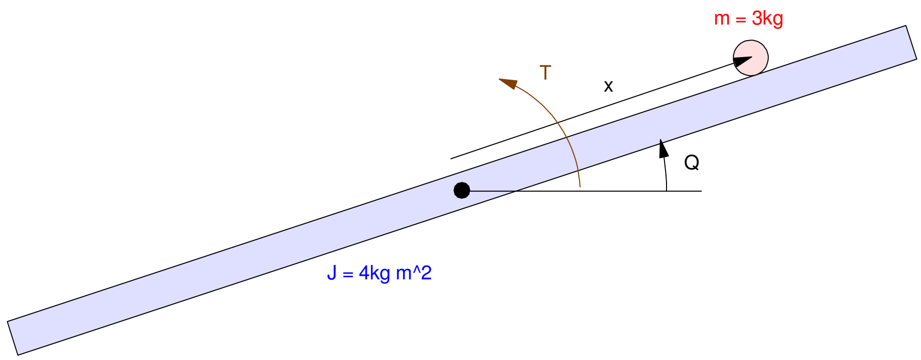
ECE 463/663 - Homework #7

Servo Compensators. Due Monday, March 8th



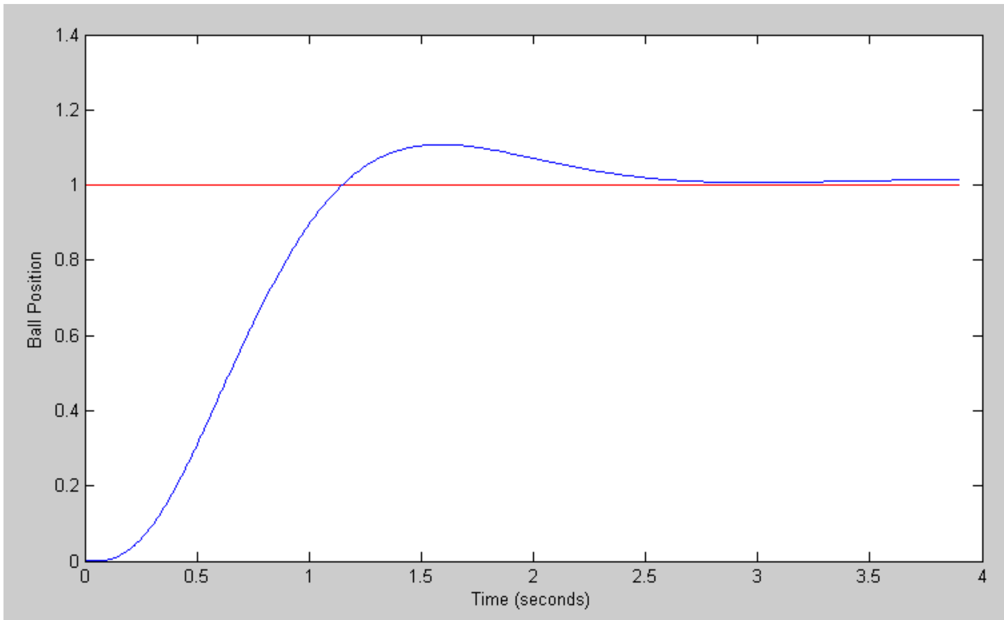
The dynamics of a Ball and Beam System (homework set #4) with a disturbance are

$$s\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -7 & 0 & 0 \\ -4.2 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.143 \end{bmatrix}T + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.143 \end{bmatrix}d$$

