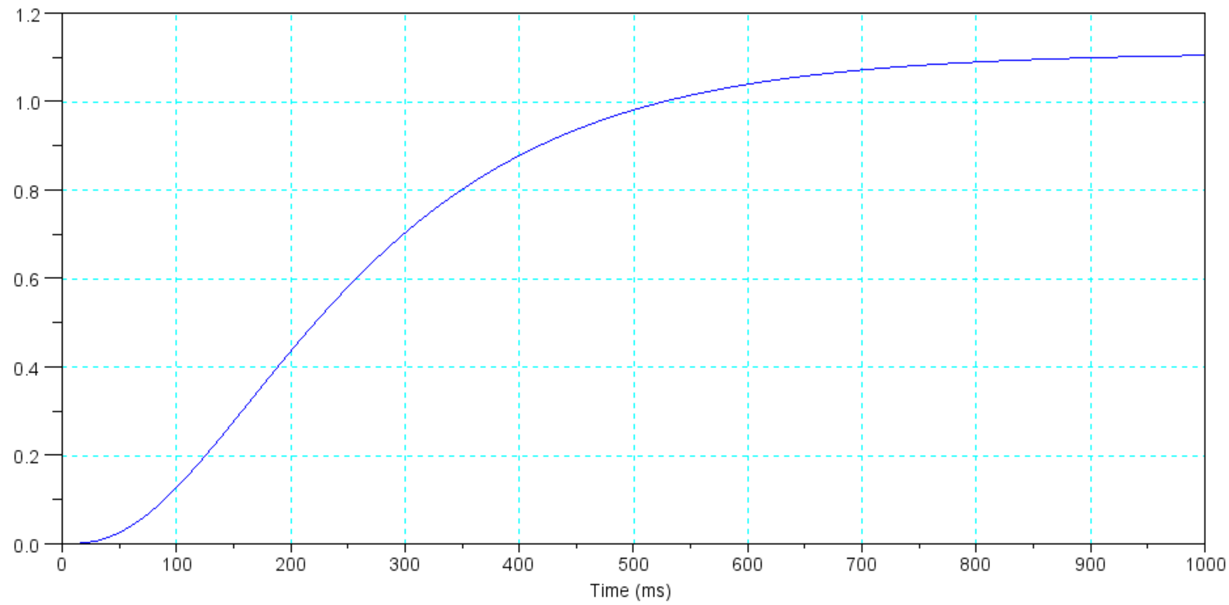
ECE 463/663 - Homework #1

LaPlace Transforms and Dominant Poles. Due Wednesday, Jan 19th

Please make the subject "ECE 463/663 HW#1" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

1) Name That System! Give the transfer function for a system with the following step response.



This is a 1st-order system (no oscillation), so

$$G(s) \approx \left(\frac{a}{s+b}\right)$$

DC gain is 1.1

$$\left(\frac{a}{b}\right) = 1.1$$

The 2% settling time is abut 700ms

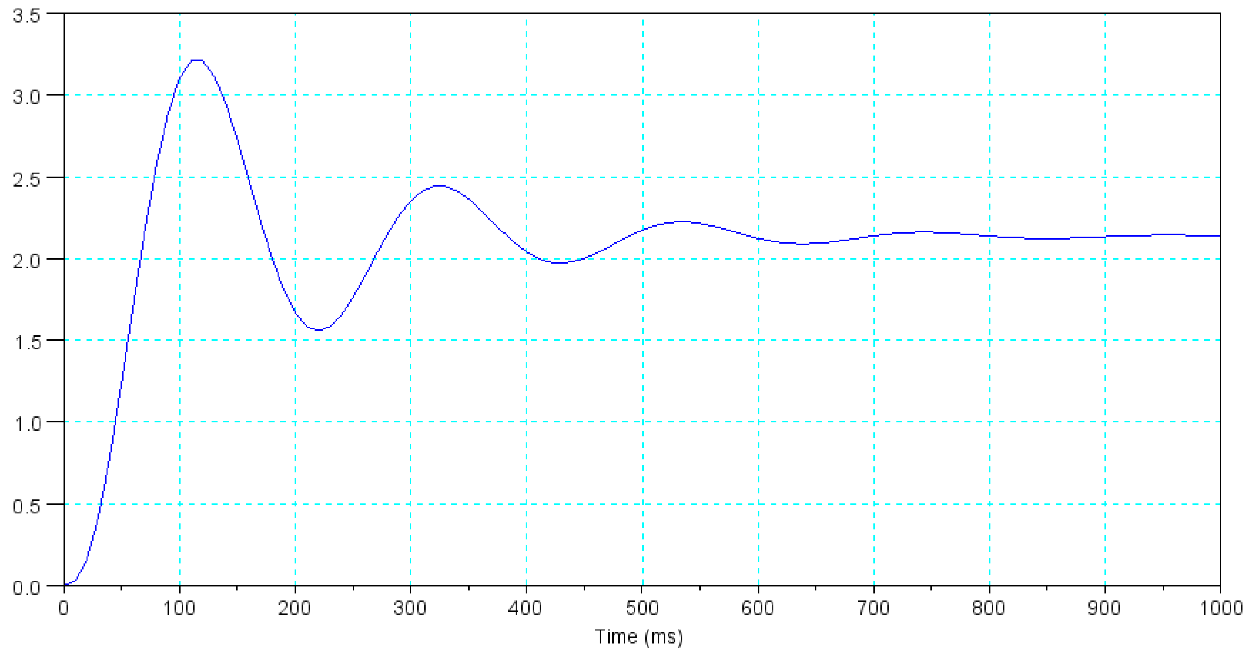
$$\left(\frac{4}{b}\right) \approx 700ms$$

