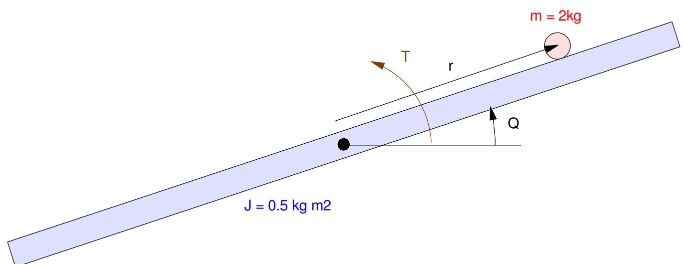
ECE 463/663 - Homework #7

Servo Compensators. Due Monday, March 6th
Please submit as a hard copy, email to jacob.glower@ndsu.edu, or submit on BlackBoard



The dynamics of a Ball and Beam System (homework set #4) with a disturbance are

$$s\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -7 & 0 & 0 \\ -7.84 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.4 \end{bmatrix}T + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.4 \end{bmatrix}d$$

Full-State Feedback with Constant Disturbances

1) For the nonlinear simulation, use the feedback control law you computed in homework #6

- With R = 1 and the mass of the ball = 2.0kg (same result you got for homework #6), and
- With R = 1 and the mass of the ball decreased to 1.5kg

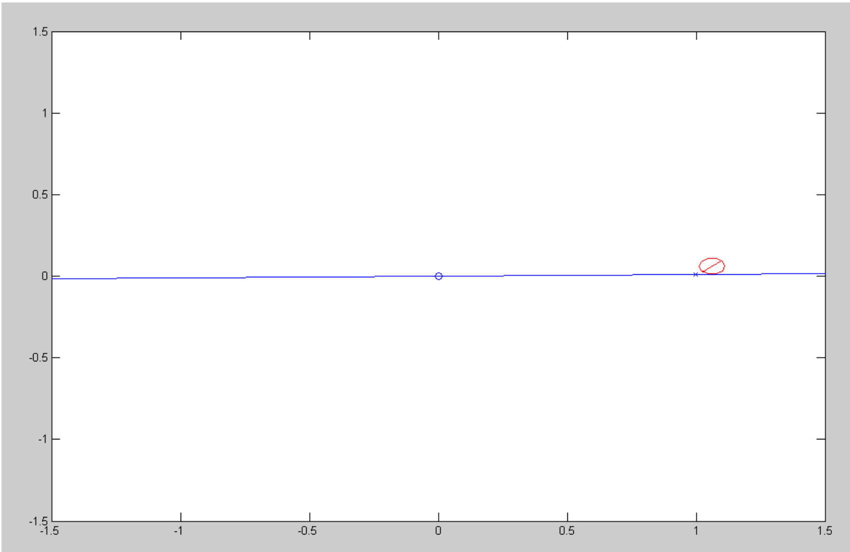
(i.e. a constant disturbance on the system due to a different mass of the ball)

Start with finding Kx and Kr:

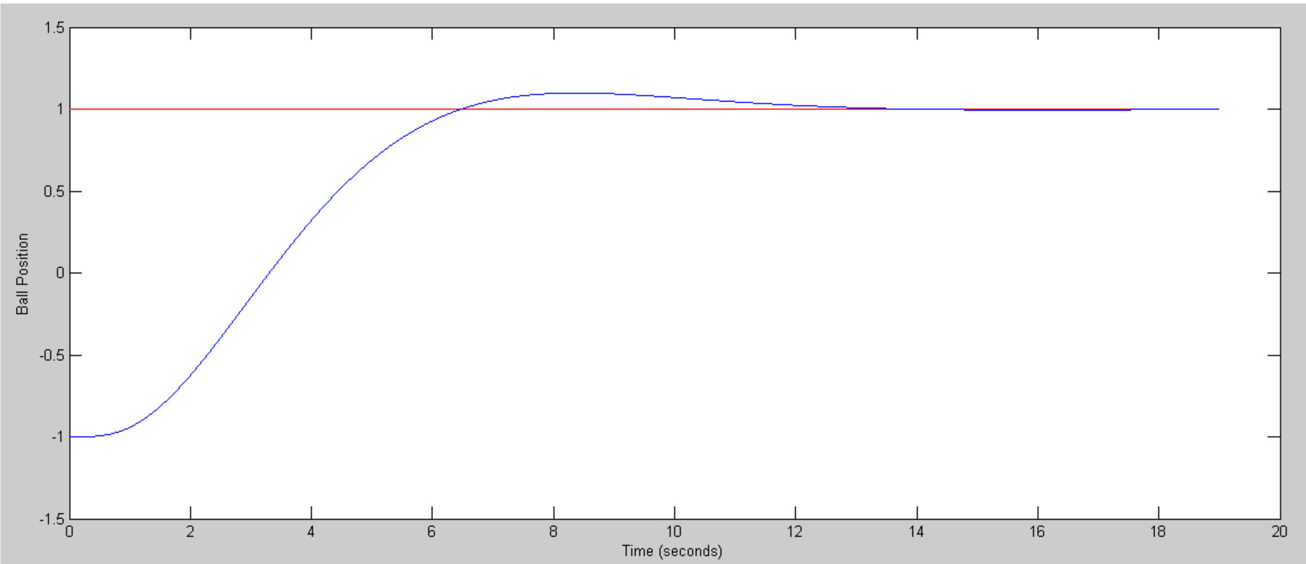
```
>> A = [0,0,1,0;0,0,0,1;0,-7,0,0;-7.84,0,0,0]

         0         0    1.0000         0
         0         0         0    1.0000
         0   -7.0000         0         0
   -7.8400         0         0         0

>> B = [0;0;0;0.4]

         0
         0
         0
    0.4000

>> C = [1,0,0,0];
>> Kx = ppl(A, B, [-0.4+j*0.42,-0.4-j*0.42,-2,-3])

Kx =  -20.3209   25.8410   -2.3150   14.5000
```

```
>> eig(A - B*Kx)

  -3.0000
  -2.0000
  -0.4000 + 0.4200i
  -0.4000 - 0.4200i

>> DC = -C*inv(A - B*Kx)*B
>> Kr = 1/DC

Kr =   -0.7209
```

