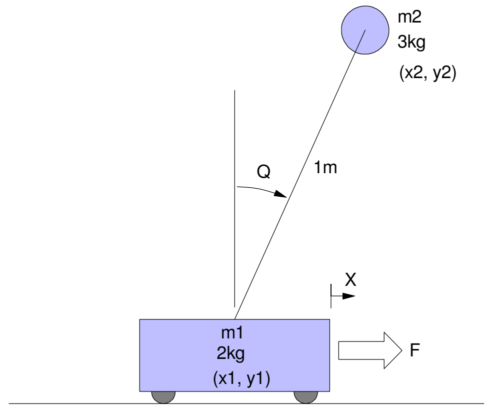
ECE 463/663 - Homework #6

Pole Placement. Due Monday, February 28th

Problem 1) (30pt) Use the dynamics of a Cart and Pendulum System from homework set #4:

$$s\begin{bmatrix} x \\ \theta \\ sx \\ s\theta \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -14.7 & 0 & 0 \\ 0 & 24.5 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta \\ sx \\ s\theta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.5 \\ -0.5 \end{bmatrix}F$$



(10pt) Design a feedback control law of the form

U = Kr * R - Kx * X

so that the closed-loop system has

- A 2% settling time of 12 seconds, and
- 10% overshoot for a step input

Translation: Place the closed-loop dominant pole at

- damping ratio = 0.5902 (10% overshoot)
- s = -0.3333 + j0.4559

In Matlab

```
>> A = [0,0,1,0;0,0,0,1;0,-14.7,0,0;0,24.5,0,0]

         0         0    1.0000         0
         0         0         0    1.0000
         0  -14.7000         0         0
         0   24.5000         0         0

>> B = [0;0;0.5;-0.5]

         0
         0
    0.5000
   -0.5000

>> Kx = ppl(A, B, [-0.3333+j*0.4559,-0.3333-j*0.4559,-2,-3])

Kx =   -0.3905  -68.6944   -1.1417  -12.4749
```

Checking that Kx is correct:

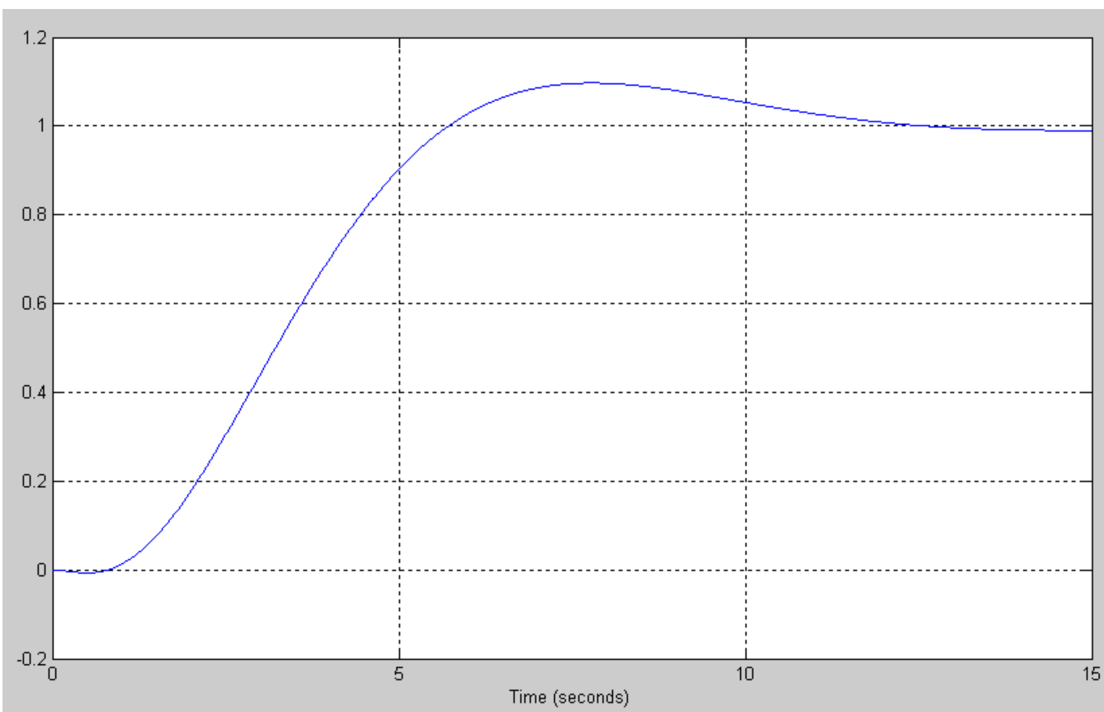
```
>> eig(A - B*Kx)

  -3.0000
  -2.0000
  -0.3333 + 0.4559i
  -0.3333 - 0.4559i
```

