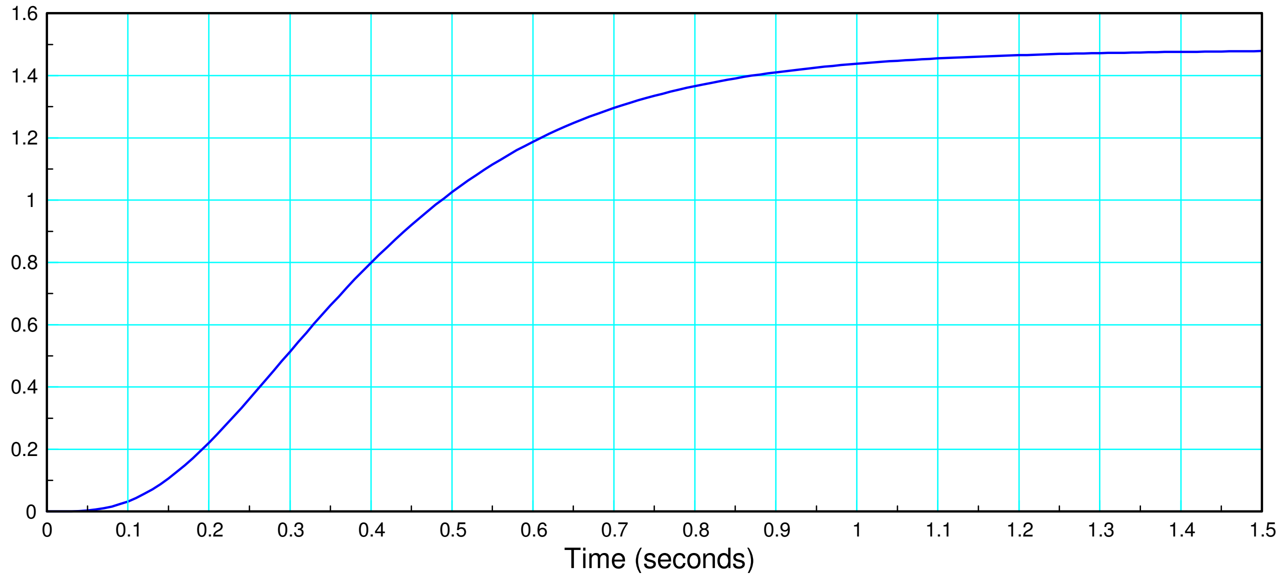
ECE 463/663 - Homework #1

LaPlace Transforms and Dominant Poles. Due Wednesday, Jan 18th

1) Name That System! Give the transfer function for a system with the following step response.



This has a (single) real dominant pole (no oscillations), so you know the answer is in the form of

$$G(s) \approx \left(\frac{a}{s+b}\right)$$

If I can get two pieces of information from the graph, I can determine 'a' and 'b'.

DC Gain:

