ECE 463/663 - Homework #6

Pole Placement. Due Monday, February 28th

Problem 1) (30pt) Use the dynamics of a Cart and Pendulum System from homework set #4:

$$s\begin{bmatrix} x \\ \theta \\ sx \\ s\theta \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -19.6 & 0 & 0 \\ 0 & 19.6 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta \\ sx \\ s\theta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.6667 \\ -0.4444 \end{bmatrix}F$$

(10pt) Design a feedback control law of the form

U = Kr * R - Kx * X

so that the closed-loop system has

- A 2% settling time of 10 seconds, and
- 5% overshoot for a step input

Translation:

- Place the closed loop dominant pole at s = -0.4 (10 second settling time)
- With a damping ration of 0.6897 (5% overshoot)
- s = -0.4 + j0.42

```
>> A = [0,0,1,0;0,0,0,1;0,-19.6,0,0;0,19.6,0,0]

         0         0    1.0000         0
         0         0         0    1.0000
         0  -19.6000         0         0
         0   19.6000         0         0


>> B = [0;0;0.6667;-0.4444]

         0
         0
    0.6667
   -0.4444

>> rank([B, A*B, A*A*B, A*A*A*B])

     4

>> P = [-0.4+j*0.42,-0.4-j*0.42,-2,-3]
>> Kx = ppl(A, B, P)
```

Kx =   -0.4632 -68.0586   -1.4877 -15.2832

```
>> C = [1,0,0,0];
>> D = 0;
>> DC = -C*inv(A - B*Kx)*B;
>> Kr = 1/DC
```

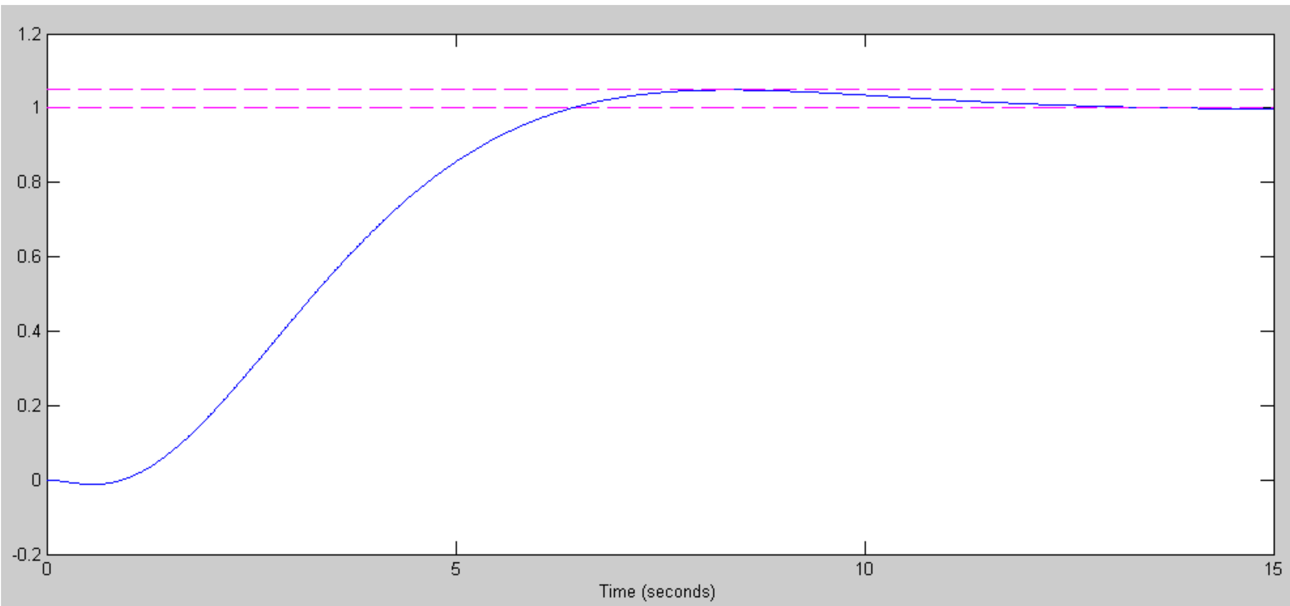
Kr =   -0.4632

(10pt) Check the step response of the linear system in Matlab

```
>> Gcl = ss(A-B*Kx, B*Kr, C, D);
>> t = [0:0.01:15]';
>> y = step(Gcl,t);
>> plot(t,y, t, t*0+1,'m--', t, t*0+1.05,'m--')
>> xlabel('Time (seconds)')
>> max(y)

ans =    1.0490


>> xlabel('Time (seconds)');
>>
```