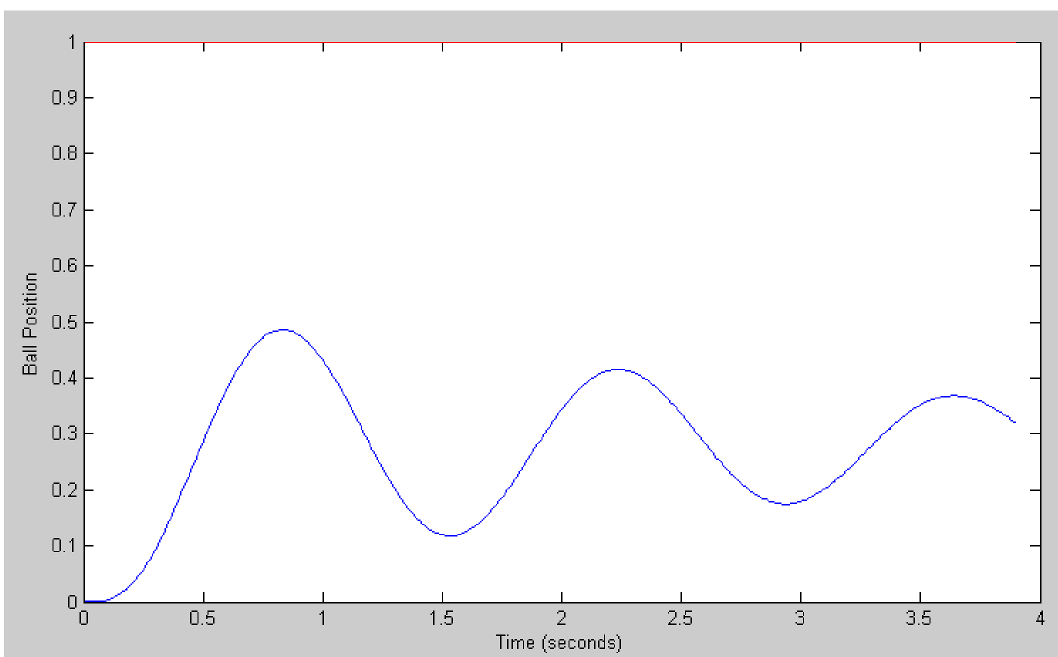
Full-State Feedback with Constant Disturbances

1) For the nonlinear simulation, use the feedback control law you computed in homework #6
   - With R = 1 and the mass of the ball = 3.0kg (same result you got for homework #6), and
   - With R = 1 and the mass of the ball increased to 3.5kg

(i.e. a constant disturbance on the system due to the extra mass of the ball)

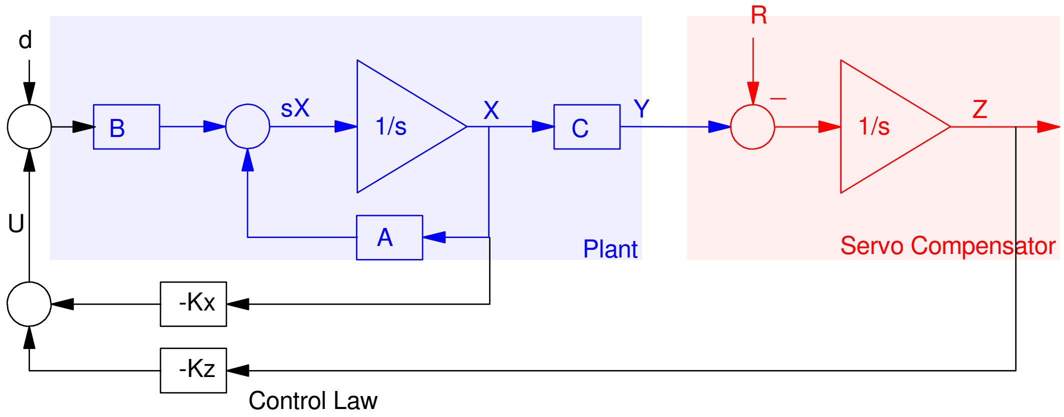


Step Response with m = 3.00kg



Step Response with m = 2.50 kg

Servo Compensators with Constant Set-Points



2) Assume a constant disturbance and/or a constant set point. Design a feedback control law that results in

- The ability to track a constant set point (R = constant)
- The ability to reject a constant disturbance (d = constant),
- A 2% settling time of 12 seconds, and
- 10% overshoot for a step input.

The augmented open-loop system is

$$\begin{bmatrix} sX \\ sZ \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \begin{bmatrix} X \\ Z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} U + \begin{bmatrix} 0 \\ -1 \end{bmatrix} R + \begin{bmatrix} B \\ 0 \end{bmatrix} d$$

The augmented closed-loop system is

$$\begin{bmatrix} sX \\ sZ \end{bmatrix} = \begin{bmatrix} A - BK_x & -BK_z \\ C & 0 \end{bmatrix} \begin{bmatrix} X \\ Z \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} R + \begin{bmatrix} B \\ 0 \end{bmatrix} d$$

Finding Kx and Kz:

