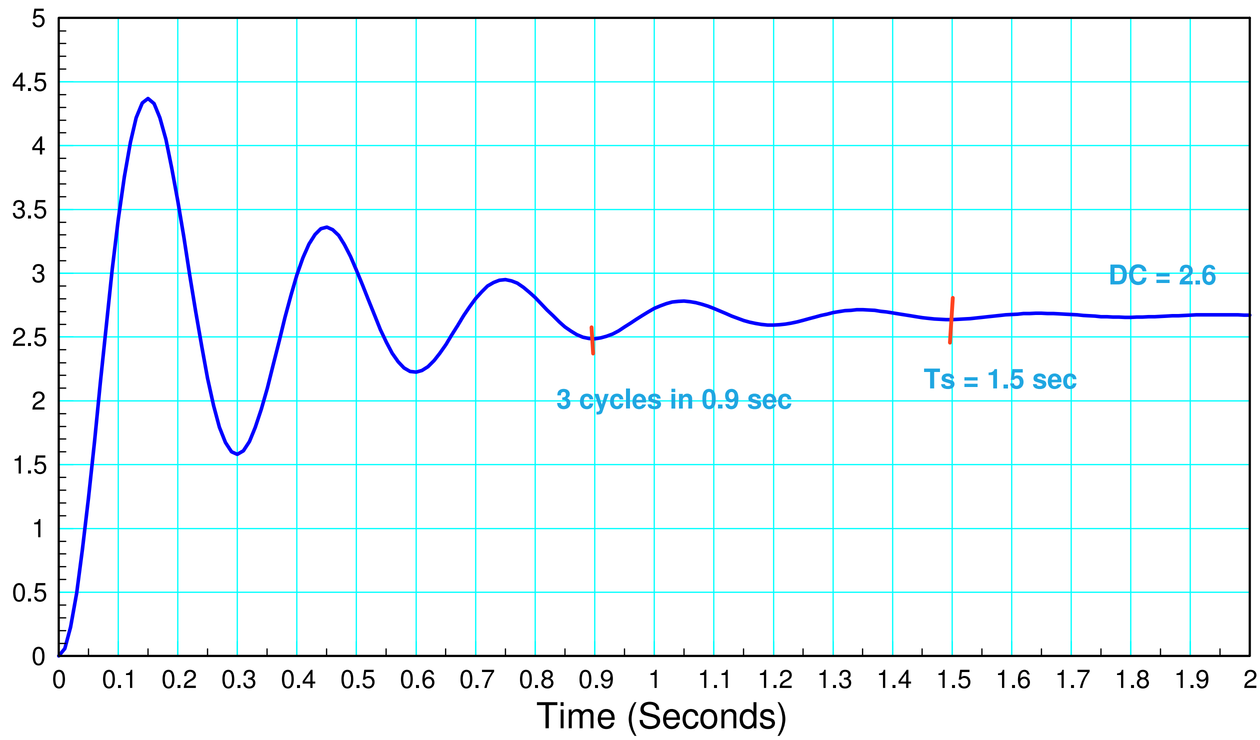
ECE 463/663: Test #1.   Name _______________

Spring 2023. Calculators allowed. Individual Effort

1) Find the transfer funciton for a system with the following step response



DC gain = 2.6

Frequency of oscillation

$$\omega_d = \left(\frac{3\text{ cycles}}{0.9\text{ sec}}\right)2\pi = 20.9$$

2% Settling Time = 1.5 sec (approx)

$$\sigma = \frac{4}{1.5} = 2.67$$

$$G(s) \approx \left(\frac{1185}{(s+2.67+j20.9)(s+2,67-j20.9)}\right) = \left(\frac{1185}{s^2+5.34+443.9}\right)$$

2) Determine a 2nd-order system which has approximately the same step response as the following system

$$Y = \left(\frac{10,000}{(s+3)(s+4)(s+12)(s+15)(s+22)}\right)X$$

Keep the two slowest poles

$$Y = \left(\frac{k}{(s+3)(s+4)}\right)X$$

Pick 'k' to match the DC gain

$$\left(\frac{10,000}{(s+3)(s+4)(s+12)(s+15)(s+22)}\right)_{s=0} = 0.2104$$

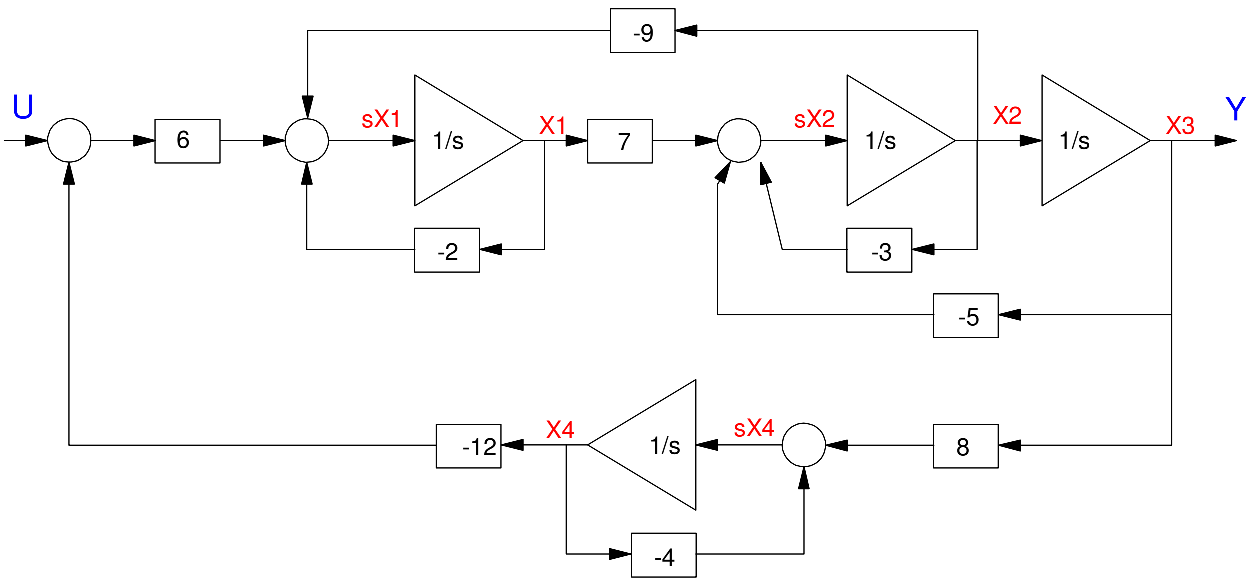
$$\left(\frac{k}{(s+3)(s+4)}\right)_{s=0} = 0.2104$$