Find Kr to make the DC gain 1.000

```
>> C = [1,0,0,0];
>> DC = -C*inv(A-B*Kx)*B

DC =    -2.5606

>> Kr = 1/DC

Kr =    -0.3905
```

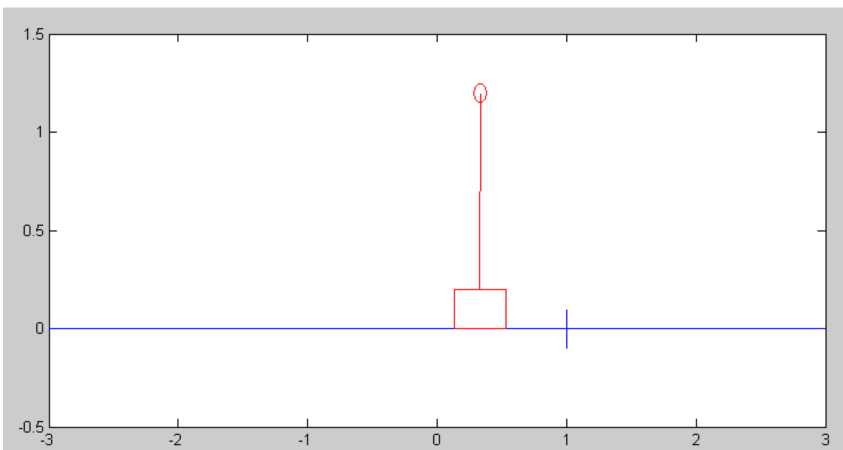
(10pt) Check the step response of the linear system in Matlab

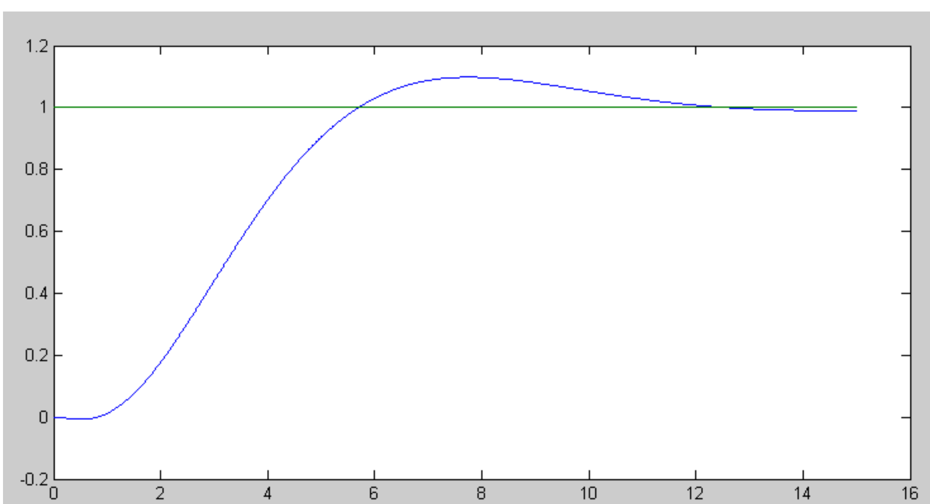
```
>> D = 0;

>> G = ss(A - B*Kx, B*Kr, C, D);

>> t = [0:0.01:15]';
>> y = step(G, t);
>> plot(t,y)
>> xlabel('Time (seconds)')
>> grid on
```