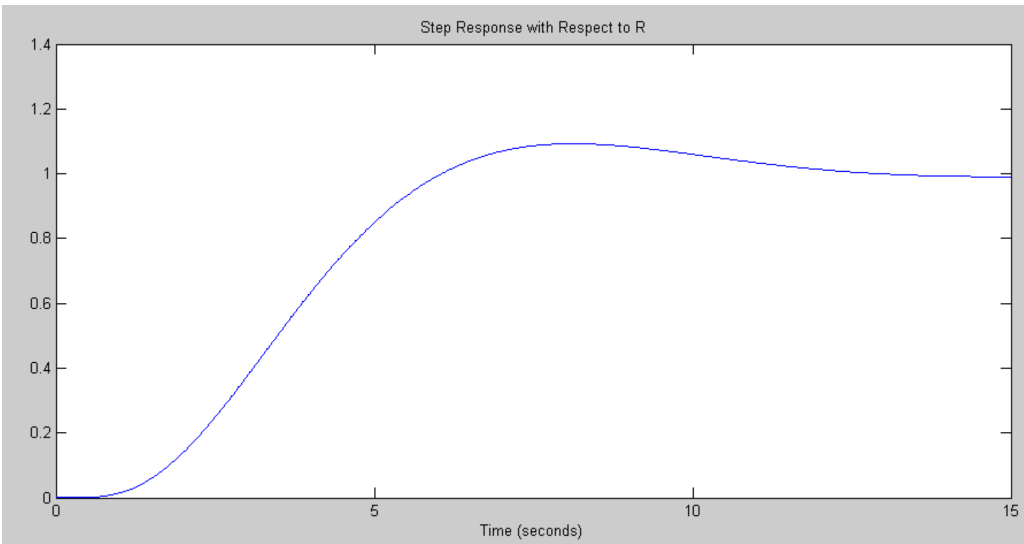
```
>> A5 = [A, B*0 ; C,0]

         0         0    1.0000         0         0
         0         0         0    1.0000         0
         0   -7.0000         0         0         0
   -4.2000         0         0         0         0
    1.0000         0         0         0         0

>> B5 = [B;0]

         0
         0
         0
    0.1430
         0
```

```
>> K5 = ppl(A5, B5, [-0.3333+j*0.4559,-0.3333-j*0.4559,-2,-3,-4])

K5 =  -53.6370  226.0023  -44.1578   67.5986   -7.6468

>> Kx = K5(1:4)

Kx =  -53.6370  226.0023  -44.1578   67.5986

>> Kz = K5(5)

Kz =   -7.6468
```

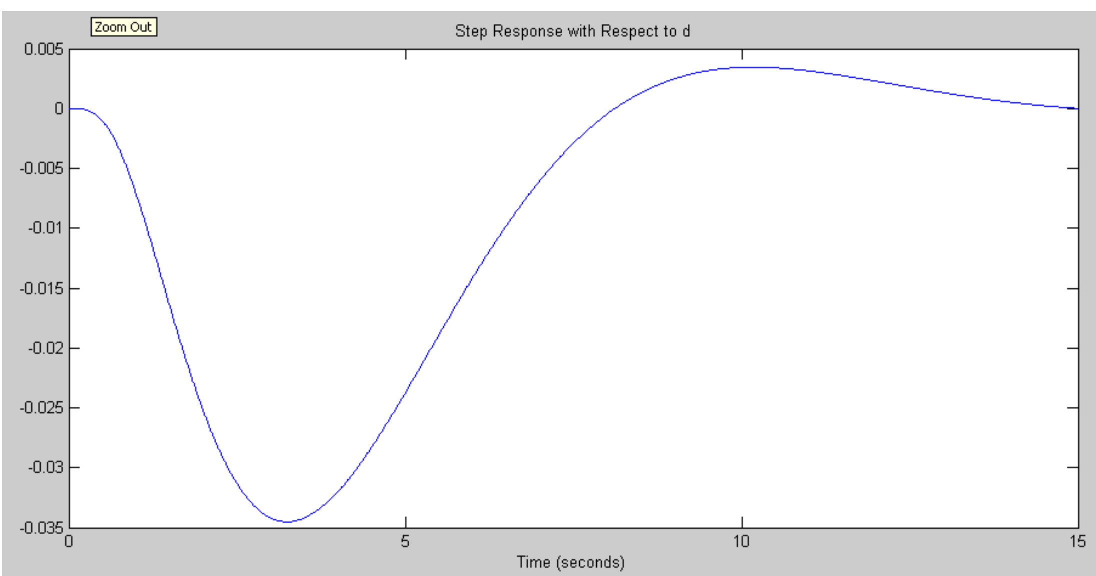
3) For the linear system, plot the step response

- With respect to a step change in R, and
- With respect to a step change in d

```
>> C5 = [1,0,0,0,0];
>> D5 = 0;
>> B5r = [0;0;0;0;-1];
>> B5d = [B;0];
>> G5 = ss(A5 - B5*K5, B5r, C5, D5);
>> t = [0:0.01:15]';
>> y = step(G5,t);
>> plot(t,y);
>> xlabel('Time (seconds)');
>> title('Step Response with Respect to R')
```