The design is for 5% overshoot

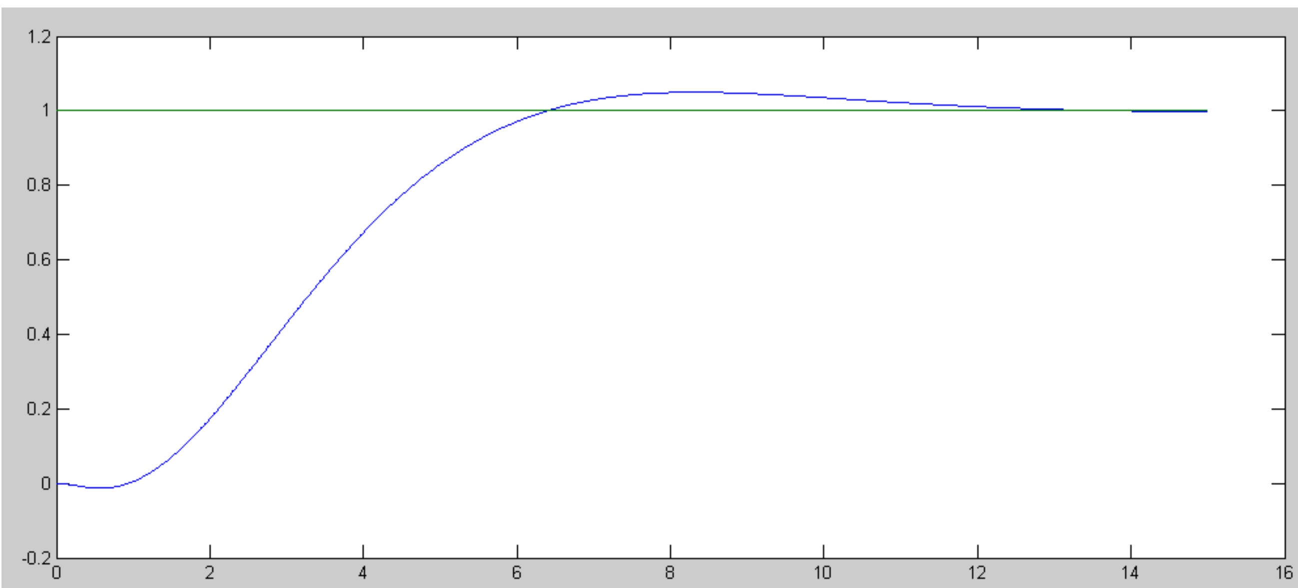
The actual overshoot is 4.90% (pretty close)



Step response of linear model of cart & pendulum: 5% overshoot

(10pt) Check the step response of the nonlinear system

```
% Cart and Pendulum ( Sp23 version)
X = [0; 0 ; 0 ; 0];
dX = zeros(4,1);
Ref = 1;
dt = 0.01;
t = 0;
Kx = [-0.4632  -68.0586   -1.4877  -15.2832];
Kr = -0.4632;

y = [];
while(t < 15)
 Ref = 1;
 U = Kr*Ref - Kx*X;
 dX = CartDynamics(X, U);
 X = X + dX * dt;
 t = t + dt;

 CartDisplay(X, Ref);
 y = [y ; X(1), Ref];
end

t = [1:length(y)]' * dt;
plot(t,y);
max(y)

ans =      1.0498                <<<<  4.98% overshoot
same overshoot as the linear model predicts
```