Now apply that to the ball & beam system with m = 2.0kg

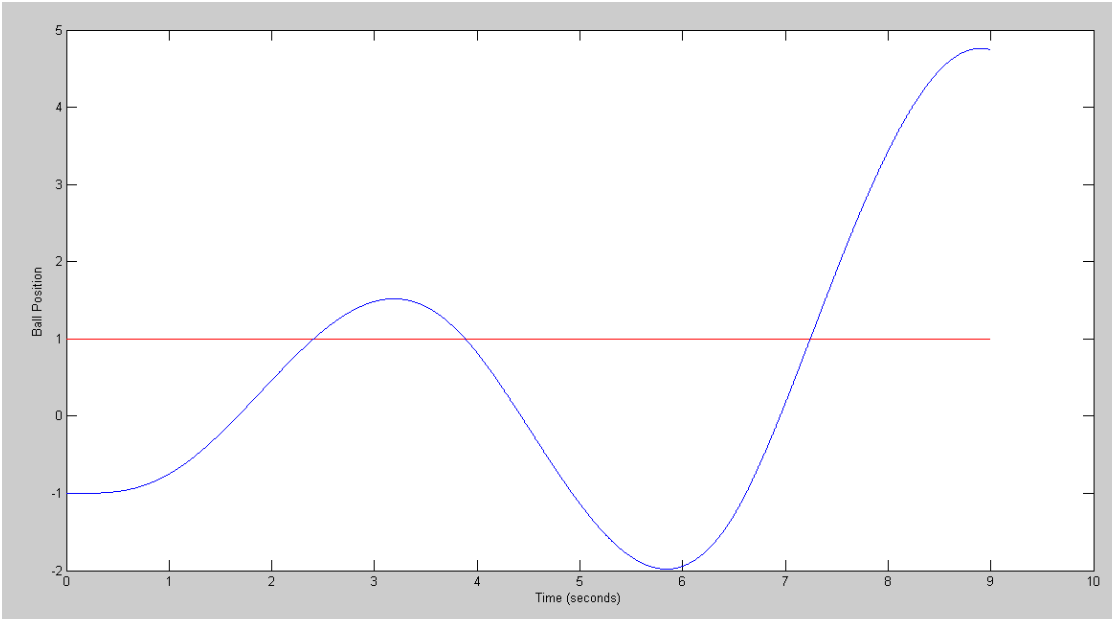


m = 2.0kg. Tracks the set point



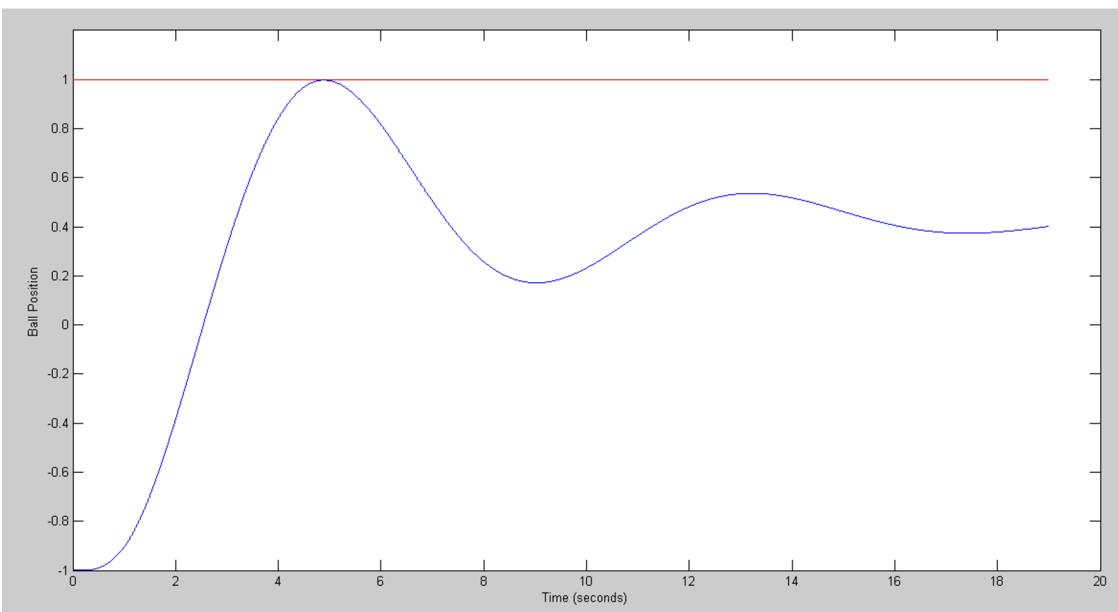
Step Response with m = 2.0kg

Next, repeat with m = 1.5kg



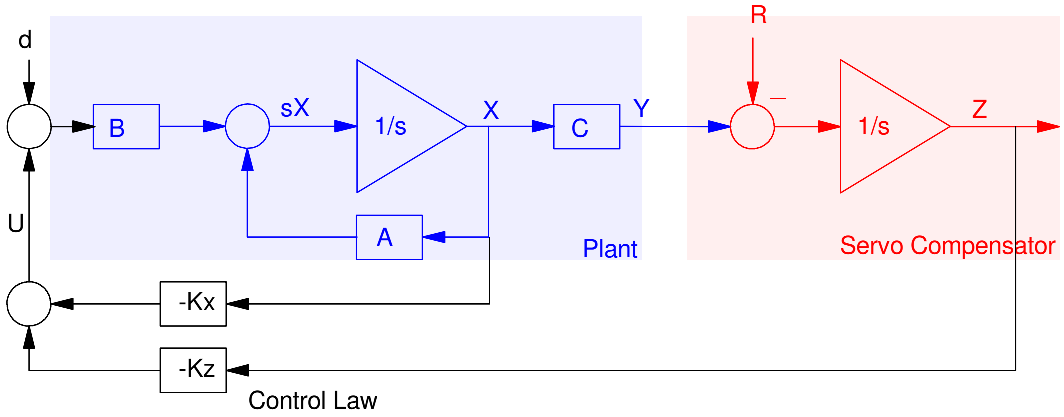
Step Response with ,m = 1.5kg (unstable)

The closed-loop system is unstable. Changing the mass to 1.9kg, it misses the target (doesn't go to 1.00)



Step Response with m = 1.9kg (misses the target)

Servo Compensators with Constant Set-Points



2) Assume a constant disturbance and/or a constant set point. Design a feedback control law that results in

- The ability to track a constant set point (R = constant)