$$b \approx 5.71$$

meaning

$$\boxed{G(s) \approx \left(\frac{6.28}{s+5.71}\right)}$$

2) Name That System! Give the transfer function for a system with the following step response.



This is a 2nd-order system (it oscillates, meaning a complex pole along with its complex conjugate)

$$G(s) \approx \left(\frac{k}{(s+\sigma+j\omega_d)(s+\sigma-j\omega_d)}\right)$$

The frequency of oscillationis about

$$\omega_d \approx \left(\frac{2 \text{ cycles}}{420 \text{ ms}}\right) 2\pi = 29.92 \frac{rad}{\text{sec}}$$

The 2% settling time is about 700ms

$$\sigma = \frac{4}{700ms} = 5.71$$

The DC gain is 2.15. Add a gain in the numerator to make the DC gain 2.15

$$\boxed{G(s) \approx \left(\frac{1994.9}{(s+5.71+j29.92)(s+5.71-j29.92)}\right)}$$

Problem 3 - 6) Assume

$$Y = \left(\frac{500}{(s+2)(s+12)(s+15)}\right)X$$

3) What is the differential equation relating X and Y?

In Matlab (not necessary, but it works...)

```
>> G = zpk([],[-2,-12,-15],500)

         500
-------------------
(s+2) (s+12) (s+15)

>> tf(G)

             500
--------------------------
s^3 + 29 s^2 + 234 s + 360
```

or

$$Y = \left(\frac{500}{s^3+29s^2+234s+360}\right)X$$

Cross multiplying:

$$(s^3 + 29s^2 + 234s + 360)Y = 500X$$

sY means *the dervative of Y*

$$y''' + 29y'' + 234y' + 360y = 500x$$

or

$$\frac{d^3y}{dt^3} + 29\frac{d^2y}{dt^2} + 234\frac{dy}{dt} + 360y = 500x$$

4) Determine y(t) assuming x(t) is a sinusoidal input:

$$x(t) = 4\cos(5t) + \sin(5t)$$

This is a phasor problem. Since x(t) is on for all time, you're at steady-state. If you change the problem to

$$x(t) = \begin{cases} 0 & t < 0 \\ 4\cos(5t) + \sin(t) & t > 0 \end{cases}$$

then it would be a LaPlace Transform problem - which would be a lot harder to solve...

The gain everywhere is

$$Y = \left(\frac{500}{(s+2)(s+12)(s+15)}\right)X$$

At s = j5

$$X = 4 - j$$ *real = cosine, imag = -sine*

$$Y = \left(\frac{500}{(s+2)(s+12)(s+15)}\right)_{s=j5} \cdot (4 - j)$$

$$Y = -1.022 - j1.557$$

meaning

$$y(t) = -1.022\cos(5t) + 1.557\sin(5t)$$

real = cosine

imag = -sine

5) Determine y(t) assuming x(t) is a step input:

$$x(t) = u(t)$$

This is a LaPlace transform problem since x(t) = 0 for t<0

At all frequencies, X and Y are related by

$$Y = \left(\frac{500}{(s+2)(s+12)(s+15)}\right)X$$

Replace X with the LaPlace transform of x(t)

$$Y = \left(\frac{500}{(s+2)(s+12)(s+15)}\right)\left(\frac{1}{s}\right)$$

Use partial fractions

$$Y = \left(\frac{1.389}{s}\right) + \left(\frac{-1.923}{s+2}\right) + \left(\frac{1.389}{s+12}\right) + \left(\frac{-0.855}{s+15}\right)$$

Take the inverse-LaPlace transform

$$y(t) = 1.389 - 1.923e^{-2t} + 1.389e^{-12t} - 0.855e^{-15t} \qquad t > 0$$

6a) Determine a 1st-order approximation for this system

$$Y = \left(\frac{500}{(s+2)(s+12)(s+15)}\right)X$$

Simplifying...