(10pt) Check the step response of the nonlinear system

```
% Cart and Pendulum ( version)
X = zeros(4,1);
dX = zeros(4,1);
Ref = 1;
dt = 0.01;
t = 0;
Kx = [-0.3905  -68.6944   -1.1417  -12.4749];
Kr =  -0.3905;

y = [];
while(t < 15)
   Ref = 1;
   U = Kr*Ref - Kx*X;
   dX = CartDynamics(X, U);

   X = X + dX * dt;
   t = t + dt;

   CartDisplay(X, Ref);
   y = [y ; X(1), Ref];
end

t = [1:length(y)]' * dt;
plot(t,y);
```





Problem 2) (30pt) Use the dynamics for the Ball and Beam system from homework set #4.

$$s\begin{bmatrix} r \\ \theta \\ sr \\ s\theta \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -7 & 0 & 0 \\ -4.2 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} r \\ \theta \\ sr \\ s\theta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.143 \end{bmatrix} T$$

(10pt) Design a feedback control law so that the closed-loop system has

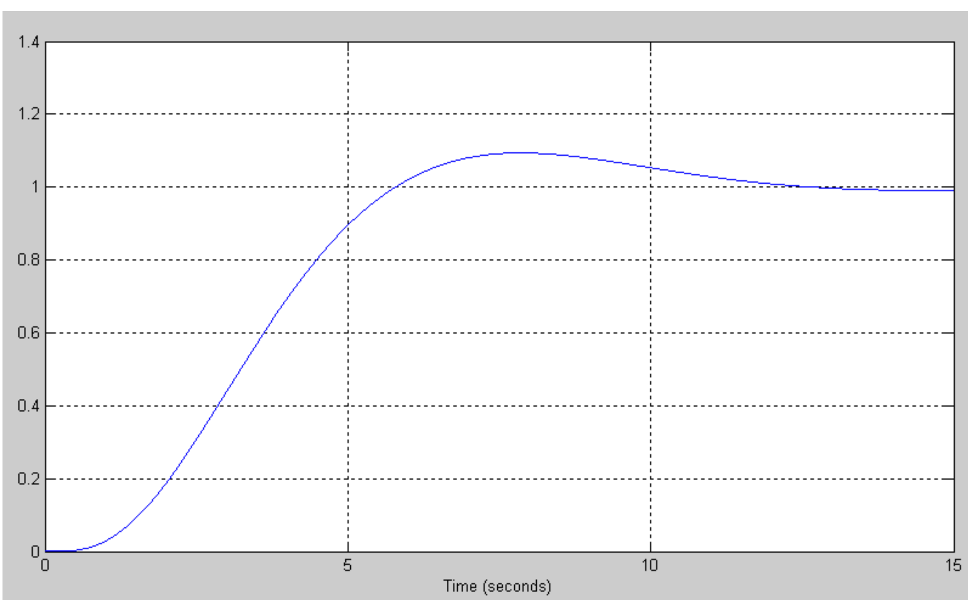
- A 2% settling time of 12 seconds, and
- 10% overshoot for a step input

Following the same procedure:

