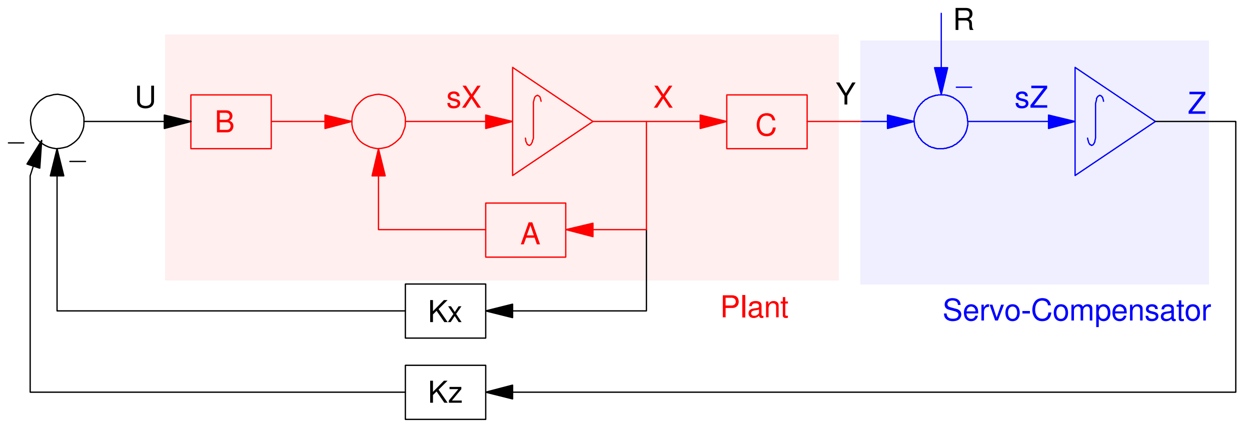
ECE 463/663 - Homework #10

LQG Control with Servo Compensators. Due Monday, April 17th
Please submit as a hard copy, email to jacob.glower@ndsu.edu, or submit on BlackBoard



Cart and Pendulum (HW #4): For the cart and pendulum system of homework #4

$$s\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -29.4 & 0 & 0 \\ 0 & 26.133 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ -0.667 \end{bmatrix} F$$

Use LQG methods to design a full-state feedback control law of the form

$$F = U = -K_z Z - K_x X$$

$$\dot{Z} = (x - R)$$

for the cart and pendulum system from homework #4 using LQG control so that

- You track constant setpoints,
- You reject constant disturbances,
- The 2% settling time is 8 seconds, and
- There is less than 10% overshoot for a step input.

1) Give the control law (Kx and Kz) and explain how you chose Q and R

Adjust Q until the response matches the desired response

- Increase the weighting on Z until it's fast enough: Q(5,5))
- Then increase the weighting on X to reduce the overshoot: Q(1,1)

Matlab Code

```
A = [0,0,1,0;0,0,0,1;0,-29.4,0,0;0,26.133,0,0];
B = [0;0;1;-0.667];
C = [1,0,0,0];

A5 = [A, zeros(4,1) ; C, 0];
B5 = [B;0];
C5 = [C, 0];
D5 = 0;
B5r = [0*B; -1];

Gd = tf(0.5,[1,1,0.5]);
t = [0:0.01:14]';
Yd = step(Gd,t);

K5 = lqr(A5, B5, diag([5,0,0,0,5]), 1);
G = ss(A5-B5*K5, B5r, C5, D5);
Y = step(G,t);

plot(t+0.5,Yd,'r',t,Y,'b');
```