Solving for k

k = 2.525

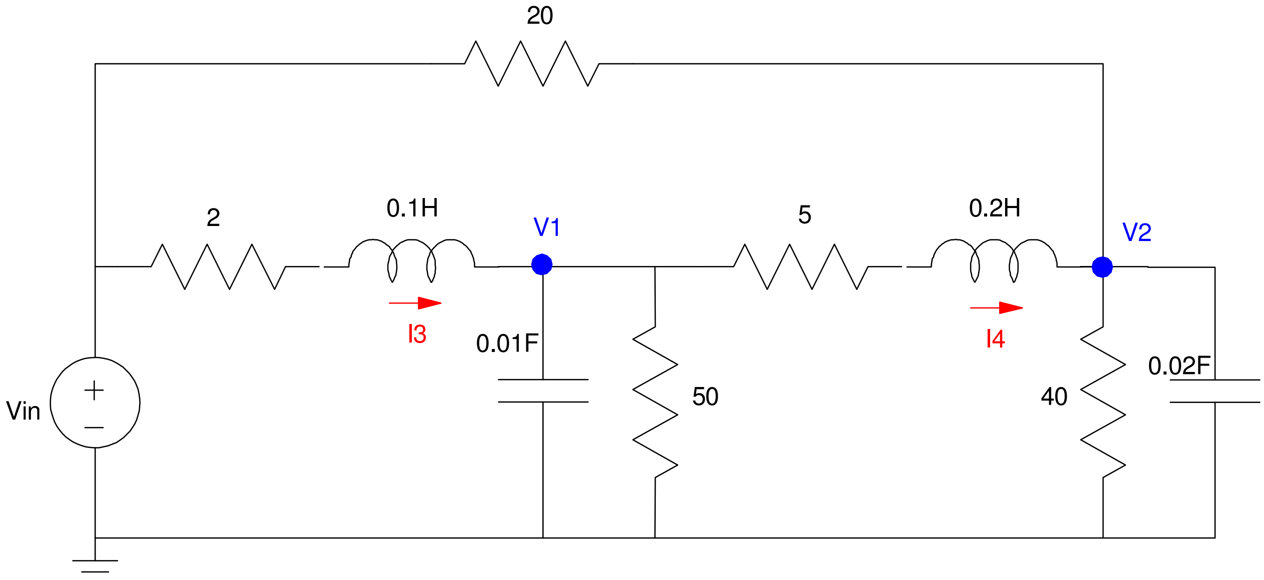
$$Y = \left(\frac{2.523}{(s+3)(s+4)}\right)X$$

3) Give {A and B} for the the state-space model for the following system



$$\begin{bmatrix} sX1 \\ sX2 \\ sX3 \\ sX4 \end{bmatrix} = \begin{bmatrix} -2 & -9 & 0 & -72 \\ 7 & -3 & -5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 8 & -4 \end{bmatrix} \begin{bmatrix} X1 \\ X2 \\ X3 \\ X4 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \\ 0 \\ 0 \end{bmatrix} U$$

4) Write four coupled differential equations to describe the following circuit. Assume the states are {V1, V2, I3, I4}. Note: For capacitors: $I = C\frac{dV}{dt}$, For inductors: $V = L\frac{dI}{dt}$



$$0.01sV_1 = I_3 - I_4 - \left(\frac{V_1}{50}\right)$$

$$0.02sV_2 = I_4 + \left(\frac{V_{in} - V_2}{20}\right) - \left(\frac{V_2}{40}\right)$$

$$0.1sI_3 = V_{in} - 2I_3 - V_1$$

$$0.2sI_4 = V_1 - 5I_4 - V_2$$

5) Assume the LaGrangian is:

$$L = 3x^2\dot{x}^3\dot{\theta}^4 + 2x\sin(\theta)$$

Determine

$$F = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \left(\frac{\partial L}{\partial x}\right)$$

$$F = \frac{d}{dt}\left(9x^2\dot{x}^2\dot{\theta}^4\right) - \left(6x\dot{x}^3\dot{\theta}^4 + 2\sin(\theta)\right)$$

Chain rule: take the derivative with respect to each term

$$F = \left(18x\dot{x}^3\dot{\theta}^4\right) + \left(18x^2\dot{x}\ddot{x}\dot{\theta}^4\right) + \left(36x^2\dot{x}^2\dot{\theta}^3\ddot{\theta}\right) - \left(6x\dot{x}^3\dot{\theta}^4 + 2\sin(\theta)\right)$$