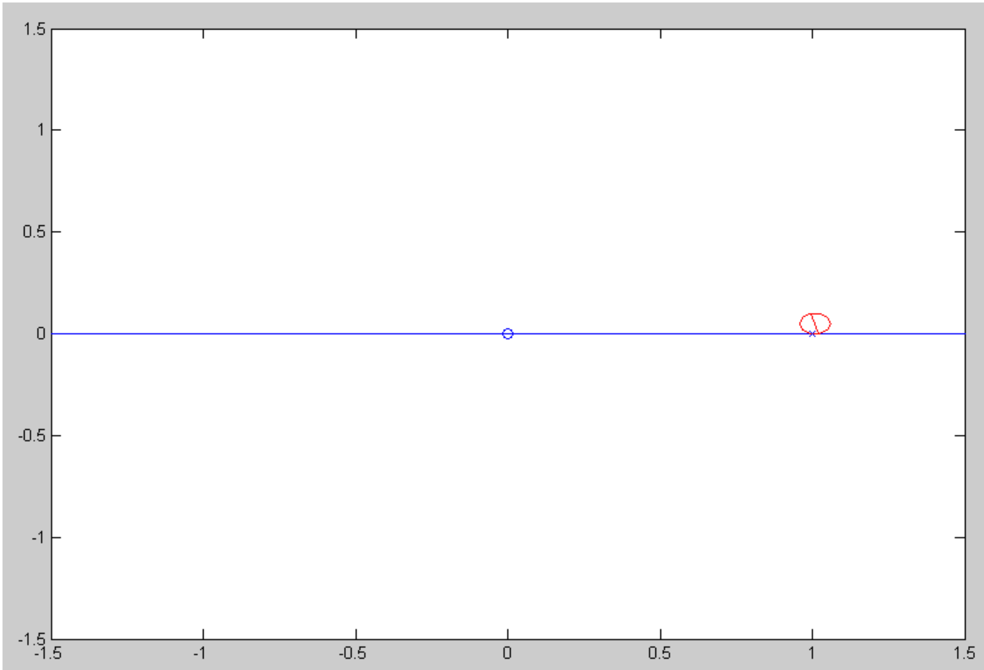
```
>> A = [0,0,1,0;0,0,0,1;0,-7,0,0;-4.2,0,0,0];
>> B = [0;0;0;0.143];
>> Kx = ppl(A, B, [-0.3333+j*0.4559,-0.3333-j*0.4559,-2,-3])

Kx =  -31.2823   67.4960   -5.5887   39.6266

>> C = [1,0,0,0];
>> D = 0;
>> DC = -C*inv(A-B*Kx)*B

DC =   -0.5231

>> Kr = 1/DC

Kr =   -1.9117
```

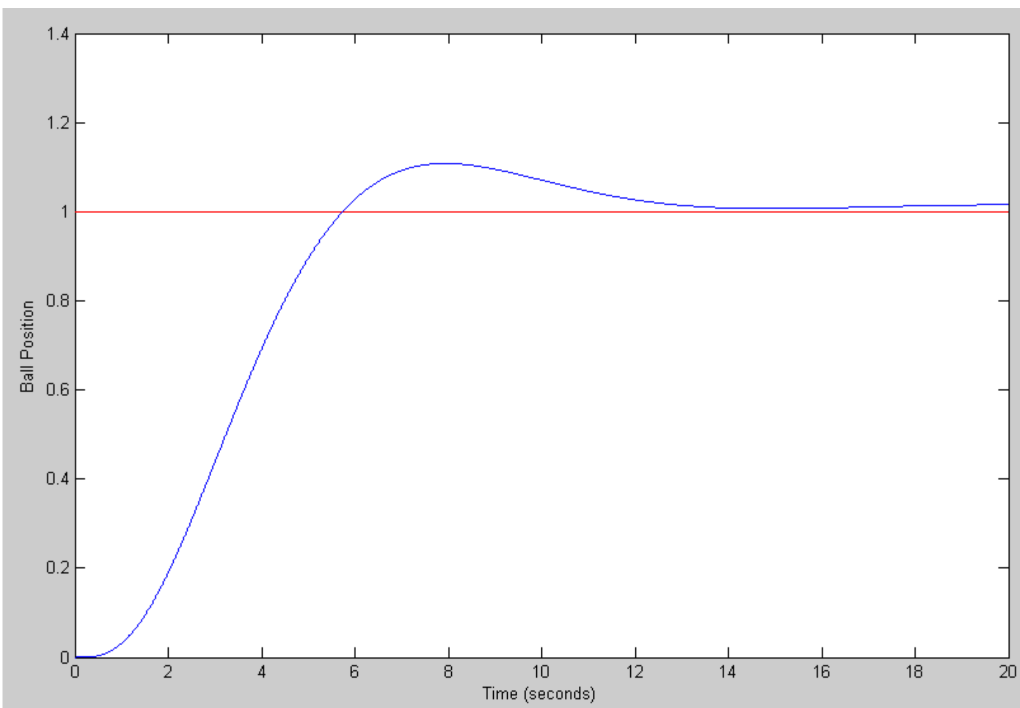
(10pt) Check the step response of the linear system in Matlab

```
>> G = ss(A - B*Kx, B*Kr, C, D);
>> t = [0:0.01:15]';
>> y = step(G, t);
>> plot(t,y)
>> xlabel('Time (seconds)')
```



