ECE 463/663 - Homework #5

Full State Feedback. Due Wednesday, February 22nd
Please submit as a hard copy, email to jacob.glower@ndsu.edu, or submit on BlackBoard

1) Write a Matlab m-file which is passed

- The system dynamics (A, B),
- The desired pole locations (P)

and then returns the feedback gains, Kx, so that roots(A - B Kx) = P

```
function [ Kx ] = ppl( A, B, P0)

N = length(A);

T1 = [];
for i=1:N
   T1 = [T1, (A^(i-1))*B];
end

P = poly(eig(A));
T2 = [];
for i=1:N
    T2 = [T2; zeros(1,i-1), P(1:N-i+1)];
end

T3 = zeros(N,N);
for i=1:N
    T3(i, N+1-i) = 1;
end

T = T1*T2*T3;

Pd = poly(P0);

dP = Pd - P;

Flip = [N+1:-1:2]';
Kz = dP(Flip);
Kx = Kz*inv(T);

end
```

Check:

```
>> A = rand(4,4);
>> B = rand(4,1);
>> Kx = ppl(A, B, [-1,-2,-3,-4])

Kx =    9.0220 -289.7033  -41.4816  318.4603

>> eig(A - B*Kx)

   -4.0000
   -1.0000
   -3.0000
   -2.0000
```

Problems 2-4) Assume the following dynamic system:

$$sX = \begin{bmatrix} -10.2 & 5 & 0 & 0 & 0 \\ 5 & -10.2 & 5 & 0 & 0 \\ 0 & 5 & -10.2 & 5 & 0 \\ 0 & 0 & 5 & -10.2 & 5 \\ 0 & 0 & 0 & 5 & -5.2 \end{bmatrix} X + \begin{bmatrix} 5 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix} X$$

2) (20 points) Find the feedback control law of the form

$$U = K_r R - K_x X$$

so that

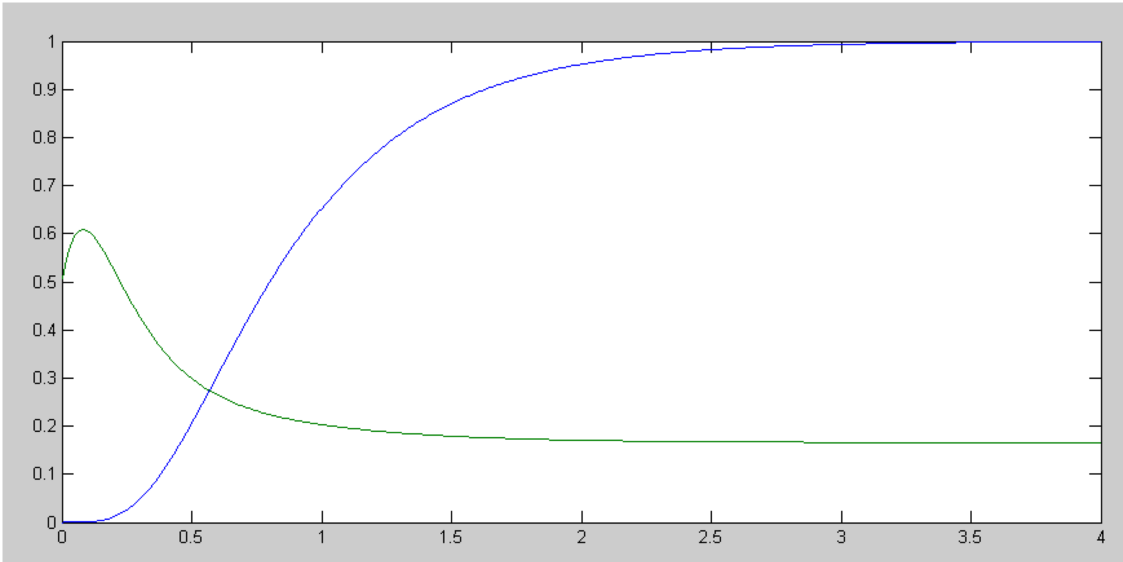
- The DC gain is 1.000 and
- The closed-loop poles are at {-2, -8, -9, -10, -11}