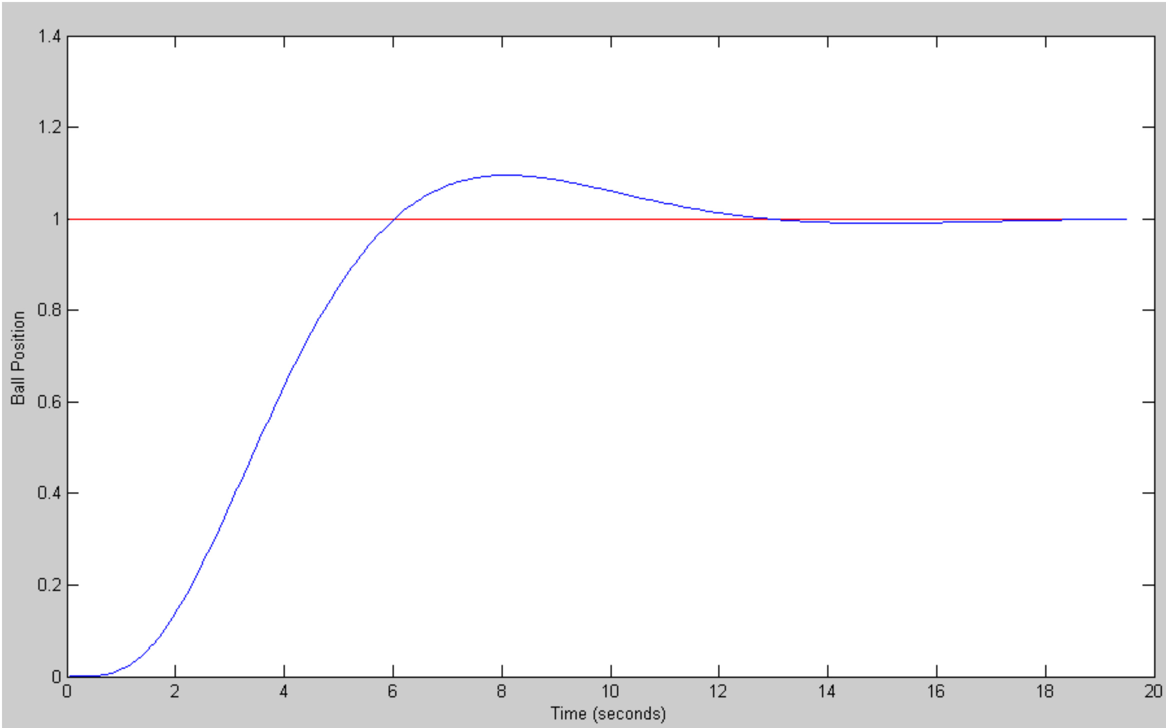



```
>> G5 = ss(A5 - B5*K5, B5d, C5, D5);
>> y = step(G5,t);
>> plot(t,y);
>> xlabel('Time (seconds)');
>> title('Step Response with Respect to d')
```