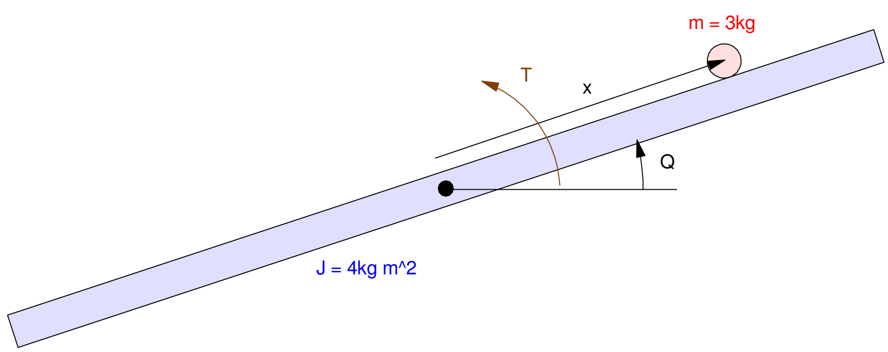
(10pt) Check the step response of the nonlinear system



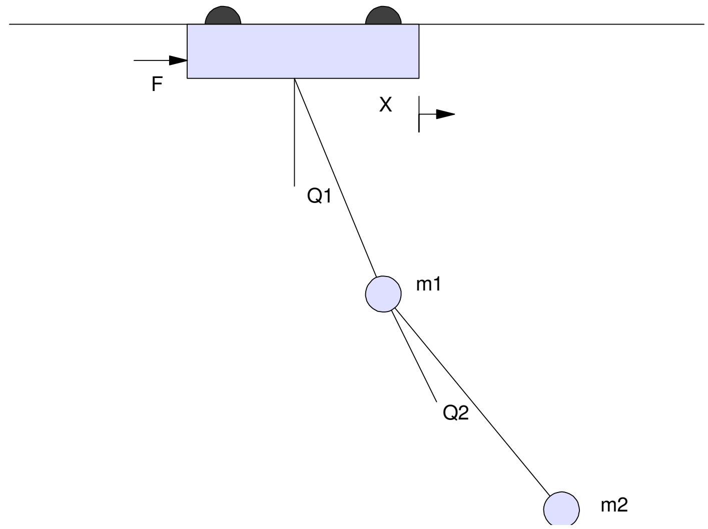


Code:

```
% Ball & Beam System
% m = 1kg
% J = 0.2 kg m^2

X = [0,0,0,0]';
dt = 0.01;
t = 0;
Kx = [ -31.2823   67.4960   -5.5887   39.6266 ];
Kr = -1.9117;
y = [];

while(t < 20)
 Ref = 1;
 U = Kr*Ref - Kx*X;
 dX = BeamDynamics(X, U);

 X = X + dX * dt;
 t = t + dt;

 y = [y ; Ref, X(1)];
 BeamDisplay(X, Ref);
 end

t = [1:length(y)]' * dt;


plot(t,y(:,1),'r',t,y(:,2),'b');
xlabel('Time (seconds)');
ylabel('Ball Position');
```



Problem #3 (30pt): The dynamics of a double gantry (Gantry2) are



$$
s\begin{bmatrix} x \\ \theta_1 \\ \theta_2 \\ \dot{x} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 2g & 0 & 0 & 0 & 0 \\ 0 & -3g & g & 0 & 0 & 0 \\ 0 & 3g & -3g & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta_1 \\ \theta_2 \\ \dot{x} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} F
$$