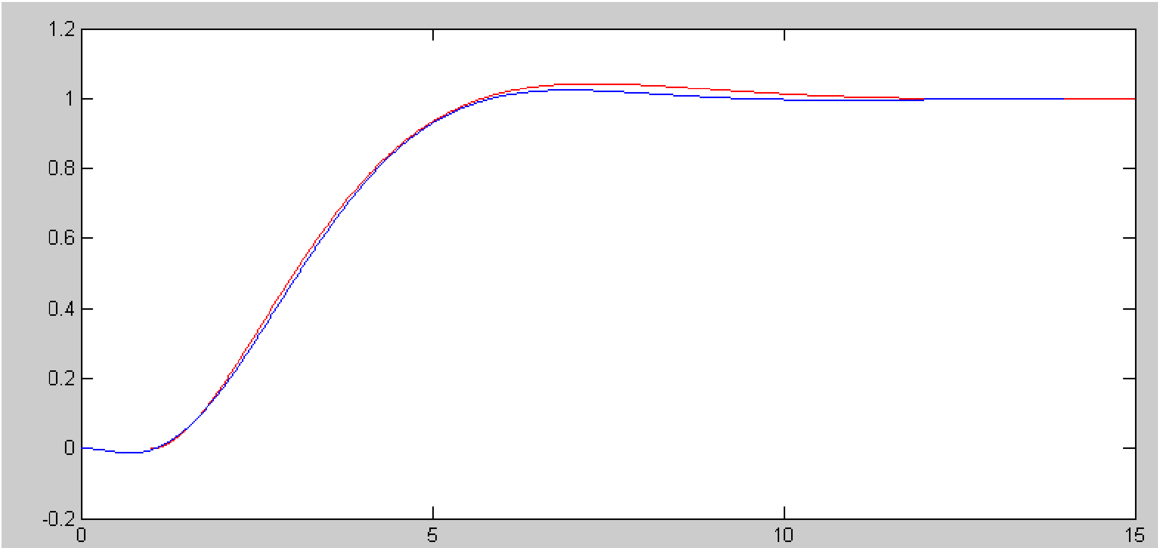
The resulting control gains are:

```
>> Kx = K5(1:4)

Kx =   -7.0346 -118.4194   -9.9473  -32.8984

>> Kz = K5(5)

Kz =   -2.2361
```

The 'optimal' closed-loop pole location is:

```
>> eig(A5 - B5*K5)

  -5.1110 + 0.1611i
  -5.1110 - 0.1611i
  -0.7156
  -0.5291 + 0.7068i
  -0.5291 - 0.7068i
```

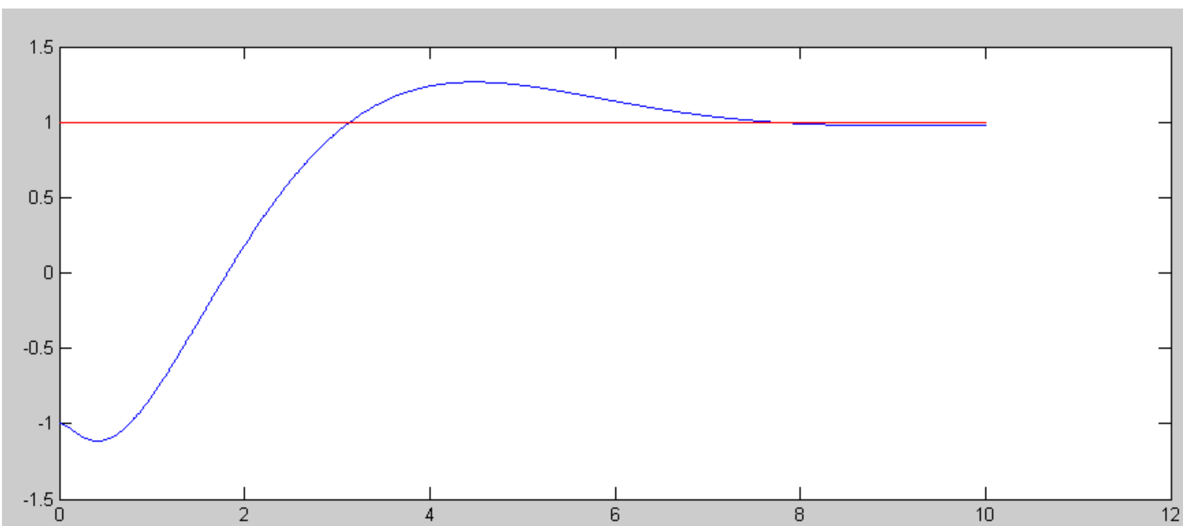
2) Plot the step response of the linear system



Desired Resposne (red) & Linear System's Response (blue)

3) Check your design with the nonlinear simulation of the cart and pendulum system.

Slightly more overshoot than expected (13.2%), but looks about the same



Matlab Code:

```
% Cart and Pendulum

X = [-1; 0 ; 0 ; 0];
Ref = 1;
dt = 0.01;
t = 0;
Kx = [-7.0346 -118.4194   -9.9473  -32.8984 ];
Kz = -2.2361;
Z = 0;

y = [];
while(t < 10)
 Ref = 1;
 U = - Kx*X - Kz*Z;

 dX = CartDynamics(X, U);
 dZ = X(1) - Ref;

 X = X + dX * dt;
 Z = Z + dZ * dt;
 t = t + dt;

 CartDisplay(X, X, Ref);
 y = [y ; X(1), Ref];
end

clf
t = [1:length(y)]' * dt;
plot(t,y(:,1),'b',t,y(:,2),'r');
```

Ball and Beam (HW #4): For the ball and beam system of homework #4

$$s\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -7 & 0 & 0 \\ -7.84 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.4 \end{bmatrix} T$$

Use LQG methods to design a full-state feedback control law of the form

$$T = U = -K_z Z - K_x X$$

$$\dot{Z} = (x - R)$$

for the ball and beam system from homework #6 using LQG control so that

- You track constant setpoints,
- You reject constant disturbances,
- The 2% settling time is 8 seconds, and
- There is less than 5% overshoot for a step input.