- The ability to reject a constant disturbance (d = constant),
- A 2% settling time of 10 seconds, and
- No overshoot for a step input.

First, form the augmented system:

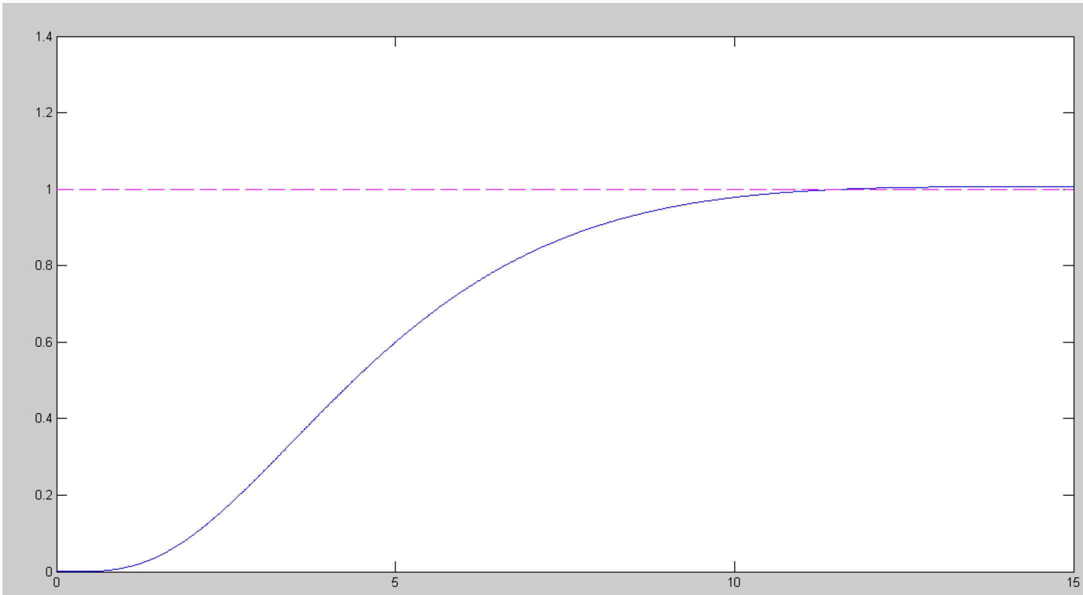
```
>> A5 = [A, zeros(4,1) ; C, 0]

         0         0    1.0000         0         0
         0         0         0    1.0000         0
         0   -7.0000         0         0         0
   -7.8400         0         0         0         0
    1.0000         0         0         0         0

>> B5u = [B;0]

         0
         0
         0
    0.4000
         0

>> B5r = [zeros(4,1);-1]

     0
     0
     0
     0
    -1
```