$$DC = \left(\frac{a}{s+b}\right)_{s \to 0} = 1.45$$

2% settling time

$$T_s = \frac{4}{b} \approx 1.3$$

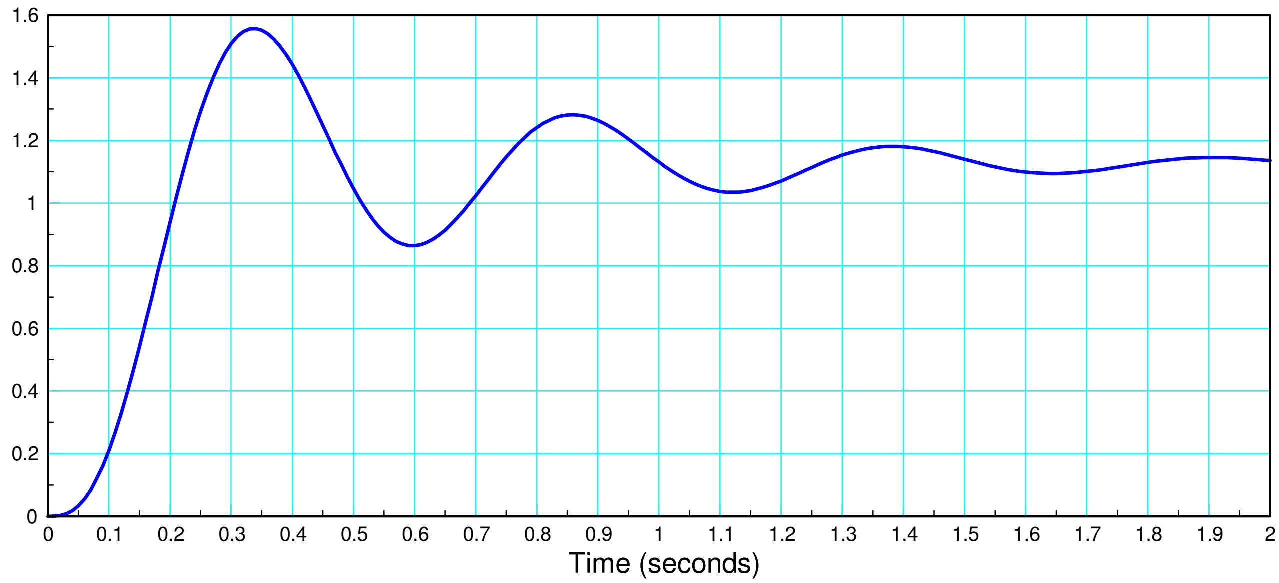
resulting in

$$b = 3.08$$

$$a = 4.46$$

$$\boxed{G(s) \approx \left(\frac{4.46}{s+3.08}\right)}$$

2) Name That System! Give the transfer function for a system with the following step response.



This has complex dominant poles (it oscillates, meaning a complex pole). This tells you that the transfer funciton is of the form

$$G(s) = \left(\frac{a}{s^2+bs+c}\right) = \left(\frac{a}{(s+\sigma+j\omega_d)(s+\sigma-j\omega_d)}\right)$$

If you can pull three pieces of information from the graph, you can solve for the three unknown terms.

DC gain

$$DC = \left(\frac{a}{(s+\sigma+j\omega_d)(s+\sigma-j\omega_d)}\right)_{s=0} = 1.15$$

Frequency of oscillation

$$\omega_d = \left(\frac{3\text{ cycles}}{1.64\text{ seconds}}\right)\cdot 2\pi = 11.49\frac{rad}{\text{sec}}$$

2% settling time

$$T_s = \frac{4}{\sigma} \approx 2\text{ sec}$$

$$\sigma = 2$$

giving

$$\boxed{G(s) \approx \left(\frac{156.42}{(s+2+j11.49)(s+2-j11.49)}\right) = \left(\frac{155.42}{s^2+4s+136.02}\right)}$$

Problem 3 - 6) Assume

$$Y = \left(\frac{50(s+7)}{(s+3)(s+9)(s+12)} \right) X$$

3) What is the differential equation relating X and Y?

Cross multiply

$$(s+3)(s+9)(s+12)Y = 50(s+7)X$$

$$(s^3 + 24s^2 + 171s + 324)Y = (50s + 350)X$$

Note that sY means *the derivative of y(t)*

$$\boxed{y''' + 24y'' + 171y' + 324y = 50x' + 350x}$$

4) Determine y(t) assuming x(t) is a sinusoidal input:

$$x(t) = 2\cos(7t) + 5\sin(7t)$$

Use phasors

$$s = j7$$

$$X = 2 - j5 \qquad \textit{real = cosine, -imag = sine}$$

$$Y = \left(\frac{50(s+7)}{(s+3)(s+9)(s+12)} \right) X$$

$$Y = \left(\frac{50(s+7)}{(s+3)(s+9)(s+12)} \right)_{s=j7} \cdot (2 - j5)$$

$$Y = -2.0506 - j0.8230$$

which means

$$\boxed{y(t) = -2.0506\cos(7t) + 0.8230\sin(7t)}$$

5) Determine y(t) assuming x(t) is a step input:

$$x(t) = u(t)$$

This one requires LaPlace transforms

$$Y = \left(\frac{50(s+7)}{(s+3)(s+9)(s+12)}\right)X$$

Substitute the LaPlace transform for X

$$Y = \left(\frac{50(s+7)}{(s+3)(s+9)(s+12)}\right)\left(\frac{1}{s}\right)$$

Do a partial fraction expansion

$$Y = \left(\frac{1.0802}{s}\right) + \left(\frac{-1.2346}{s+3}\right) + \left(\frac{-0.6173}{s+9}\right) + \left(\frac{0.7716}{s+12}\right)$$

Do an invers LaPlace tranform

$$\boxed{y(t) = (1.0802 - 1.2346e^{-3t} - 0.6173e^{-9t} + 0.7716e^{-12t})u(t)}$$

6a) Determine a 1st-order approximation for this system

$$Y = \left(\frac{50(s+7)}{(s+3)(s+9)(s+12)}\right)X \approx \left(\frac{a}{s+b}\right)X$$

Keep the dominant pole (s = -3)