Plot

- The resulting closed-loop step reponse, and
- The resulting input, U

```
>> A = [-10.2,5,0,0,0;5,-10.2,5,0,0;0,5,-10.2,5,0;0,0,5,-10.2,5;0,0,0,5,-5.2]

  -10.2000    5.0000         0         0         0
    5.0000  -10.2000    5.0000         0         0
         0    5.0000  -10.2000    5.0000         0
         0         0    5.0000  -10.2000    5.0000
         0         0         0    5.0000   -5.2000

>> B = [5;0;0;0;0];
>> C = [0,0,0,0,1];
>> Kx = ppl(A, B, [-2,-8,-9,-10,-11])

Kx =   -1.2000    3.7360   -2.8758    3.9676   -0.4432

>> eig(A - B*Kx)

  -11.0000
  -10.0000
   -9.0000
   -8.0000
   -2.0000

>> DC = -C*inv(A-B*Kx)*B

DC =    0.1973

>> Kr = 1/DC

Kr =    5.0688
```

```
>> Gcl = ss(A-B*Kx, B*Kr, [C ; -Kx], [D;Kr]);
>> y = step(Gcl,t);
>> plot(t,y * diag([1,1/10]))
>> xlabel('Seconds');
```



Step response to the output (blue) and input/10 (green)

3) (20 points) Repeat problem #2 but find Kx and Kr so that

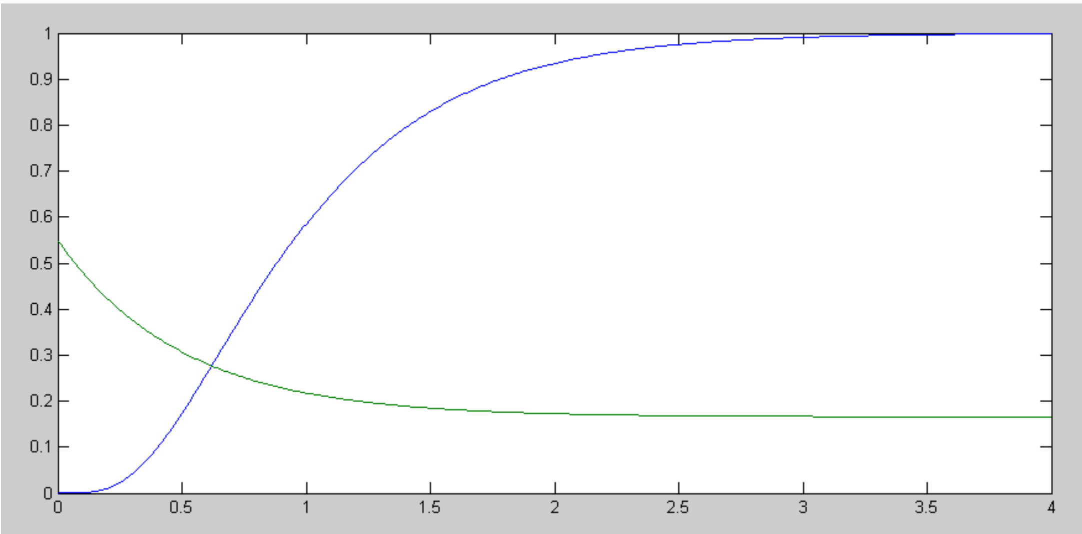
- The DC gain is 1.000 and
- The closed-loop dominant pole is at s = -2 and the other four poles don't move (the are the same as the fast four poles of the open-loop system (eigenvalues of A)