Now find the feedback gains to meet the requirements (a little trial and error)

```
>> K5 = ppl(A5, B5u, [-0.4+j*0.25,-0.4-j*0.25,-2,-3,-4])

K5 =  -28.5232   83.5562  -16.7152   24.5000   -1.9071
```

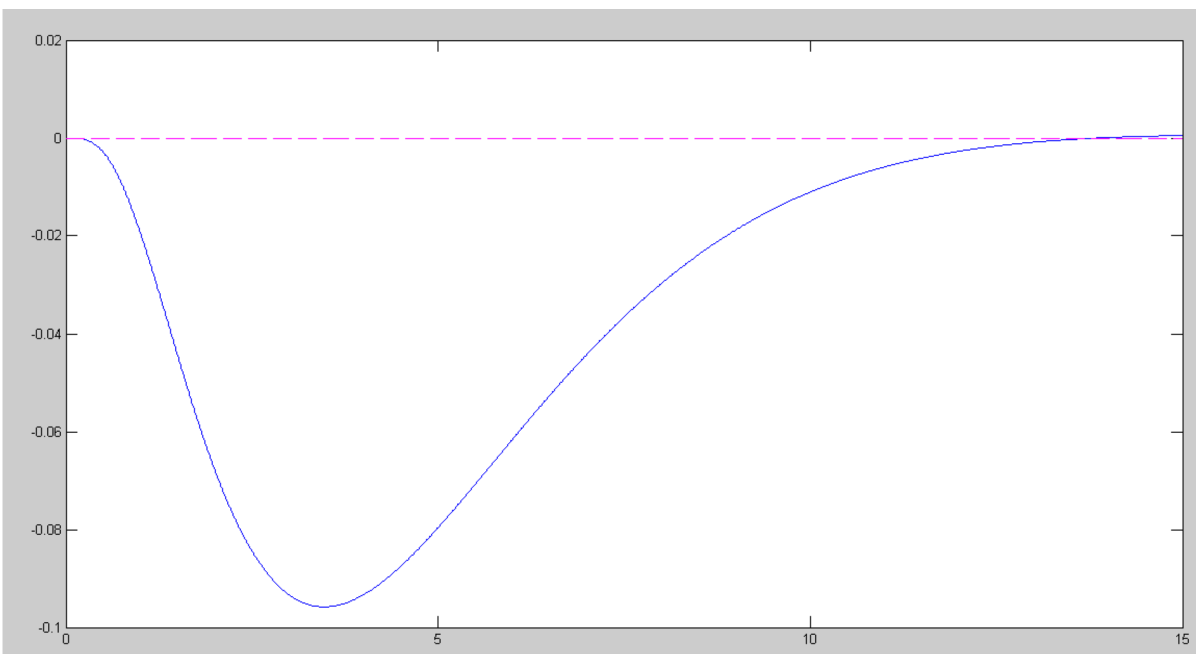
3) For the linear system, plot the step response with respect to R and d

```
>> G5 = ss(A5 - B5u*K5, B5r, C5, D5);
>> y = step(G5,t);
>> plot(t,y,'b',t,0*y+1,'m--')
```



Step Response with respect to R (tracks a constant set point)

