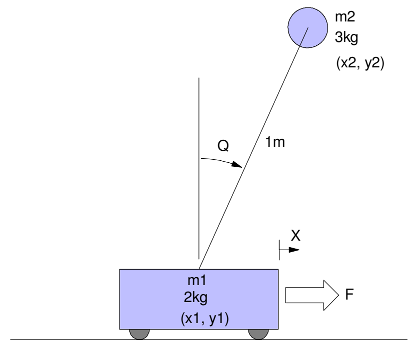
ECE 463/663 - Homework #12

LQG/LTR. Due Monday, April 25th, 2022

LQG / LTR

1) For the cart and pendulum system of homework set #4:

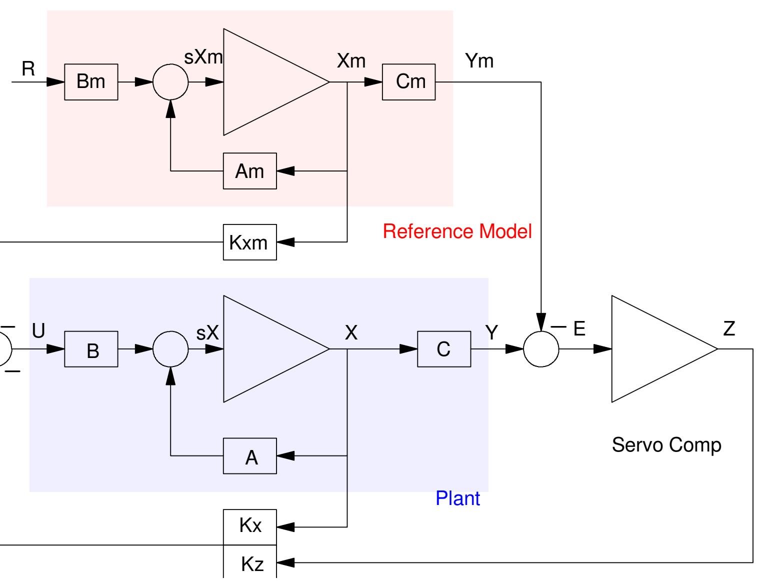


$$s\begin{bmatrix} x \\ \theta \\ sx \\ s\theta \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -14.7 & 0 & 0 \\ 0 & 24.5 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta \\ sx \\ s\theta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.5 \\ -0.5 \end{bmatrix}F$$

Design a control law so that the cart and pendulum system behaves like the following reference model:

$$y_m = \left(\frac{0.4}{s^2+0.7s+0.4}\right)R$$

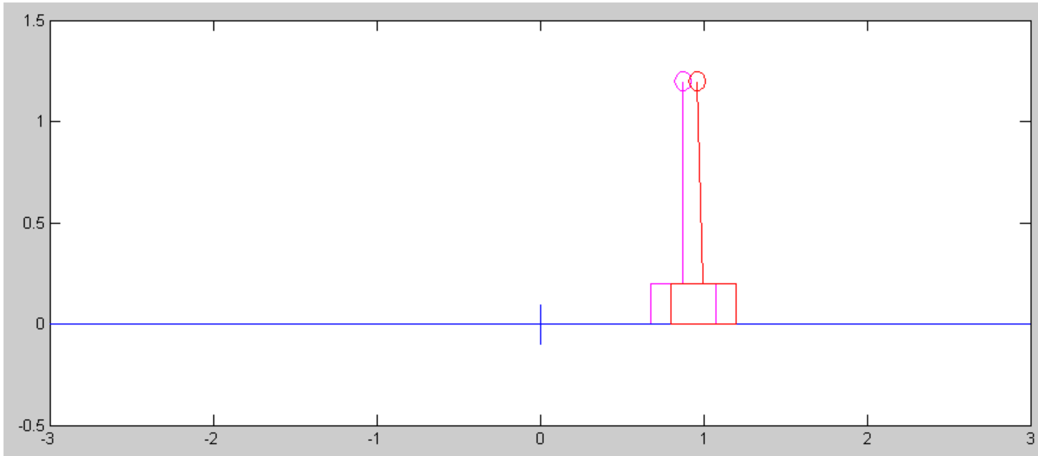
1) Give a block diagram for your controller



$$s\begin{bmatrix} X \\ Z \\ X_m \end{bmatrix} = \begin{bmatrix} A & 0 & 0 \\ C & 0 & -C_m \\ 0 & 0 & A_m \end{bmatrix}\begin{bmatrix} X \\ Z \\ X_m \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}U + \begin{bmatrix} 0 \\ 0 \\ B_m \end{bmatrix}R$$

$$U = \begin{bmatrix} -K_x & -K_z & -K_m \end{bmatrix}\begin{bmatrix} X \\ Z \\ X_m \end{bmatrix}$$

2) Plot the step response of the model and the linearlized plant for yor control law for

