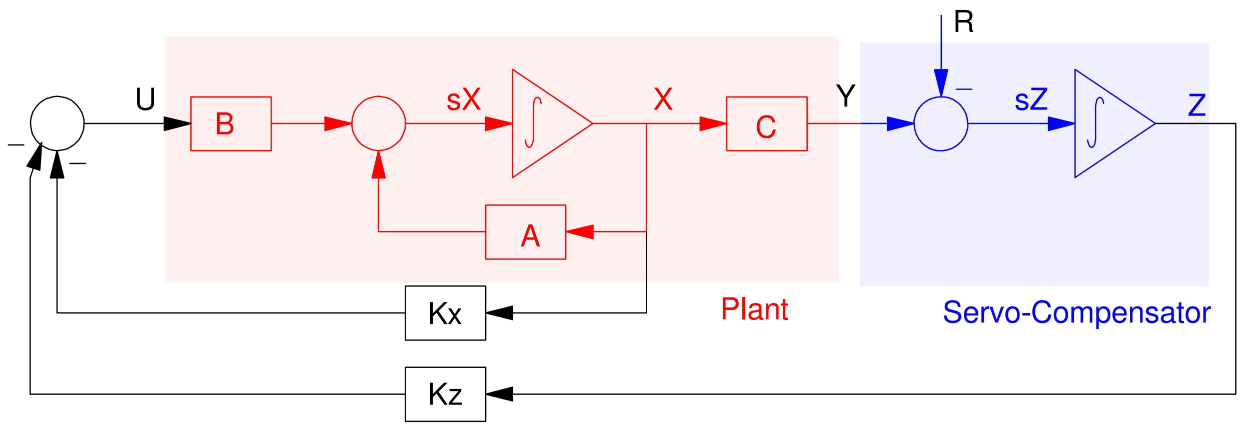
ECE 463/663 - Homework #10

LQG Control with Servo Compensators. Due Monday, April 11th, 2022



Cart and Pendulum (HW #4): For the cart and pendulum system of homework #4

$$s\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -14.7 & 0 & 0 \\ 0 & 24.5 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.5 \\ -0.5 \end{bmatrix} F$$

Use LQG methods to design a full-state feedback control law of the form

$$F = U = -K_z Z - K_x X$$

$$\dot{Z} = (x - R)$$

for the cart and pendulum system from homework #4 using LQG control so that

- You track constant setpoints,
- You reject constant disturbances,
- The 2% settling time is 10 seconds, and
- There is less than 10% overshoot for a step input.

1) Give the control law (Kx and Kz) and explain how you chose Q and R

Adjust Q until the step response is close to the desired

$$y_d = \left(\frac{0.5}{s^2+s+0.5}\right) R$$

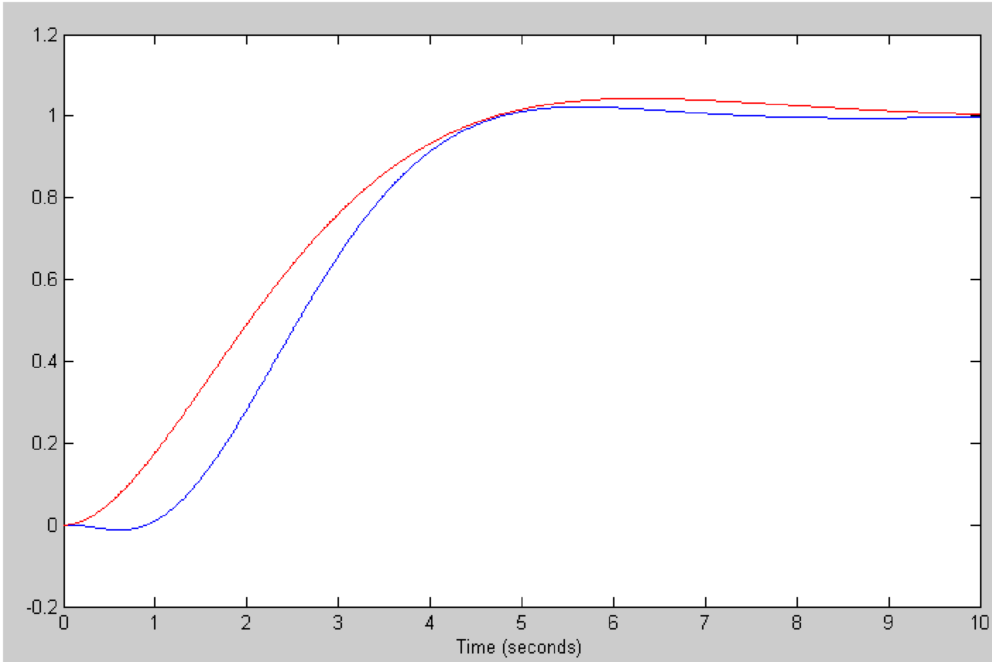
Matlab Code

```
Gd = tf(0.5,[1,1,0.5]);
t = [0:0.01:10]';
yd = step(Gd, t);

A = [0,0,1,0 ; 0,0,0,1 ; 0,-14.7,0,0 ; 0,24.5,0,0];
B = [0;0;0.5;-0.5];
C = [1,0,0,0];

A5 = [A,zeros(4,1) ; C, 0];
B5 = [B;0];
B5r = [0*B;-1];
C5 = [C,0];
D5 = 0;

K5 = lqr(A5, B5, diag([20,0,0,0,30]), 1);
G5 = ss(A5-B5*K5, B5r, C5, D5);
y = step(G5,t);

plot(t,y,'b',t,yd,'r');
```