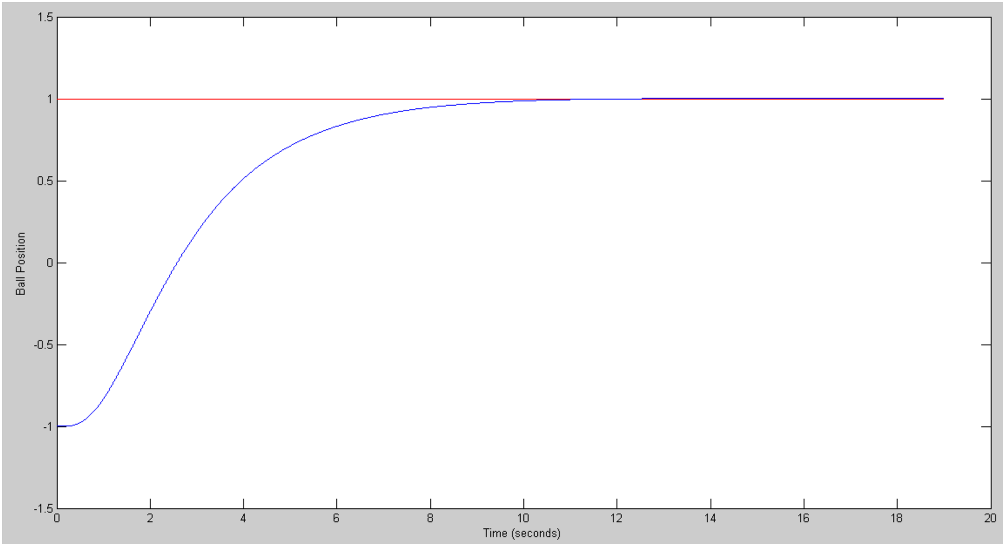
```
>> G5 = ss(A5 - B5u*K5, B5u, C5, D5);
>> y = step(G5,t);
>> plot(t,y,'b',t,0*y,'m--')
```



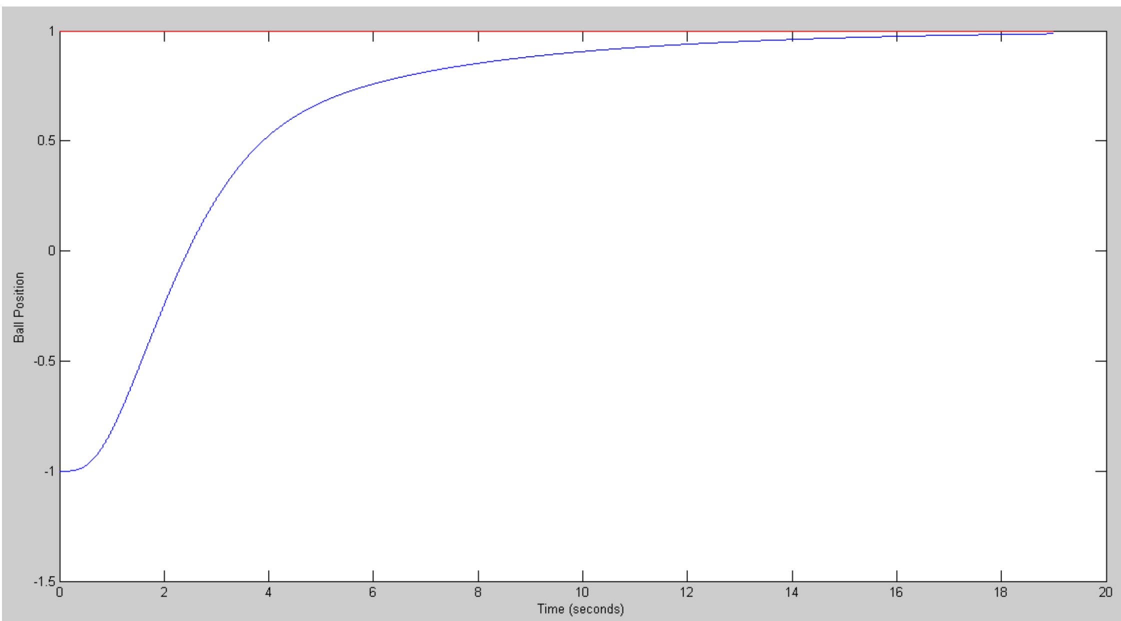
Step Response with Respect to d (rejects a constant disturbance)

4) Implement your control law on the nonlinear ball and beam system

- With R = 1 and the mass of the ball being 2.0kg, and
- With R = 1 and the mass of the ball being 1.5kg