(10pt) Design a feedback control law of the form

U = Kr * R - Kx * X

so that the closed-loop system has

- A 2% settling time of 12 seconds, and
- 10% overshoot for a step input

```
>> g = 9.8;
>> a11 = zeros(3,3);
>> a12 = eye(3,3);
>> a21 = [0,2,0;0,-3,1;0,3,-3]*g;
>> a22 = zeros(3,3);
>> A = [a11,a12 ; a21,a22]

     0         0         0    1.0000         0         0
     0         0         0         0    1.0000         0
     0         0         0         0         0    1.0000
     0   19.6000         0         0         0         0
     0  -29.4000    9.8000         0         0         0
     0   29.4000  -29.4000         0         0         0
```

```
>> B = [0;0;0;1;-1;1]

     0
     0
     0
     1
    -1
     1

>> Kx = ppl(A, B, [-0.3333+j*0.4559,-0.3333-j*0.4559,-2,-2.5,-3,-3.5])
```

Kx = 0.0872 23.4553 16.9696 0.3146 -5.1406 6.2114

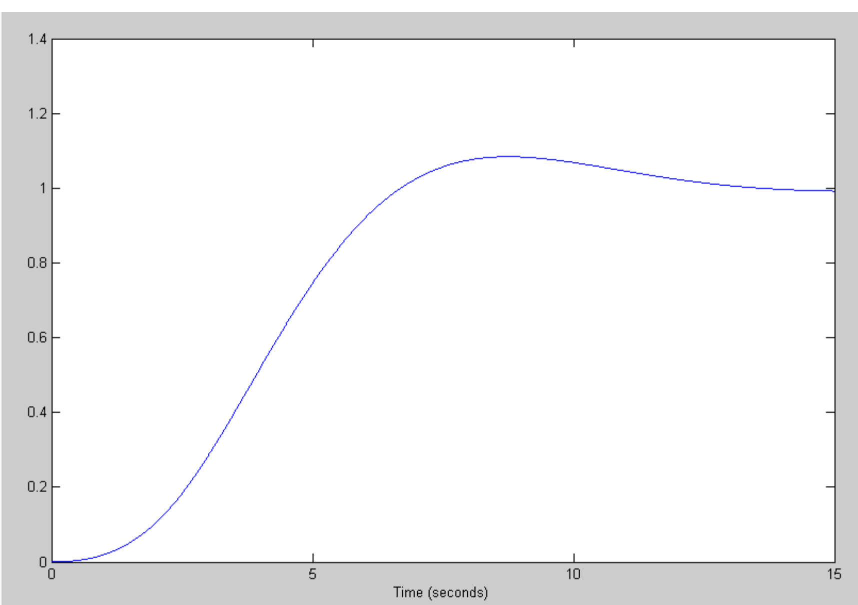
```
>> eig(A - B*Kx)

  -3.5000
  -3.0000
  -2.5000
  -2.0000
  -0.3333 + 0.4559i
  -0.3333 - 0.4559i

>> C = [1,0,0,0,0,0];
>> D = 0;
>> DC = -C*inv(A-B*Kx)*B

DC =   11.4716

>> Kr = 1/DC
```