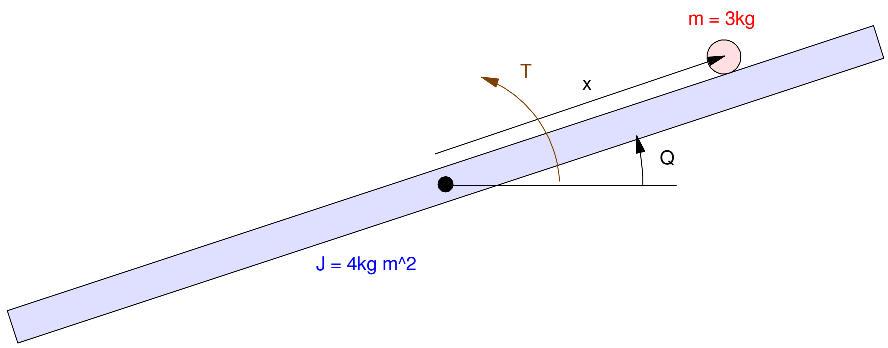
Step response of nonlinear model of cart & pendulum system. 5% overshoot

Problem 2) (30pt) Use the dynamics for the Ball and Beam system from homework set #4.

$$s\begin{bmatrix} r \\ \theta \\ sr \\ s\theta \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -7 & 0 & 0 \\ -7.84 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} r \\ \theta \\ sr \\ s\theta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.4 \end{bmatrix} T$$

(10pt) Design a feedback control law so that the closed-loop system has

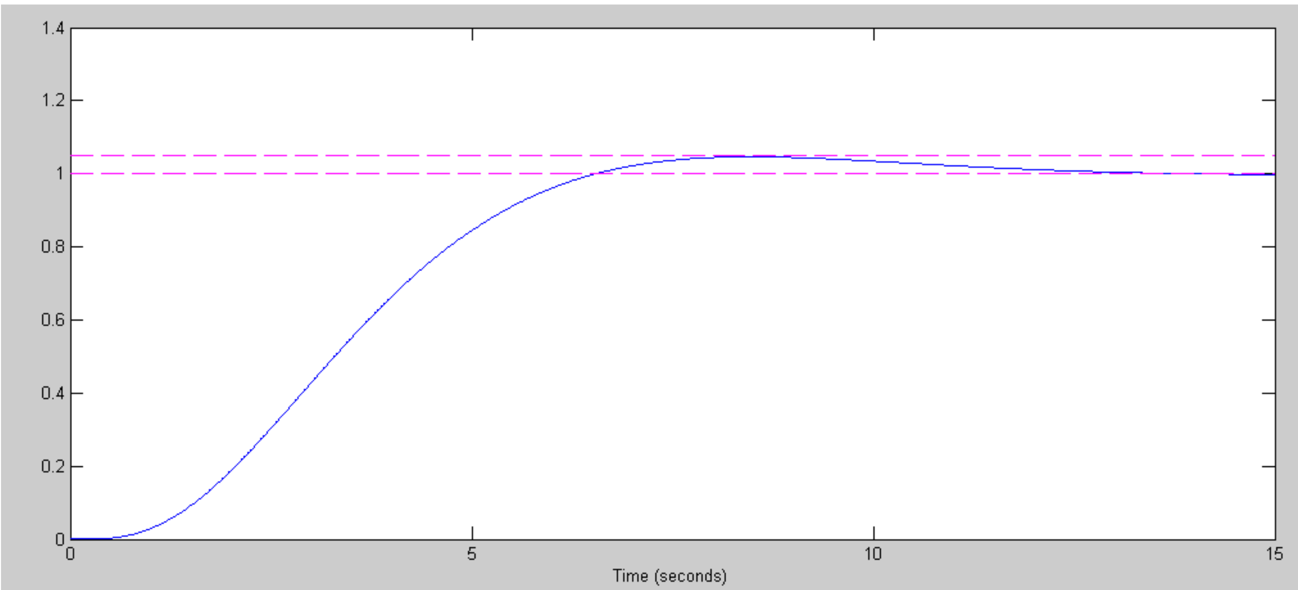
- A 2% settling time of 10 seconds, and
- 5% overshoot for a step input




```
>> A = [0,0,1,0;0,0,0,1;0,-7,0,0;-7.84,0,0,0]

         0         0    1.0000         0
         0         0         0    1.0000
         0   -7.0000         0         0
   -7.8400         0         0         0

>> B = [0;0;0;0.4]

         0
         0
         0
    0.4000

>> P = [-0.4+j*0.42,-0.4-j*0.42,-2,-3]
>> Kx = ppl(A, B, P)

Kx =  -20.3209   25.8410   -2.3150   14.5000

>> C = [1,0,0,0];
>> D = 0;
>> DC = -C*inv(A - B*Kx)*B;
>> Kr = 1/DC

Kr =   -0.7209
```