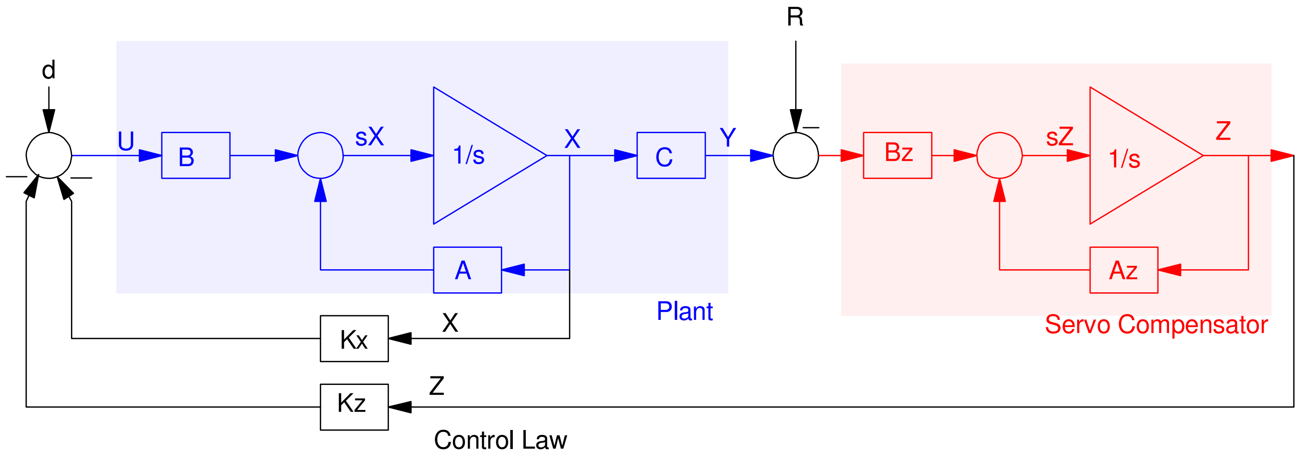


Step Response: m = 2.0kg (tracks a constant set point)



Step Response (m = 1.9kg)
(tracks a constant set point, rejects a constant disturbance)

Servo Compensators with Sinulsoidal Set-Points



5) Assume a 0.6 rad/sec disturbance and/or set point (R). Design a feedback control law that results in

- The ability to track a constant set point (R = sin(0.6t))
- The ability to reject a constant disturbance (d = sin(0.6t)),
- A 2% settling time of 10 seconds

Form the augmented system:

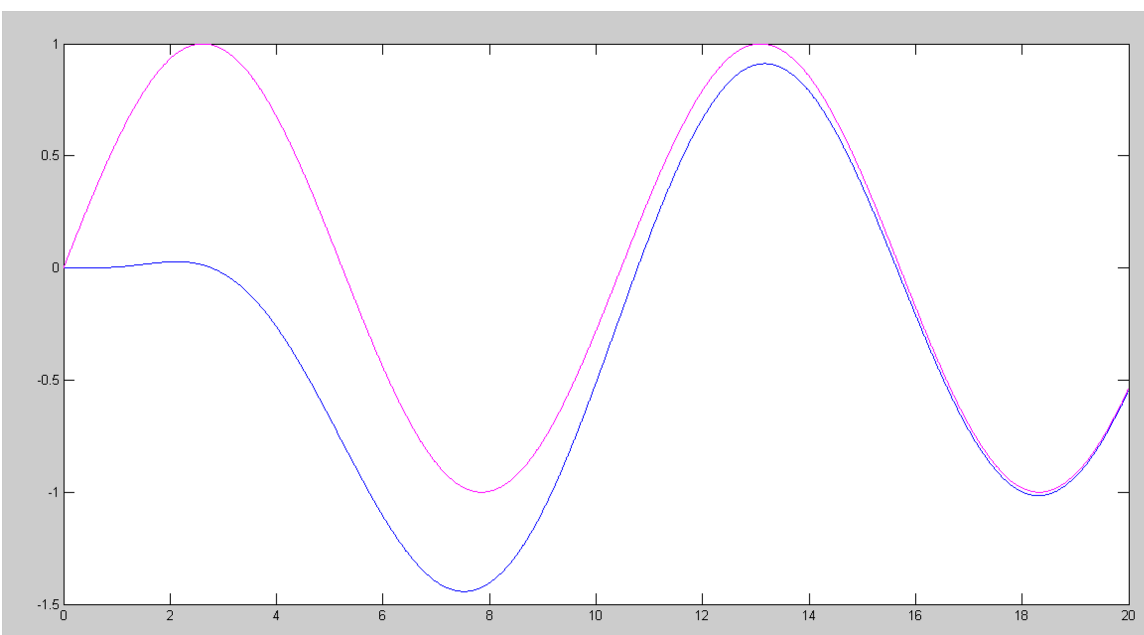
```
>> Az = [0,0.6;-0.6,0]

         0    0.6000
   -0.6000         0

>> Bz = [1;1]

     1
     1

>> A6 = [A, zeros(4,2) ; Bz*C,Az]

         0         0    1.0000         0         0         0
         0         0         0    1.0000         0         0
         0   -7.0000         0         0         0         0
   -7.8400         0         0         0         0         0
    1.0000         0         0         0         0    0.6000
    1.0000         0         0         0   -0.6000         0

>> B6u = [B ; 0*Bz]

         0
         0
         0
    0.4000
         0
         0
```

```
>> B6r = [0*B ; -Bz]

     0
     0
     0
     0
    -1
    -1

>> C6 = [1,0,0,0,0,0];
>> D6 = 0;
>> K6 = ppl(A6, B6u, [-0.4,-0.4,-2,-3,-4,-5])

K6 =  -99.9714  205.0000  -74.1829   37.0000   10.2072  -26.5871

>> Kx = K6(1:4)

Kx =  -99.9714  205.0000  -74.1829   37.0000

>> Kz = K6(5:6)

Kz =   10.2072  -26.5871
```