Plot

- The resulting closed-loop step reponse, and
- The resulting input, U

In Matlab

```
>> P = eig(A)

  -18.6125   -14.3542   -8.7769   -3.6514   -0.6051

>> P(5) = -2

  -18.6125  -14.3542   -8.7769   -3.6514   -2.0000

>> Kx = ppl(A, B, P)

Kx =     0.2790     0.5354     0.7484     0.9008     0.9802

>> DC = -C*inv(A-B*Kx)*B

DC =     0.1825

>> Kr = 1/DC

Kr =     5.4797

>> Gcl = ss(A-B*Kx, B*Kr, [C ; -Kx], [D;Kr]);
>> y = step(Gcl,t);
>> plot(t,y * diag([1,0.1]))
```



Step response to the output (blue) and input/10 (green)
Note that the output is about the same as before, but
The input is much smaller

4) (20 points) Repeat problem #2 but find Kx and Kr so that

- The DC gain is 1.000
- The 2% settling time is 2 seconds, and
- There is 10% overshoot for a step input.

Plot

- The resulting closed-loop step reponse, and
- The resulting input, U

```
>> P = eig(A)'

P =  -18.6125  -14.3542   -8.7769   -3.6514   -0.6051

>> P(4)  = -2+j*2.73;
>> P(5)  = -2-j*2.73;
```

`P = -18.6125     -14.3542      -8.7769      -2.0000 + 2.7300i   -2.0000 - 2.7300i`

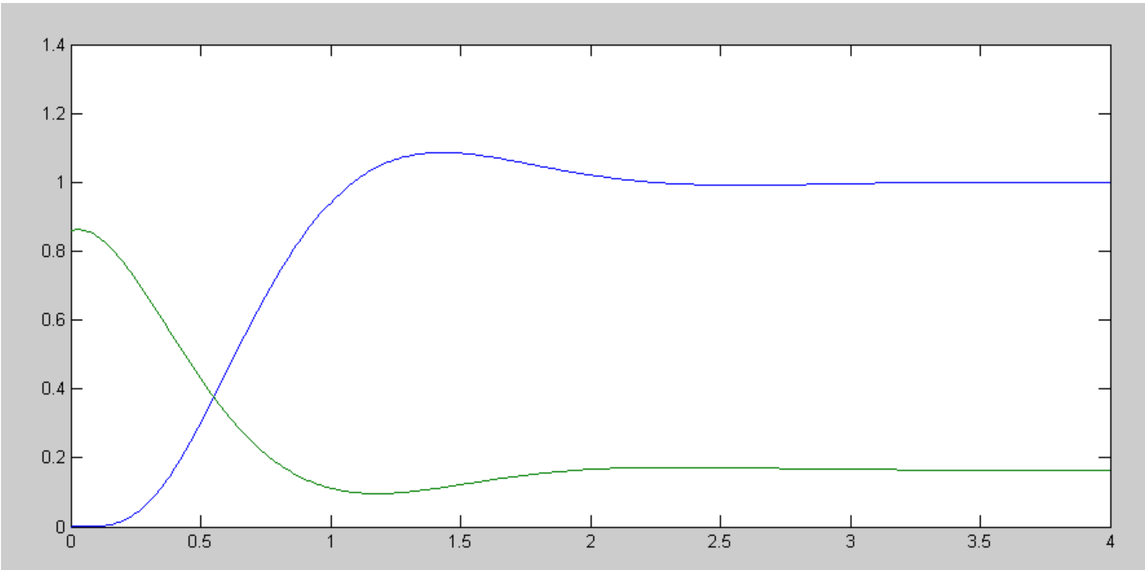
```
>> Kx = ppl(A, B, P)
```

`Kx =   -0.0513    0.3088    1.1771    2.2414    2.9724`

```
>> DC = -C*inv(A-B*Kx)*B

DC =    0.1164

>> Kr = 1/DC
```

`Kr =    8.5938`

```
>> Gcl = ss(A-B*Kx, B*Kr, [C ; -Kx], [D;Kr]);
>> y = step(Gcl,t);
>> plot(t,y)
>> plot(t,y * [1;0.1])
>> plot(t,y * diag([1;0.1]))
>>
```



Step response to the output (blue) and input/10 (green)
Note: With full-state feedback, you can place the closed-loop poles anywhere