Kr = 0.0872

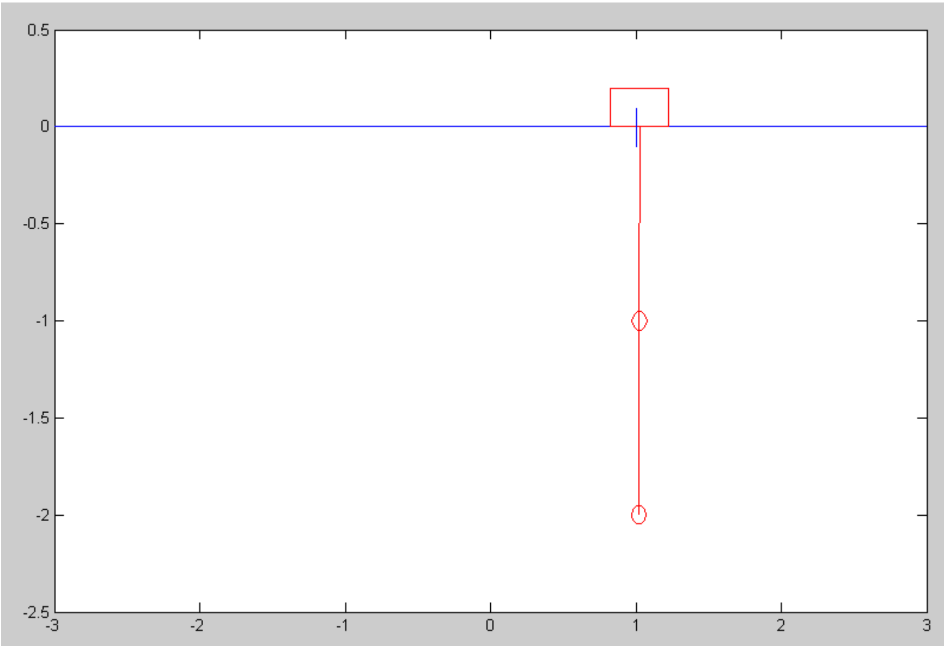
```
>>
```

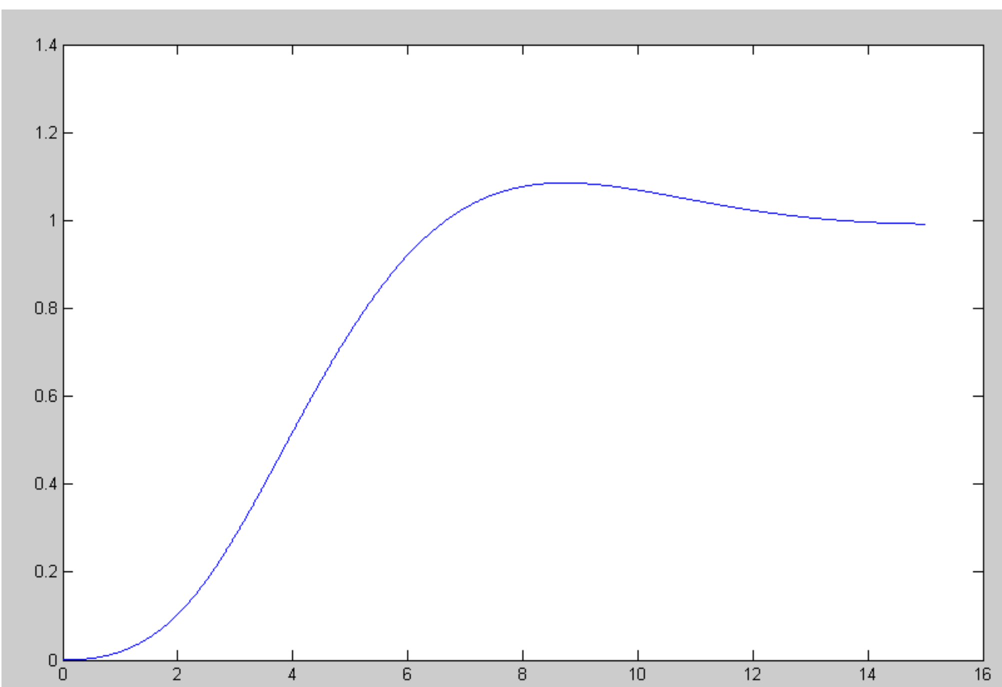
(10pt) Determine the step response of the linear system in Matlab

```
>> G = ss(A - B*Kx, B*Kr, C, D);
>> t = [0:0.01:15]';
>> y = step(G, t);
>> plot(t,y)
>> xlabel('Time (seconds)')
```



(10pt) Determine the step response of the nonlinear system





Code:

```
% Double Pendulum system - main routine
% ECE 463 Lecture #10
% cart = 1kg
% m1 = m2 = 1kg
% L1 = L2 = 1m
% X = [x, q1, q2, dx, dq1, dq2]

X = [0, 0, 0, 0, 0, 0]';
Ref = 1;
dt = 0.01;
U = 0;
t = 0;
Kx = [0.0872   23.4553   16.9696    0.3146   -5.1406    6.2114];
Kr =  0.0872;

y = [];

while(t < 15)
 Ref = 1;
 U = Kr*Ref - Kx*X;
 dX = Gantry2Dynamics(X, U);
 X = X + dX * dt;
 t = t + dt;
 Gantry2Display(X, Ref);
 plot([Ref, Ref],[-0.1,0.1],'b');
 y = [y; X(1)];
end

t = [1:length(y)]' * dt;
plot(t,y);
```