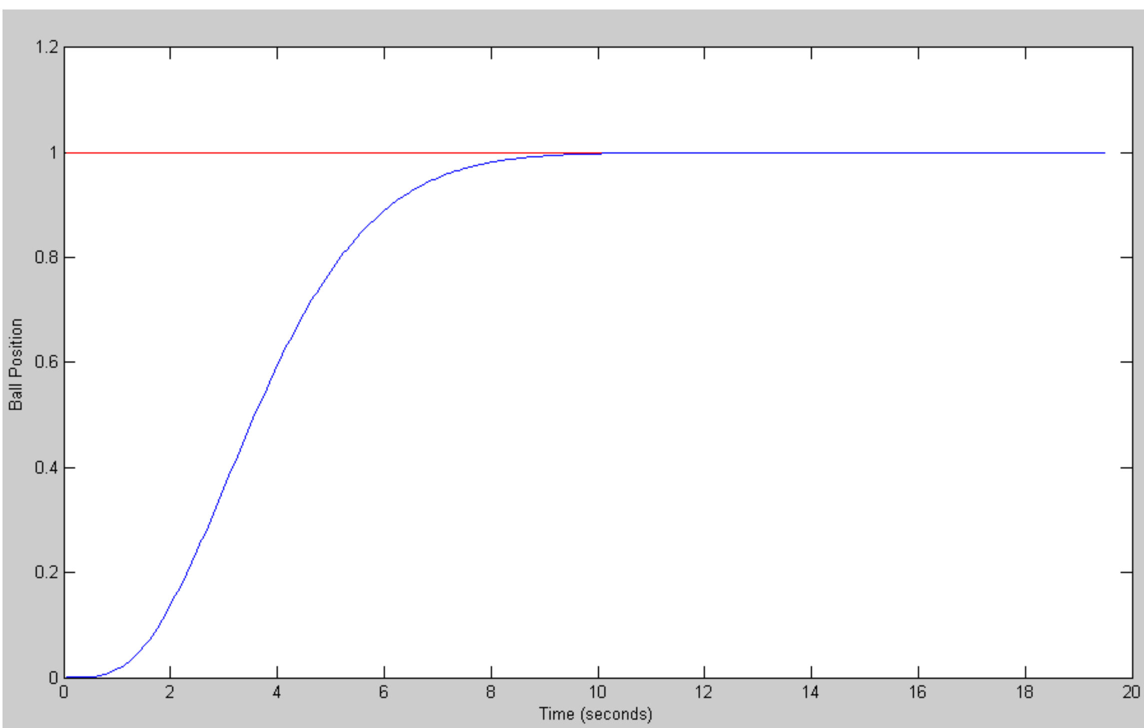


4) Implement your control law on the nonlinear ball and beam system

- With R = 1 and the mass of the ball being 3.0kg, and
- With R = 1 and the mass of the ball being 2.5kg



Step Response with m = 3.00 kg

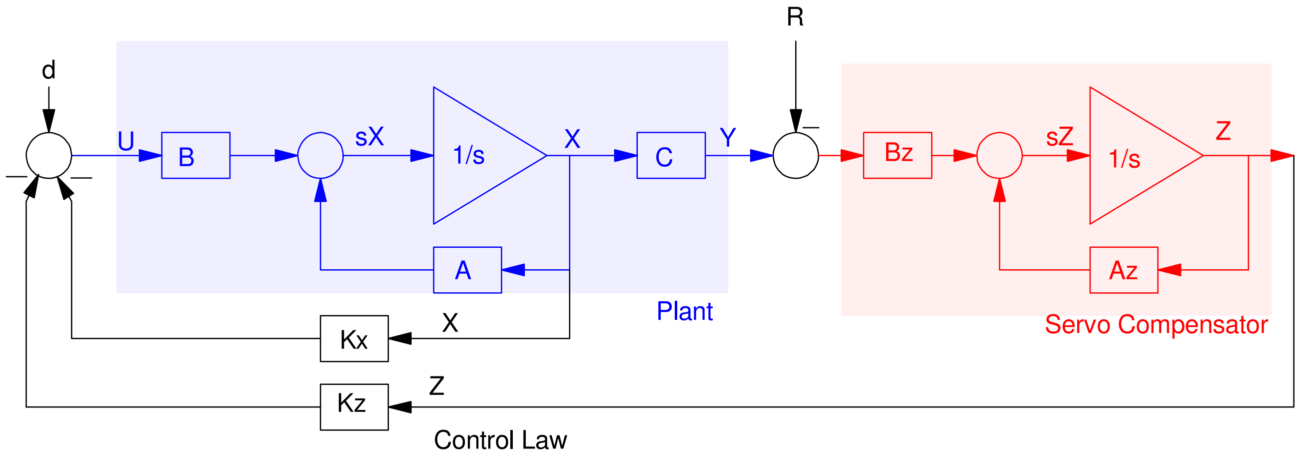


Step Response with m = 2.5kg

Matlab Code

```
% Ball & Beam System
% homework #7: Servo compensator

X = [0,0,0,0]';
Z = 0;

dt = 0.01;
t = 0;
Kx = [-53.6370  226.0023  -44.1578   67.5986];
Kz = -7.6468;
y = [];
n = 0;

while(t < 19.5)
 Ref = 1;
 U = -Kz*Z - Kx*X;
 dX = BeamDynamics(X, U);
 dZ = X(1) - Ref;

 X = X + dX * dt;
 Z = Z + dZ * dt;
 t = t + dt;

 n = mod(n+1, 5);
 if(n == 0)
    y = [y ; Ref, X(1)];
    BeamDisplay(X, Ref);
 end
 end

t = [1:length(y)]' * dt*5;


plot(t,y(:,1),'r',t,y(:,2),'b');
xlabel('Time (seconds)');
ylabel('Ball Position');
```

Servo Compensators with Sinulsoidal Set-Points



5) Assume a 0.5 rad/sec disturbance and/or set point (R). Design a feedback control law that results in