Match the DC gain (1.0802)

$$\boxed{G(s) \approx \left(\frac{3.2406}{s+3}\right)}$$

6b) Compare the step response of your 1st-order model to the actual 3rd-order system

```
>> G3 = zpk(-7,[-3,-9,-12],50)

     50 (s+7)
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(s+3) (s+9) (s+12)

>> G1 = zpk([],[-3],[3.2406])

3.2406
------
(s+3)

>> t = [0:0.01:2]';
>> y1 = step(G1,t);
>> y3 = step(G3,t);
>> plot(t,y1,'b',t,y3,'r')
>> xlabel('Time (Seconds)');
```

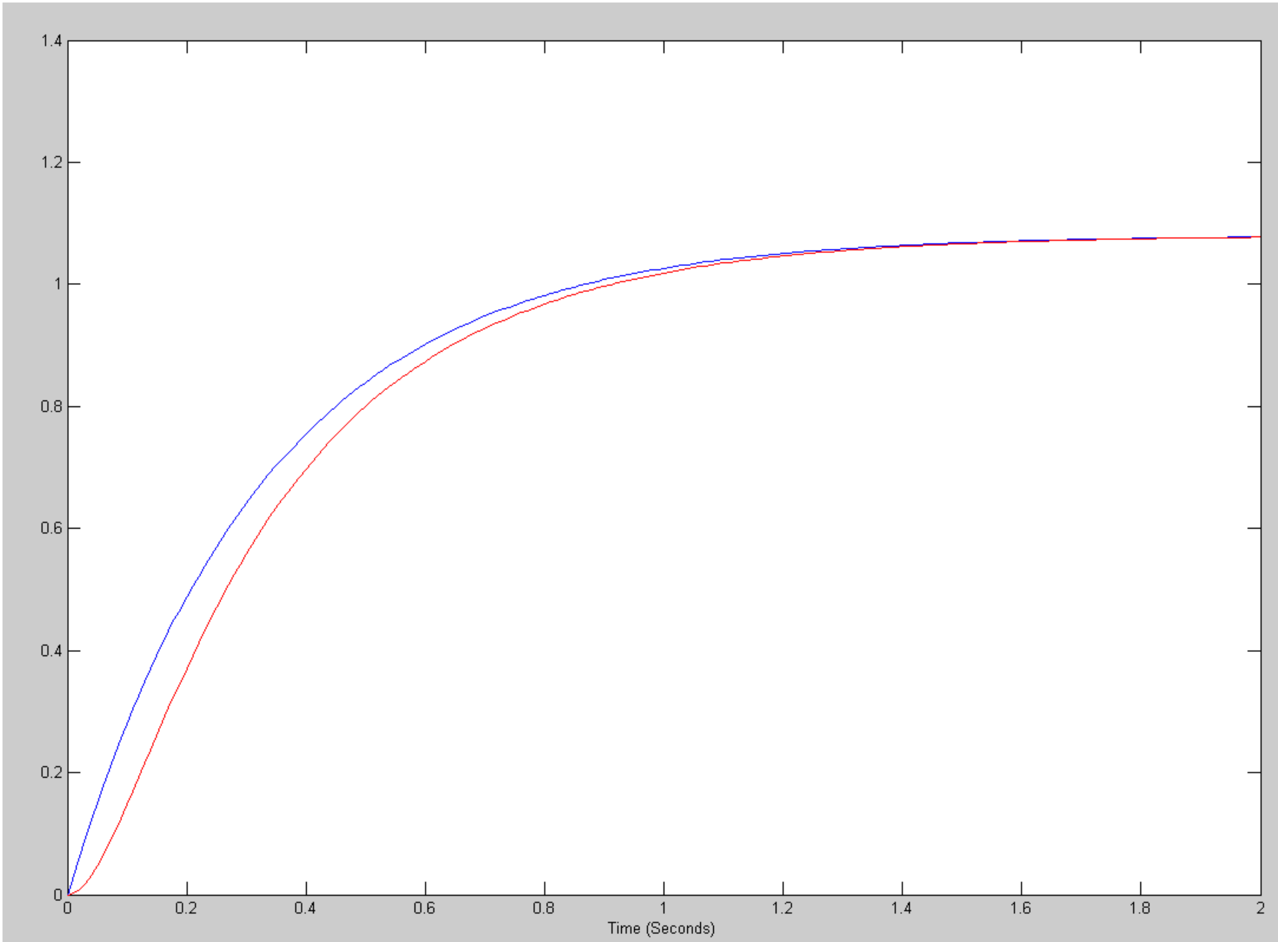

Red: 3rd-Order Sytem. Blue: 1st-Order Approximation