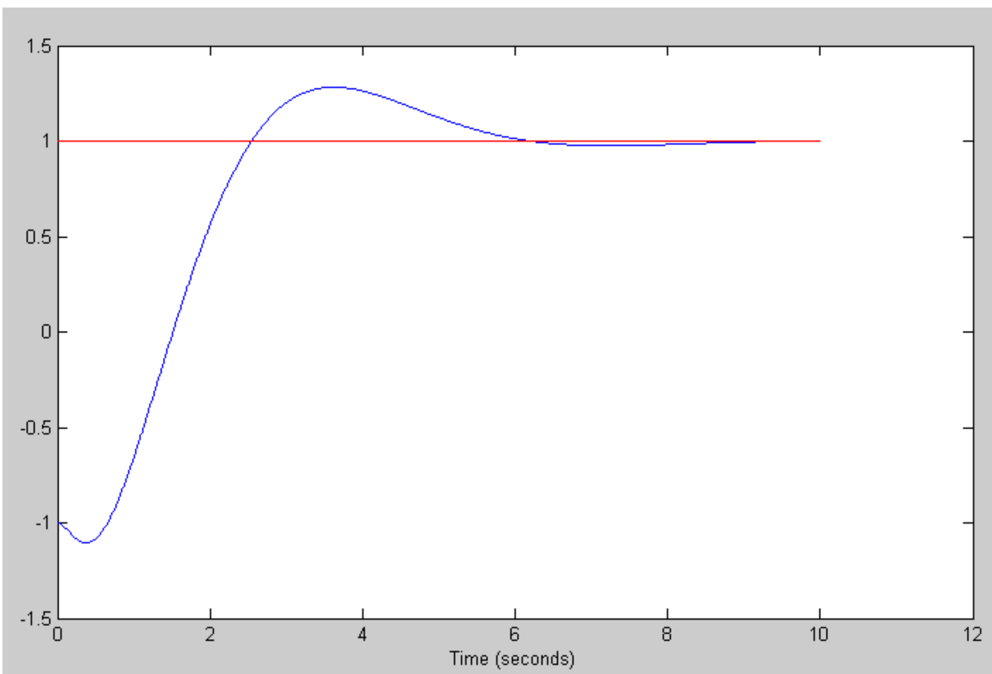
Final result:

```
>> K5

K5 =  -14.2571 -160.9249  -16.7297  -40.9510   -5.4772

>> eig(A5 - B5*K5)

  -4.9466 + 0.1315i
  -4.9466 - 0.1315i
  -0.8929
  -0.6623 + 0.8882i
  -0.6623 - 0.8882i

>>
```

2) Plot the step response of the linear system



3) Check your design with the nonlinear simulation of the cart and pendulum system.