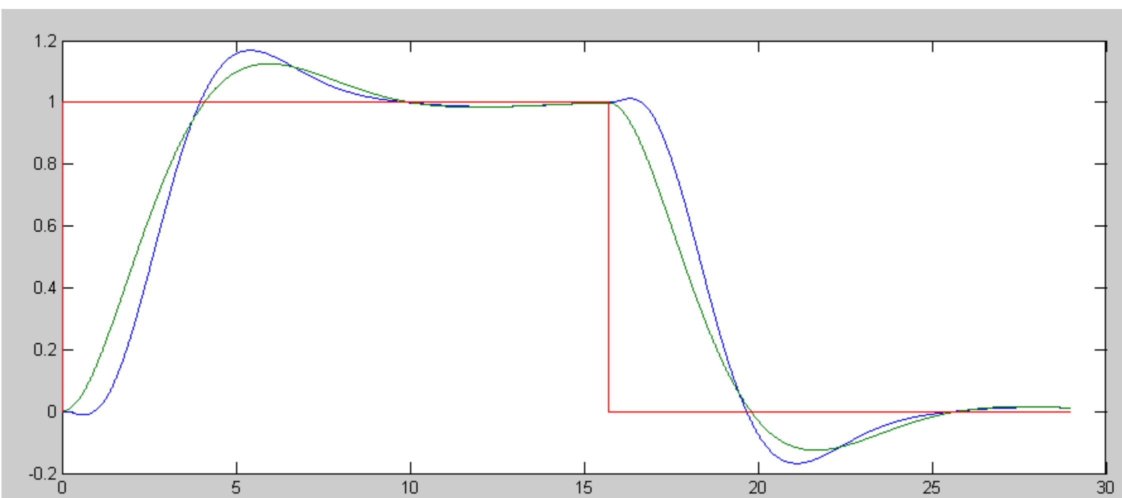


Matlab Code:

```
A = [0,0,1,0;0,0,0,1;0,-14.7,0,0;0,24.5,0,0];
B = [0;0;0.5;-0.5];
C = [1,0,0,0];

Ref = 1;
dt = 0.01;
t = 0;

%Reference Model
Gm = tf( 0.4, [1, 0.7, 0.4]);

X = ss(Gm);
Am = X.a;
Bm = X.b;
Cm = X.c;
[n,m] = size(Am);

A7 = [ A, zeros(4,1), zeros(4,n) ;
       C, 0, -Cm;
       zeros(n,4), zeros(n,1), Am];
B7 = [B; 0; zeros(n,1)];
B7r = [zeros(4,1); 0; Bm];

C7 = [0*C, 1, 0*Cm];

Q = C7' * C7;
R = 1;

K7 = lqr(A7, B7, Q*1e2, 1);

Kx = K7(1:4);
Kz = K7(5);
Km = K7(6:5+n);

X = zeros(4,1);
Xm = zeros(n,1);

Z = 0;
```

```
n = 0;
y = [];
while(t < 29)
    Ref = 1*(sin(0.2*t) > 0);
    U = -Km*Xm - Kx*X - Kz*Z;

    dX = CartDynamics(X, U);
    dXm = Am*Xm + Bm*Ref;
    dZ = X(1) - Cm*Xm;

    X = X + dX * dt;
    Xm = Xm + dXm * dt;
    Z = Z + dZ*dt;

    t = t + dt;
    n = mod(n+1, 5);
    if(n == 0)
       CartDisplay(X, [Cm*Xm;0;0;0], Ref);
       end
    y = [y ; X(1), Cm*Xm, Ref];
    end

hold off;
t = [1:length(y)]' * dt;
plot(t,y);
```

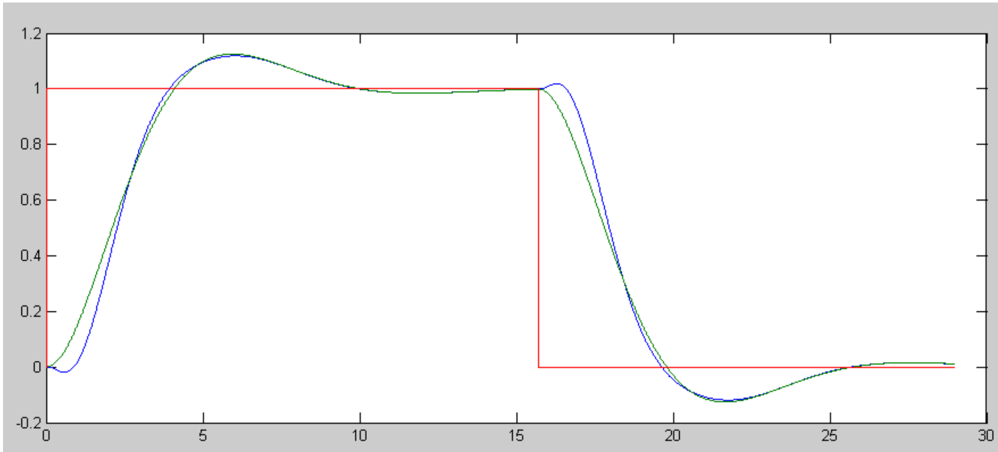
$Q = 100 z^2$



Step Response with $Q = 100 z^2$

```
Kx = -20.4126 -175.6025  -20.8336  -45.7487
Kz = -10.0000
Km =  4.9221   13.1982
```

