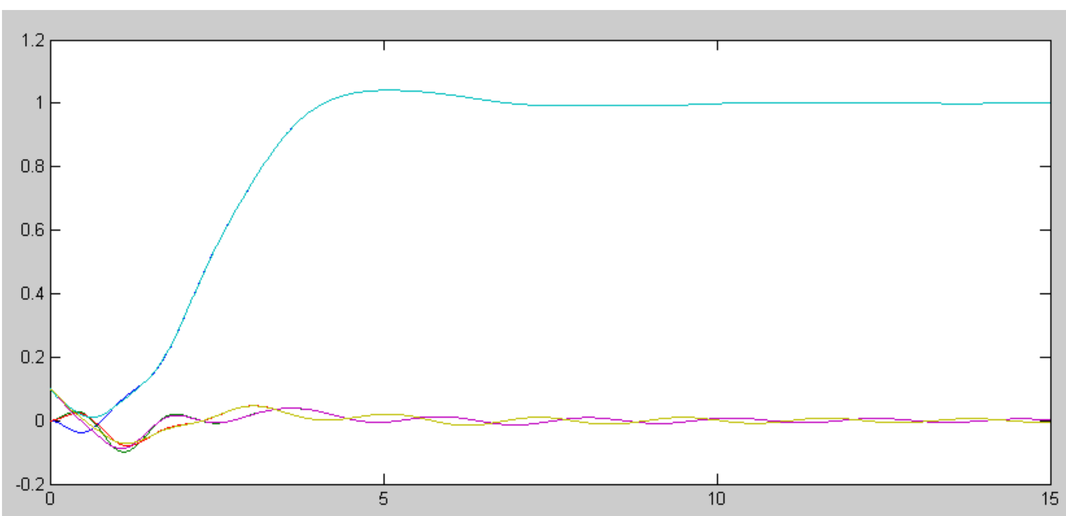
```
>> eig(A - H*C)

  -1.7223 + 4.9982i
  -1.7223 - 4.9982i
  -3.2382
  -1.0459
  -1.6739 + 2.5006i
  -1.6739 - 2.5006i

>>
```

Linear System

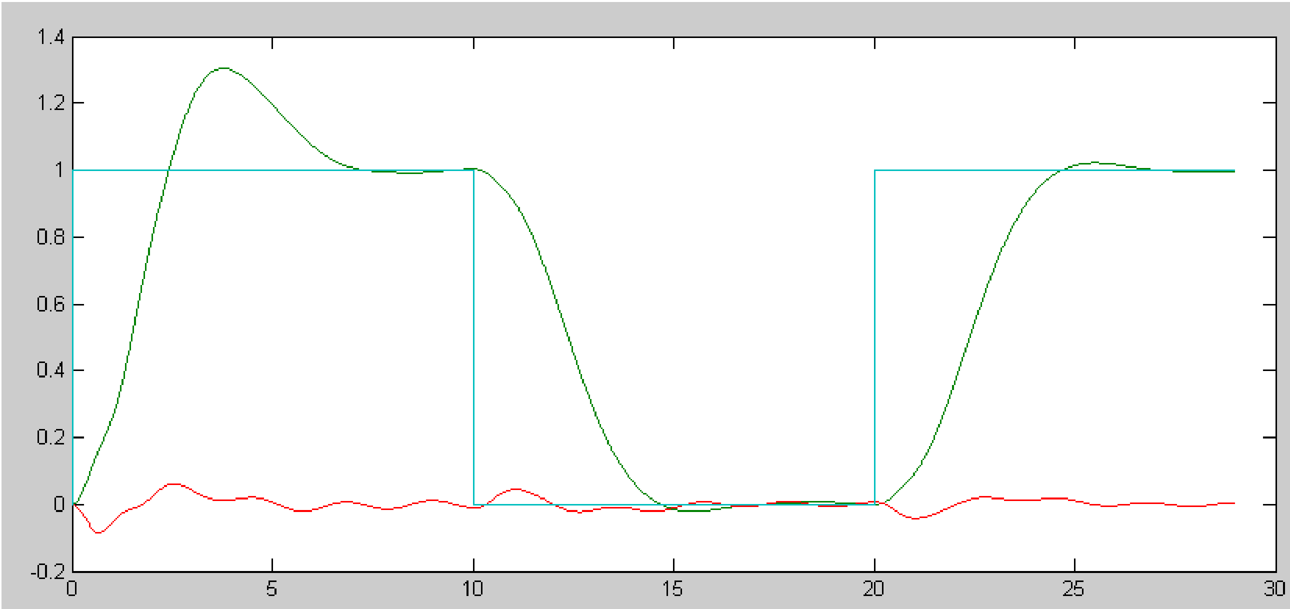
$$s\begin{bmatrix} X \\ Z \\ X_e \end{bmatrix} = \begin{bmatrix} A & -BK_z & -BK_x \\ C & 0 & 0 \\ HC & -BK_z & A-HC-BK_x \end{bmatrix}\begin{bmatrix} X \\ Z \\ X_e \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} R:$$

Checking the step response of the linear system

```
>> Kz = K7(7);
>> Kx = K7(1:6);
>> A13 = [A, -B*Kz, -B*Kx ; Cx, 0, zeros(1,6) ; H*C, -B*Kz, A-H*C-B*Kx];
>> B13r = [0*B; -1; 0*B];
>> C13 = [C,zeros(3,1),0*C ; 0*C, zeros(3,1), C];
>> D13  = zeros(6,1);
>> X0 = [0;0;0;0;0;0; 0; 0.1;0.1;0.1;0.1;0.1;0.1];
>> y = step3(A13, B13r, C13, D13, t, X0, R);
>> plot(t,y)
```



Checking the nonlinear system



Code:

```
% ECE 463/663 Final Exam
% Cart with two pendulums

Ref = 1;
dt = 0.01;
t = 0;
n = 0;
y = [];
X = [0 0 0 0 0 0]';
Z = 0;

Kx = [7.5004   -2.5600   -2.7131    7.3137    2.0521   -1.4036];
Kz = 3.1623;

H = [
    4.7503   -0.4871   -0.3095
   -0.4871    3.2428   -0.2064
   -0.3095   -0.2064    3.0833
    6.4494    0.3264    1.3896
   -4.1559    0.3980   -1.2554
   -3.7136    0.1005   -0.1773 ];

Ae = A;
Be = B;
Ce = C;
Xe = X;

while(t < 29)
 Ref = 1*(sin(0.314*t) > 0);
 U = -Kz*Z - Kx*Xe;

 dX = Cart2Dynamics(X, U + 1);
 dXe = Ae*Xe + Be*U - H*(C*Xe - C*X);
  dZ = X(1) - Ref;

 X = X + dX * dt;
 Z = Z + dZ * dt;
```

```
    Xe = Xe + dXe * dt;

    t = t + dt;
    n = mod(n+1, 5);
    if(n == 0)
       Cart2Display(X, Ref);
    end
    y = [y ; X(1), X(1), X(3), Ref];
  end

  hold off;
  t = [1:length(y)]' * dt;
  plot(t,y);
```

A Level (max 100 points)

Assume

- No noise
- Only positions and angles are measured $\{x, \theta_1, \theta_2\}$
- A constant set point, and
- An input disturbance (d = 1)

Bonus!

Derive the dynamics for this system