- The ability to track a constant set point (R = sin(0.5t))
- The ability to reject a constant disturbance (d = sin(0.5t)),
- A 2% settling time of 12 seconds, and

Open-Loop Augmented System

$$\begin{bmatrix} sX \\ sZ \end{bmatrix} = \begin{bmatrix} A & 0 \\ B_z C & A_z \end{bmatrix} \begin{bmatrix} X \\ Z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} U + \begin{bmatrix} 0 \\ -1 \end{bmatrix} R + \begin{bmatrix} B \\ 0 \end{bmatrix} d$$

Closed-Loop Augmented System

$$\begin{bmatrix} sX \\ sZ \end{bmatrix} = \begin{bmatrix} A - BK_x & -BK_z \\ B_z C & A_z \end{bmatrix} \begin{bmatrix} X \\ Z \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} R + \begin{bmatrix} B \\ 0 \end{bmatrix} d$$

To find Kx and Kz

First, define {Az, Bz} so that

- Az has roots at +/- j0.5
- {Az, Bz} is controllable

$$sZ = \begin{bmatrix} 0 & 0.5 \\ -0.5 & 0 \end{bmatrix} Z + \begin{bmatrix} 1 \\ 1 \end{bmatrix} (y - R)$$

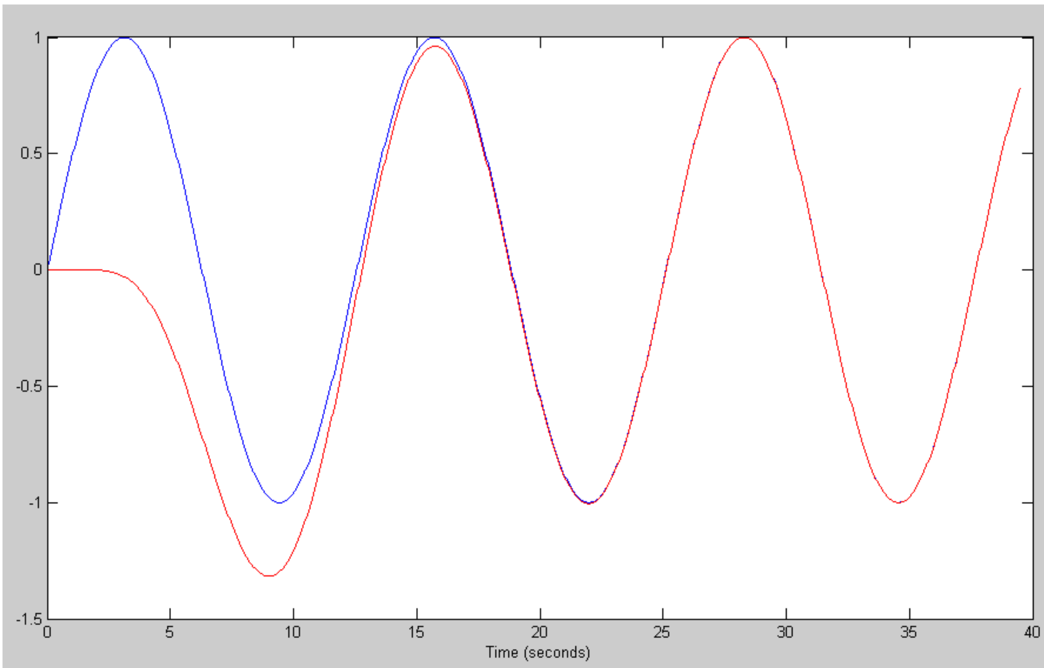
Second, create the autmented system (6x6 now) and find stabilizaing feedback gains

```
>> Az = [0,0.5;-0.5,0]

         0    0.5000
   -0.5000         0

>> Bz = [1;1];
>> A6 = [A, zeros(4,2) ; Bz*C, Az]

         0         0    1.0000         0         0         0
         0         0         0    1.0000         0         0
         0   -7.0000         0         0         0         0
   -4.2000         0         0         0         0         0
    1.0000         0         0         0         0    0.5000
    1.0000         0         0         0   -0.5000         0

>> B6r = [0*B ; -Bz]

     0
     0
     0
     0
    -1
    -1

>> B6d = [B ; 0*Bz]

         0
         0
         0
    0.1430
         0
         0

>> B6 = [B ; 0*Bz]

         0
         0
         0
    0.1430
         0
         0

>> K6 = ppl(A6, B6, [-4/12,-1,-1.1,-1.2,-1.3,-1.4])

                   Kx                                  Kz
K6 =  -41.1862  112.5874  -20.2797   44.2890    1.7985   -2.5093
```