(10pt) Check the step response of the linear system in Matlab

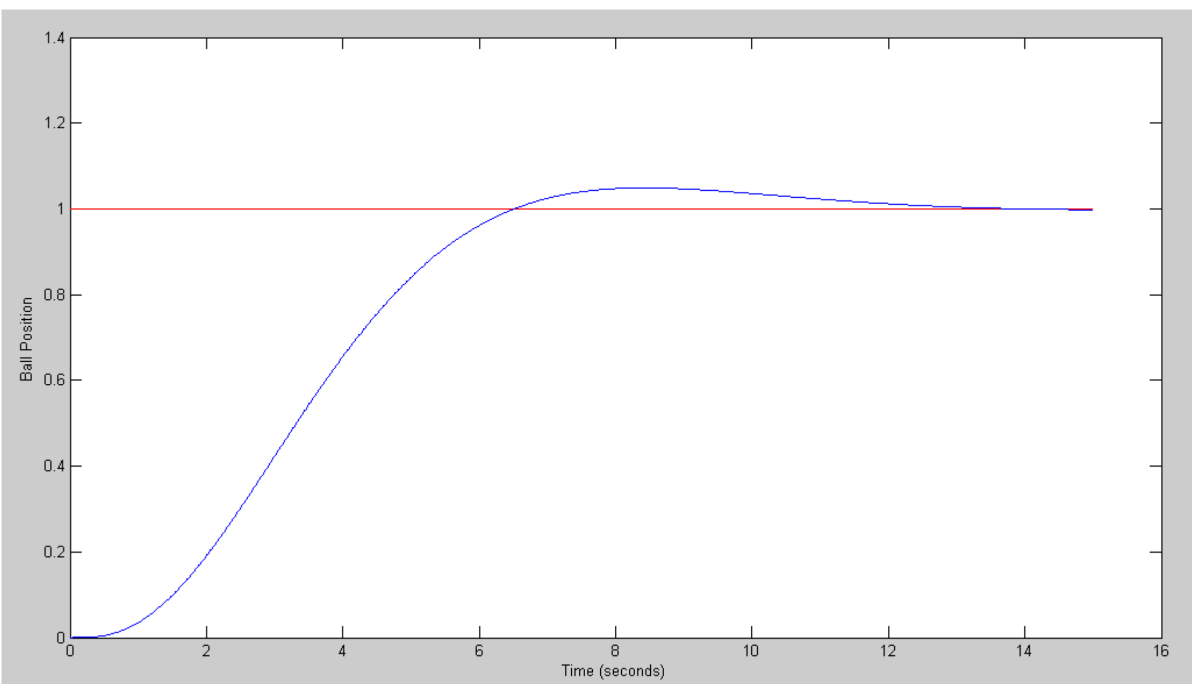
```
>> Gcl = ss(A-B*Kx, B*Kr, C, D);
>> t = [0:0.01:15]';
>> y = step(Gcl,t);
>> plot(t,y, t, t*0+1,'m--', t, t*0+1.05,'m--')
>> xlabel('Time (seconds)')
>> max(y)

ans =    1.0465      <<<<< 4.65% overshoot
```