4) Give the control law (Kx and Kx) and explain how you chose Q and R

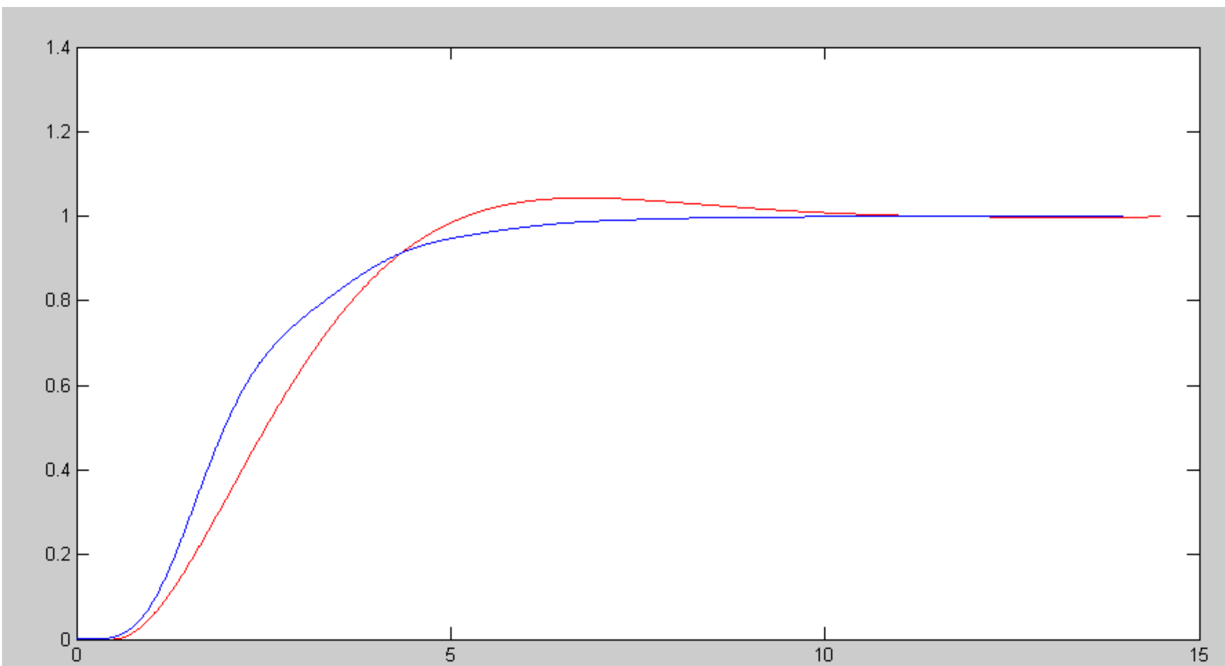
- Start with weighting Z: Q(5,5)
- Increase the weighting until it's fast enough
- Play with the ther gains to get rid of the oscillations: weighting q' seems to help

```
A = [0,0,1,0;0,0,0,1;0,-7,0,0;-7.84,0,0,0];
B = [0;0;0;0.4];
C = [1,0,0,0];

A5 = [A, zeros(4,1) ; C, 0];
B5 = [B;0];
C5 = [C, 0];
D5 = 0;
B5r = [0*B; -1];

Gd = tf(0.5,[1,1,0.5]);
t = [0:0.01:14]';
Yd = step(Gd,t);

K5 = lqr(A5, B5, diag([0,0,80,0,200]), 1);
G = ss(A5-B5*K5, B5r, C5, D5);
Y = step(G,t);

plot(t+0.5,Yd,'r',t,Y,'b');
```

The resulting gains and eigenvalues:

```
 Kx = K5(1:4)

Kx =  -53.1198   70.9530  -26.1422   18.8352

>> Kz = K5(5)

Kz =  -14.1421

>> eig(A5 - B5*K5)

  -0.8318 + 2.8809i
  -0.8318 - 2.8809i
  -3.4527
  -1.6401
  -0.7777

>>
```

5) Plot the step response of the linear system



6) Check your design with the nonlinear simulation of the cart and pendulum system.