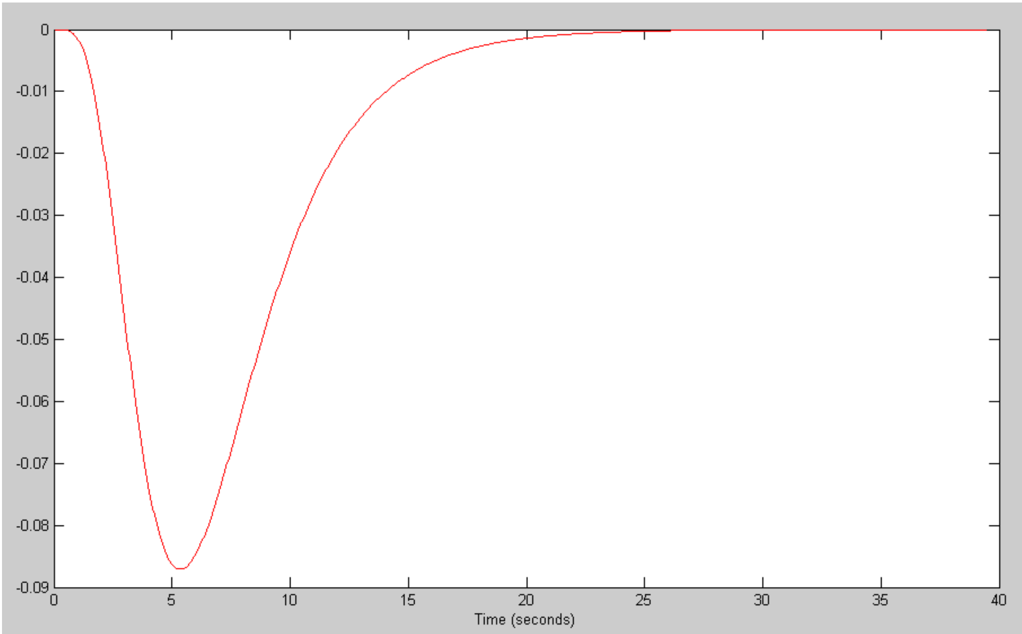
6) For the linear system, plot the response

- With R(t) = sin(0.5t), and
- With d(t) = sin(0.5t)

```
>> R = sin(0.5*t);
>> y = step3(A6-B6*K6, B6r, C6, D6, t, X0, R);
>> plot(t,R,'b',t,y,'r')
>> xlabel('Time (seconds)');
```

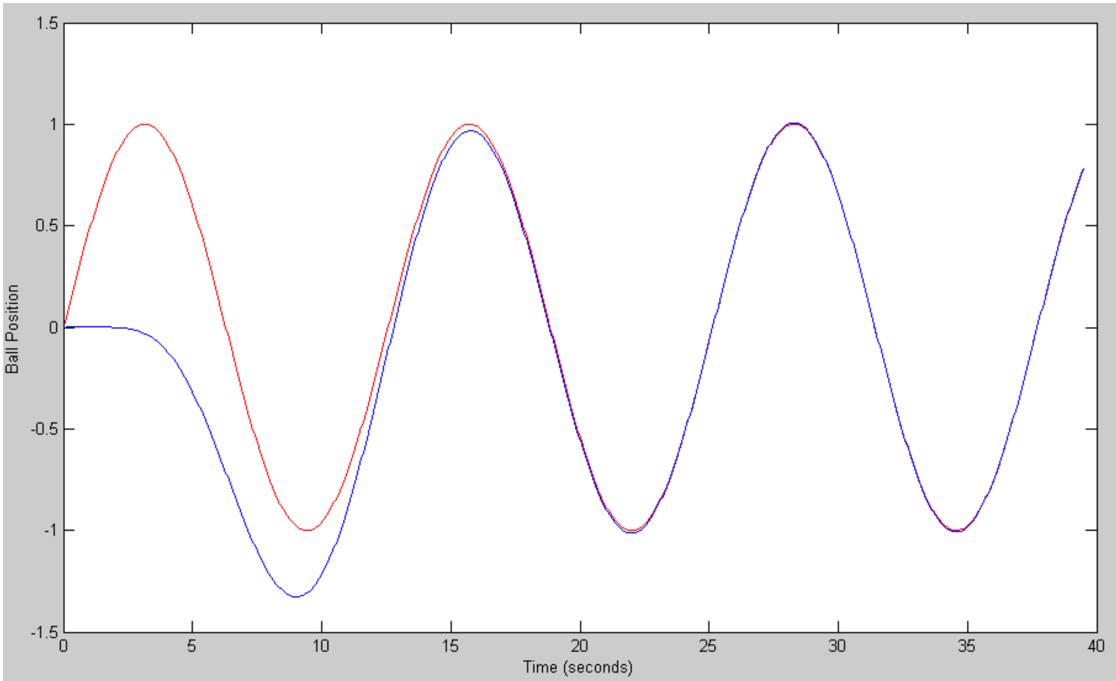



```
>> y = step3(A6-B6*K6, B6d, C6, D6, t, X0, R);
>> plot(t,y,'r')
>> xlabel('Time (seconds)');
```