Step Response of Linear Model of Ball & Beam System: 5% Overshoot

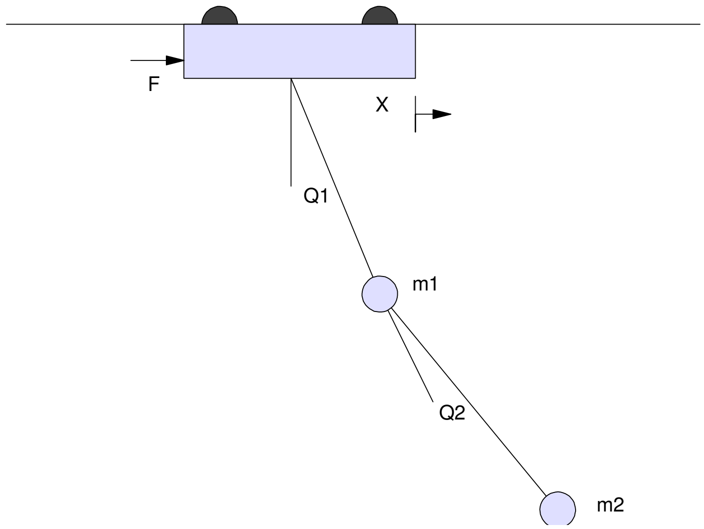
(10pt) Check the step response of the nonlinear system

```
% Ball & Beam System
% m = 1kg
% J = 0.2 kg m^2

X = [0, 0, 0, 0]';
dt = 0.01;
t = 0;
Kx = [ -20.5817   26.1453   -2.5324   14.5000 ];
Kr = -0.9817;
n = 0;
y = [];

while(t < 15)
  Ref = 1;
  U = Kr*Ref - Kx*X;
  dX = BeamDynamics(X, U);

  X = X + dX * dt;
  t = t + dt;

  y = [y ; Ref, X(1)];
  n = mod(n+1,5);
  if(n == 0)
     BeamDisplay(X, Ref);
     end
  end

t = [1:length(y)]' * dt;


plot(t,y(:,1),'r',t,y(:,2),'b');
xlabel('Time (seconds)');
ylabel('Ball Position');
```



Step Response of Ball & Beam System: 5% overshoot

Problem #3 (30pt): The dynamics of a double gantry (Gantry2) are



$$s\begin{bmatrix} x \\ \theta_1 \\ \theta_2 \\ \dot{x} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 2g & 0 & 0 & 0 & 0 \\ 0 & -3g & g & 0 & 0 & 0 \\ 0 & 3g & -3g & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta_1 \\ \theta_2 \\ \dot{x} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} F$$

(10pt) Design a feedback control law of the form

U = Kr * R - Kx * X

so that the closed-loop system has

- A 2% settling time of 10 seconds, and
- 5% overshoot for a step input

```
>> Z = zeros(3,3);
>> I = eye(3,3);
>> g = 9.8;
>> K = [0,2,0;0,-3,1;0,3,-3]*g;
>> A = [Z,I ; K,Z]

         0         0         0    1.0000         0         0
         0         0         0         0    1.0000         0
         0         0         0         0         0    1.0000
         0   19.6000         0         0         0         0
         0  -29.4000    9.8000         0         0         0
         0   29.4000  -29.4000         0         0         0
```

```
>> eig(A)

        0
        0
   0.0000 + 6.8099i
   0.0000 - 6.8099i
   0.0000 + 3.5250i
   0.0000 - 3.5250i

>> B = [0;0;0;1;-1;1]

     0
     0
     0
     1
    -1
     1

>> rank([B,A*B,A^2*B,A^3*B,A^4*B,A^5*B])

ans =     6

>> P = [-0.4+j*0.42,-0.4-j*0.42,-2,-3,-4,-5];
>> Kx = ppl(A, B, P)
```