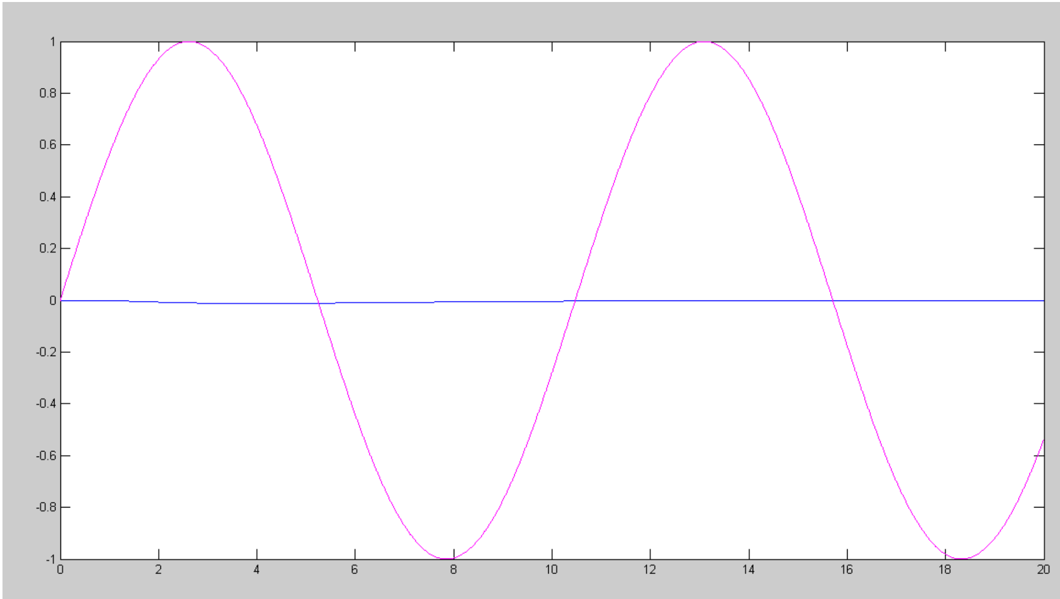
6) For the linear system, plot the response

- With R(t) = sin(0.6t), and
- With d(t) = sin(0.6t)

```
>> X0 = zeros(6,1);
>> t = [0:0.01:20]';
>> R = sin(0.6*t);

>> y = step3(A6-B6u*K6,B6r,C6,D6,t,X0,R);
>> plot(t,y,'b',t,R,'m')

>> y = step3(A6-B6u*K6,B6u,C6,D6,t,X0,R);
>> plot(t,y,'b',t,R,'m')
```



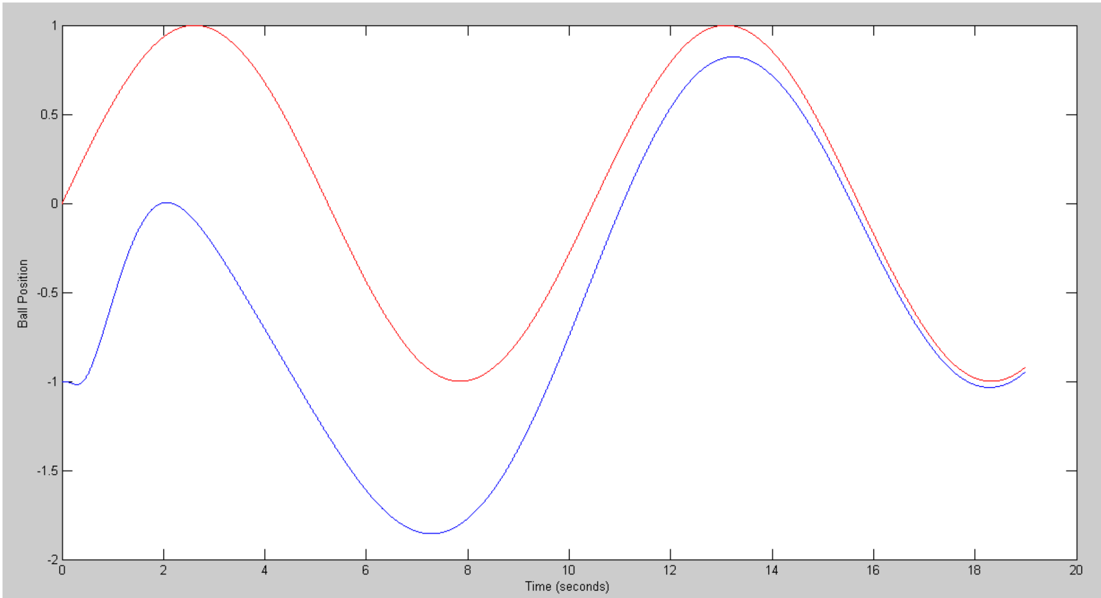
Linear System: Tracks a 0.6 rad/sec set point