Ball and Beam (HW #4): For the ball and beam system of homework #4

$$s\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -7 & 0 & 0 \\ -4.2 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.143 \end{bmatrix} T$$

Use LQG methods to design a full-state feedback control law of the form

$$T = U = -K_z Z - K_x X$$

$$\dot{Z} = (x - R)$$

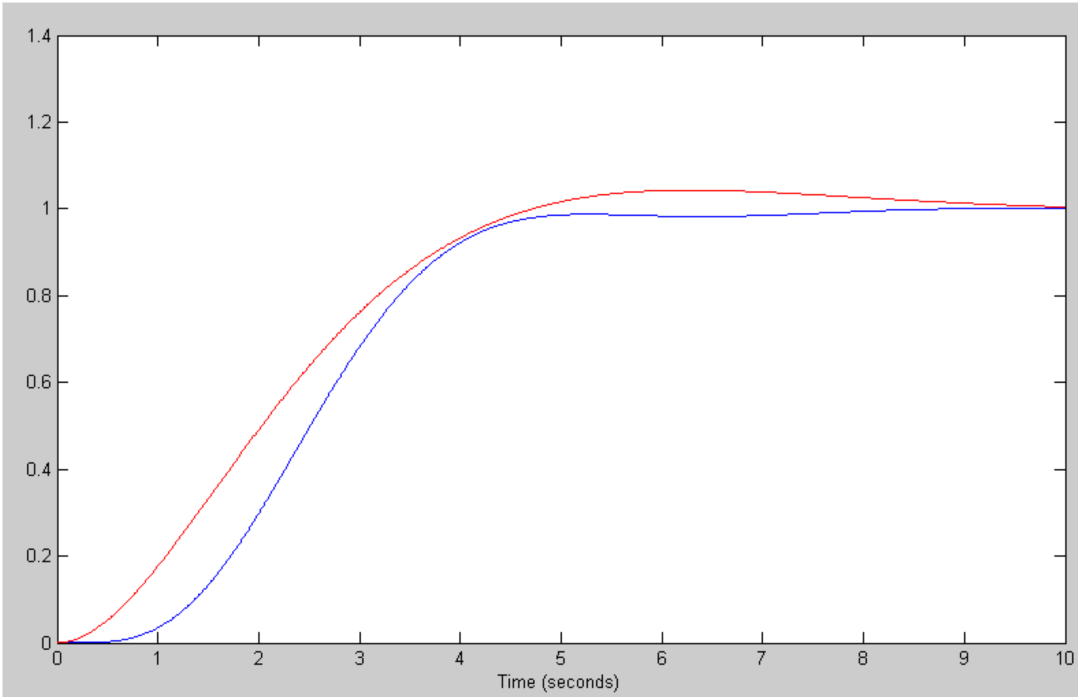
for the ball and beam system from homework #6 using LQG control so that

- You track constant setpoints,
- You reject constant disturbances,
- The 2% settling time is 6 seconds, and
- There is less than 5% overshoot for a step input.

4) Give the control law (Kx and Kx) and explain how you chose Q and R

```
% ball and beam (Problem 4)

Gd = tf(0.5,[1,1,0.5]);
t = [0:0.01:10]';
yd = step(Gd, t);

A = [0,0,1,0 ; 0,0,0,1 ; 0,-7,0,0 ; -4.2,0,0,0];
B = [0;0;0;0.143];
C = [1,0,0,0];

A5 = [A,zeros(4,1) ; C, 0];
B5 = [B;0];
B5r = [0*B;-1];
C5 = [C,0];
D5 = 0;

K5 = lqr(A5, B5, diag([0,0,0,7000,700]), 1);
G5 = ss(A5-B5*K5, B5r, C5, D5);
y = step(G5,t);

plot(t,y,'b',t,yd,'r');

>> K5

K5 =  -98.4866  318.0728  -73.9749  106.9980  -26.4575

>> eig(A5 - B5*K5)

 -11.9813
  -1.0352 + 0.0840i
  -1.0352 - 0.0840i
  -0.6245 + 1.2881i
  -0.6245 - 1.2881i
```

5) Plot the step response of the linear system



6) Check your design with the nonlinear simulation of the cart and pendulum system.