- Keep the dominant pole (s = -2)
- Keep the DC gain (1.389)

$$Y \approx \left(\frac{2.778}{s+2}\right)X$$

6b) Compare the step response of your 1st-order model to the actual 3rd-order system

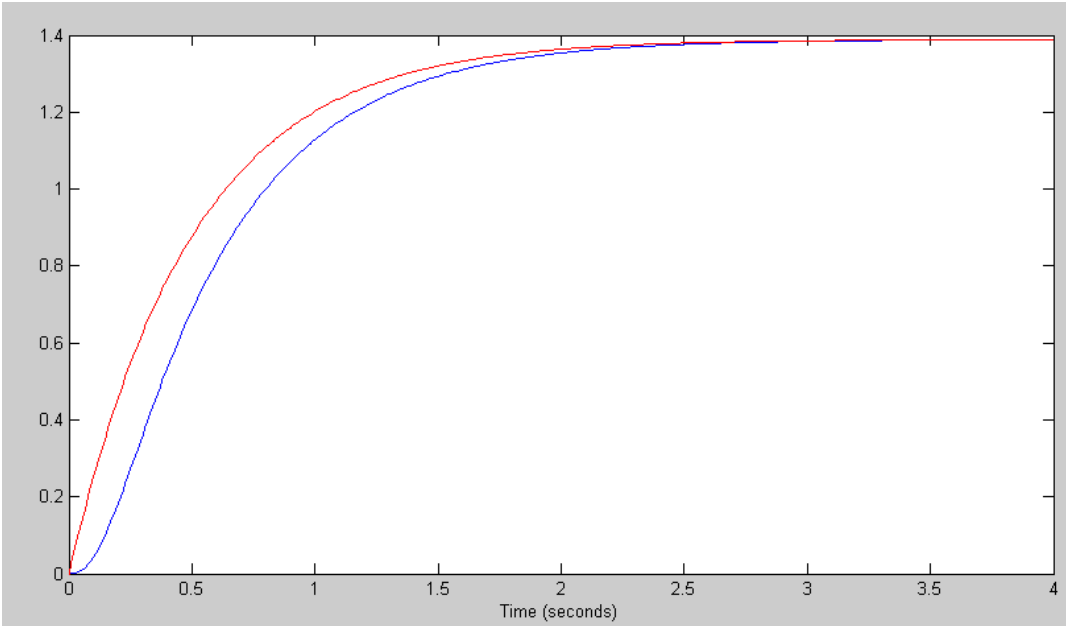
```
>> G3 = zpk([],[-2,-12,-15],500)

        500
--------------------
(s+2) (s+12) (s+15)

>> G1 = zpk([],[-2],2.778)

2.778
-----
(s+2)

>> t = [0:0.01:4]';
>> y1 = step(G1,t);
>> y3 = step(G3,t);
>> plot(t,y3,'b',t,y1,'r');
>> xlabel('Time (seconds)');
```

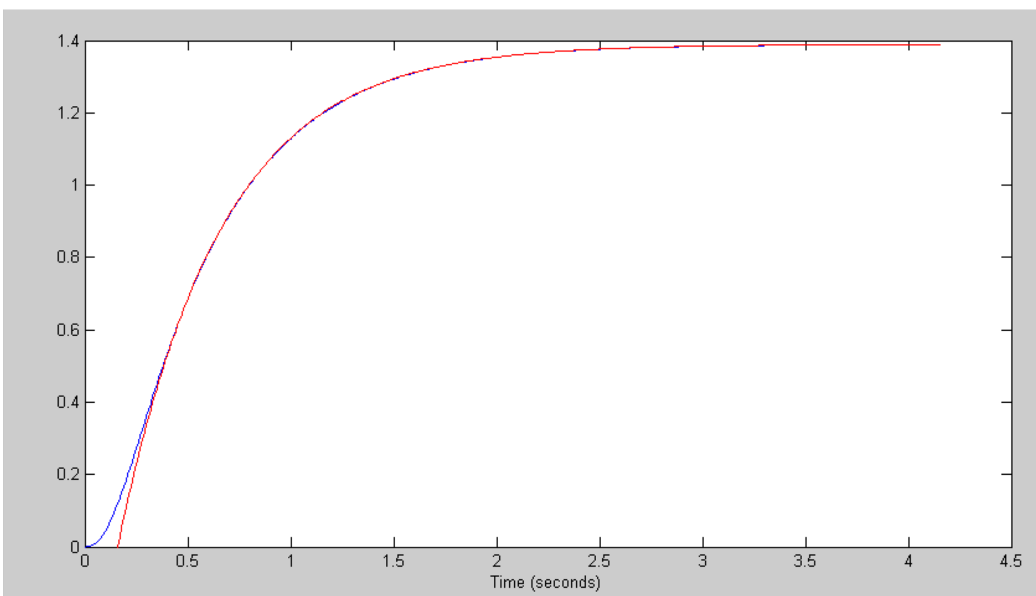


G(s) (blue) & its 1st-Order Approximation (red)

If you add a delay, you get a better model

Ignored dynamics can be approximated with a delay

```
>> plot(t,y3,'b',t+0.16,y1,'r');
```



G(s) (blue) & its 1st-order approximation along with a 160ms delay (red)