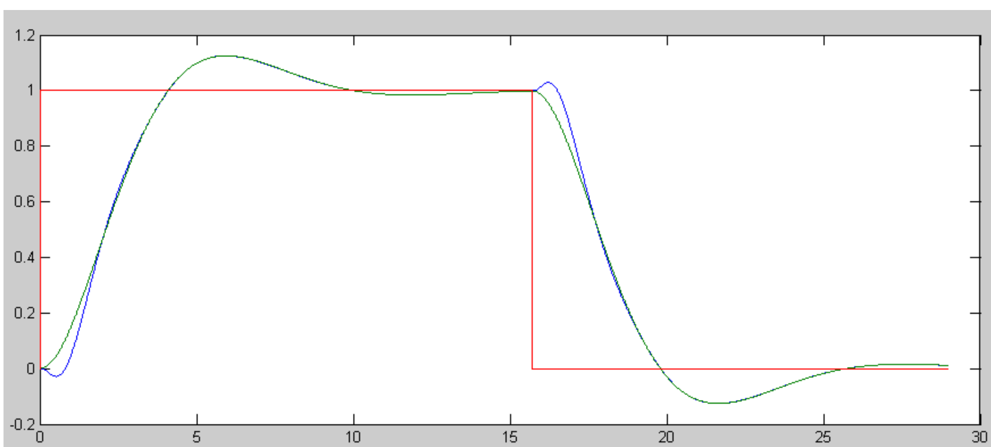
Q = 1,000 z^2



Step Response with Q = 1000 z^2

```
                         Kx                          Kz              Km
Kx =  -49.2073 -237.1821  -38.2850  -65.7058
Kz =  -31.6228
Km =   10.2908   34.6242
```

Q = 10,000 z^2



Step Response with Q = 10,000 z^2

```
Kx = -123.7426 -368.2544  -76.5611 -107.8349
Kz = -100.0000
Km =   22.1174   90.8714
```

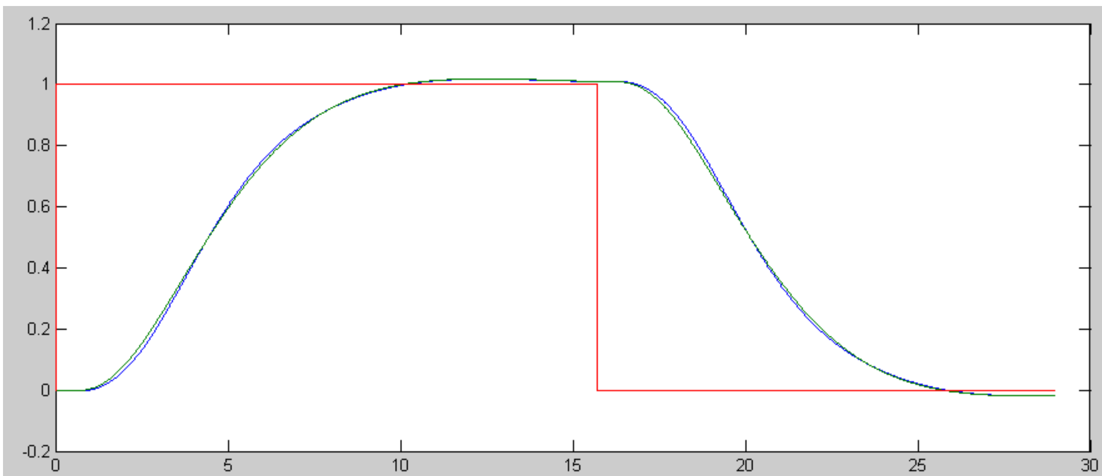
Sidelight: Q needs to be large for good tracking due to

- Trying to make a 4th-order system behave like a 2nd-order system
- The plant is a non-minimum-phase system (has a zero in the right-half plane)

A better reference model would be

$$G_m = \left(\frac{0.4}{s^2+0.7s+0.4}\right)\left(\frac{1.5(s+4)}{(s+2)(s+3)}\right)$$

This gives good tracking with Q = 100Z



Step Response with $Q = 100\ z^2$ and a 4th order reference model

```
Kx =  -20.4126 -175.6025  -20.8336  -45.7487
Kz =  -10.0000
Km =    8.7023    6.1053    4.1771    0.9673
```