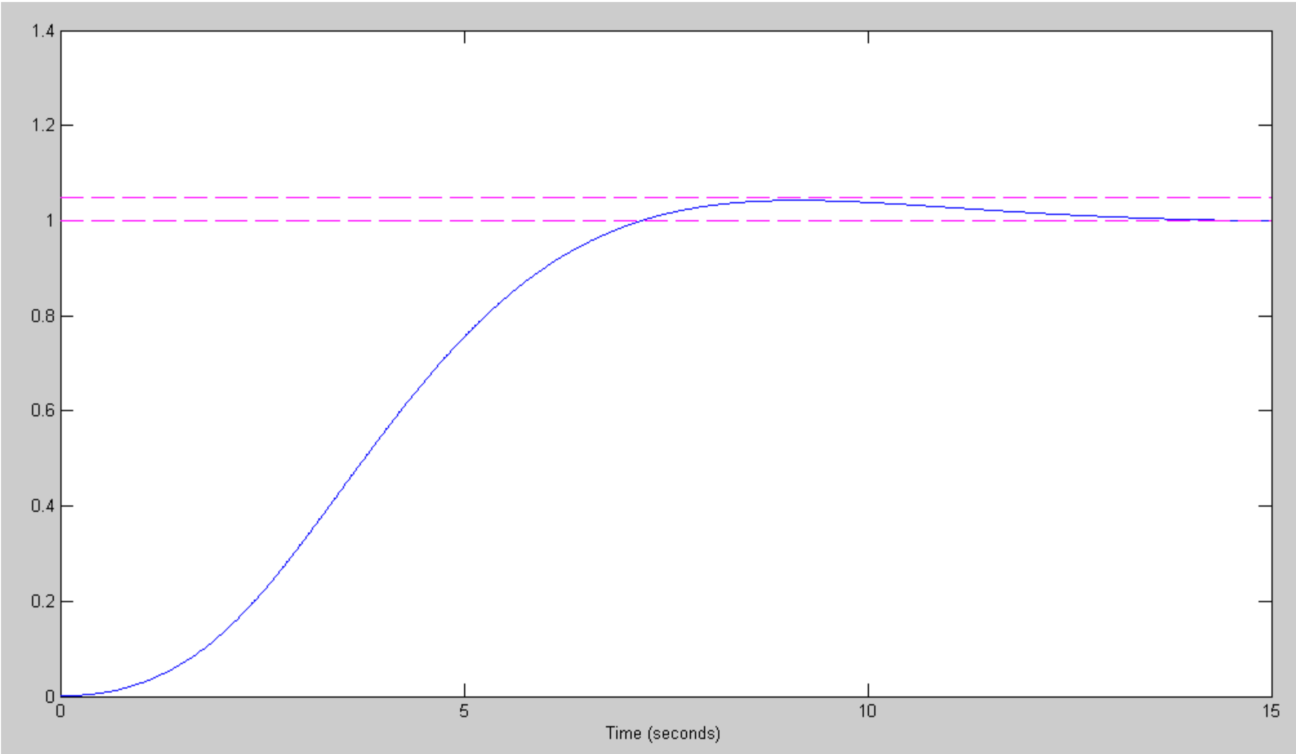
Kx =   0.2102   16.1936   39.7198   0.7695   -9.4564   4.5741

```
>> eig(A - B*Kx)

  -5.0000
  -4.0000
  -3.0000
  -2.0000
  -0.4000 + 0.4200i
  -0.4000 - 0.4200i

>> C = [1,0,0,0,0,0];
>> D = 0;
>> DC = -C*inv(A - B*Kx)*B;
>> Kr = 1/DC
```

Kr =   0.2102

(10pt) Determine the step response of the linear system in Matlab

```
>> Gcl = ss(A-B*Kx, B*Kr, C, D);
>> t = [0:0.01:15]';
>> y = step(Gcl,t);
>> plot(t,y,t,0*t+1,'m--',t,0*t+1.05,'m--')
>> xlabel('Time (seconds)');
>>
```



(10pt) Determine the step response of the nonlinear system

```
X = [0, 0, 0, 0, 0, 0]';
Ref = 1;
dt = 0.01;
U = 0;
t = 0;
Kx = [ 0.2102   16.1936   39.7198    0.7695   -9.4564    4.5741];
Kr = 0.2102;
y = [];
n = 0;
while(t < 15)
   Ref = 1;
   U = Kr*Ref - Kx*X;
   dX = Gantry2Dynamics(X, U);
   X = X + dX * dt;
   t = t + dt;
   n = mod(n+1,5);
   if(n == 0)
      Gantry2Display(X, Ref);
      plot([Ref, Ref],[-0.1,0.1],'b');
      end
   y = [y ; Ref, X(1), X(2), X(3)];
end

pause(2);
t = [1:length(y)]' * dt;
plot(t,y);
xlabel('Time (seconds)');
```