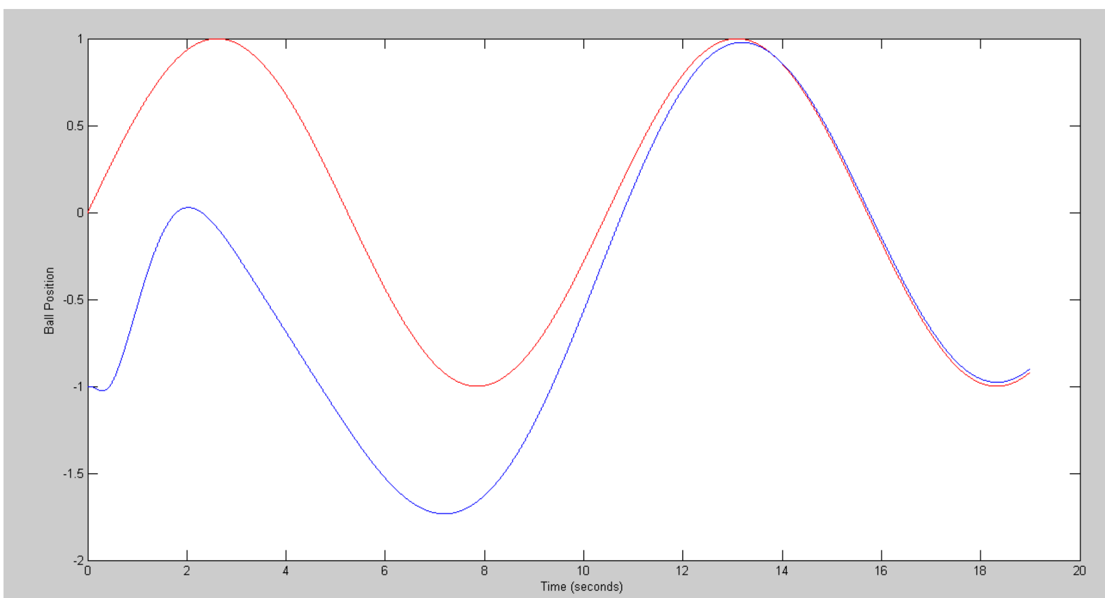
Linear System: Rejects a 0.6 rad/sec disturbance

7) Implement your control law on the nonlinear ball and beam system

- With R = sin(0.6t) and the mass of the ball being 2.0kg, and
- With R = sin(0.6t) and the mass of the ball being 1.5kg



Nonlinear System with m = 2.0kg
Tracks a 0.6 rad/sec set point



Nonlinear System with m = 1.5kg
Tracks a 0.6 rad/sec set point and rejects a 0.6 rad/sec disturbance

Final Code

```
% Ball & Beam System
% ECE 463 Homework Set #6
% Spring 2023

X = [-1, 0, 0, 0]';
dt = 0.002;
t = 0;

Kx = [ -99.9714  205.0000  -74.1829   37.0000];
Kz = [10.2072  -26.5871];
Z = zeros(2,1);
Az = [0,0.6;-0.6,0];
Bz = [1;1];

n = 0;
y = [];

while(t < 19)
   Ref =sin(0.6*t);
   U = -Kz*Z - Kx*X;
   dX = BeamDynamics(X, U);
   dZ = Az*Z + Bz*(X(1) - Ref);
   X = X + dX * dt;
   Z = Z + dZ * dt;
   t = t + dt;

   y = [y ; Ref, X(1)];
   n = mod(n+1,5);
   if(n == 0)
      BeamDisplay(X, Ref);
      end
   end

t = [1:length(y)]' * dt;

plot(t,y(:,1),'r',t,y(:,2),'b');
xlabel('Time (seconds)');
ylabel('Ball Position');
```