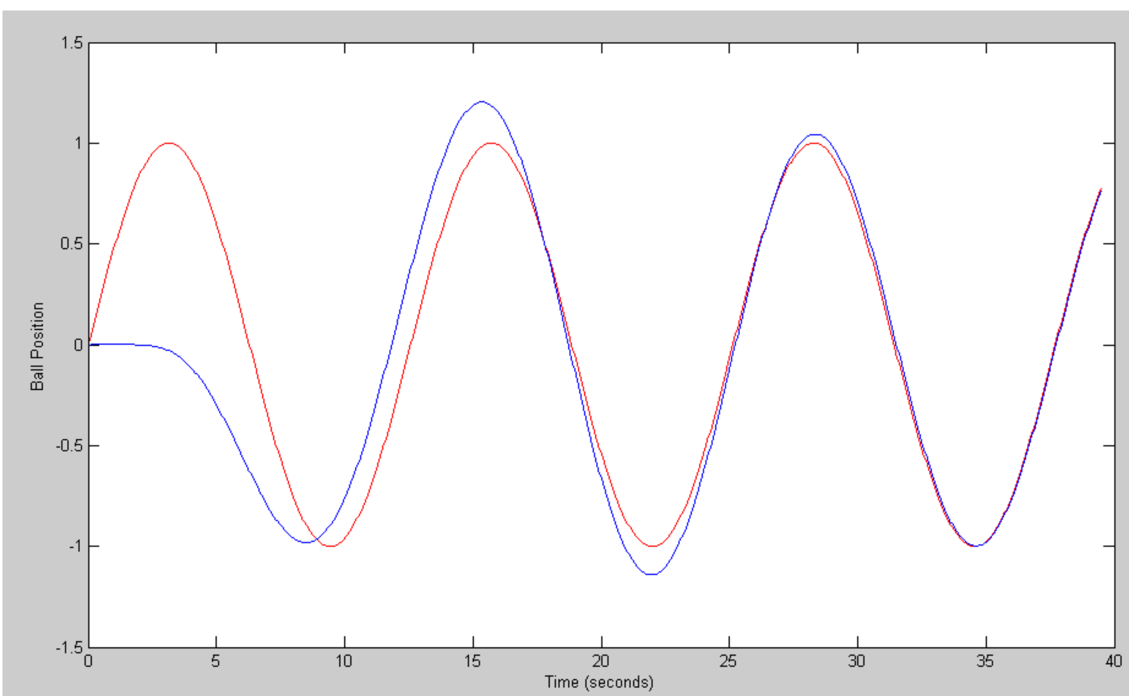

0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
-0.08
-0.09
0
5
10
15
20
25
30
35
40
Time (seconds)

7) Implement your control law on the nonlinear ball and beam system

- With R = sin(0.5t) and the mass of the ball being 3.0kg, and
- With R = sin(0.5t) and the mass of the ball being 2.5kg



Response with m = 3.0kg



Response when m = 2.5kg

Code

```
% Ball & Beam System
% homework #7: Servo compensator

X = [0,0,0,0]';
Z = [0;0];

dt = 0.01;
t = 0;

Az = [0,0.5 ; -0.5,0];
Bz = [1;1];

Kx = [-41.1862  112.5874  -20.2797   44.2890];
Kz = [1.7985   -2.5093];
y = [];
n = 0;

while(t < 39.5)
 Ref = sin(0.5*t);
 U = -Kz*Z - Kx*X;
 dX = BeamDynamics(X, U);
 dZ = Az*Z + Bz*(X(1) - Ref);

 X = X + dX * dt;
 Z = Z + dZ * dt;
 t = t + dt;

 n = mod(n+1, 5);
 if(n == 0)
    y = [y ; Ref, X(1)];
    BeamDisplay(X, Ref);
 end
 end

t = [1:length(y)]' * dt*5;


plot(t,y(:,1),'r',t,y(:,2),'b');
xlabel('Time (seconds)');
ylabel('Ball Position');
```