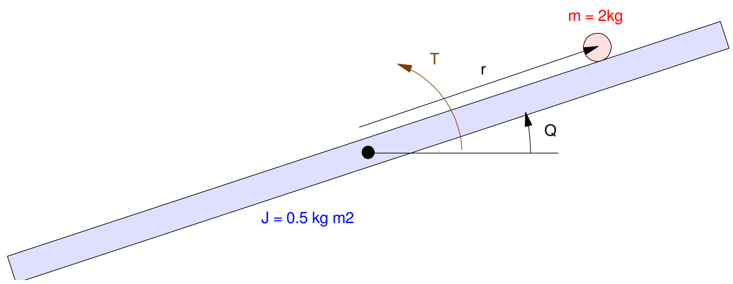
ECE 463/663 - Test #2: Name ________________

Due midnight Sunday, March 26th.   Individual Effort Only (no working in groups)



The linearized dynamics for a ball and beam system are:

$$s\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -7 & 0 & 0 \\ -7.84 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.4 \end{bmatrix}(T + d)$$

C Level (max 80 points)

Design a feedback control law for the ball and beam system assuming

- All states are measured (no observer is needed)
- A constant & sinusoidal set point (R(t) = 1 + 0.3 sin(0.4t)), and
- A constant disturbance (d(t) = 1)

Validate your feedback control law on the linear system

Validate your feedback control law on the nonlinear system

- With the ball having a mass of 2.0kg (nominal case)
- With the ball having a mass of 1.9kg (constant disturbance)

Design a servo compensator with poles at {0, +j0.4, -j0.4}

$$sZ = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0.4 \\ 0 & -0.4 & 0 \end{bmatrix} Z + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} (y - R)$$

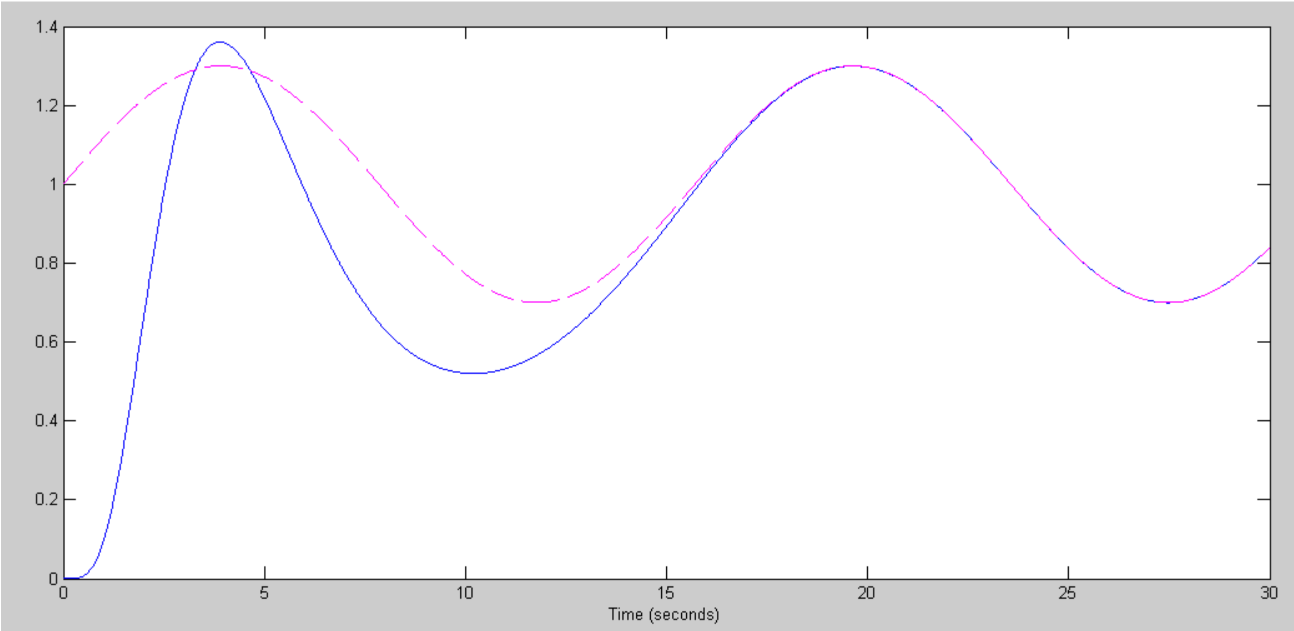
The plant plus servo compensator are:

$$\begin{bmatrix} sX \\ sZ \end{bmatrix} = \begin{bmatrix} A & 0 \\ B_z C & A_z \end{bmatrix} \begin{bmatrix} X \\ Z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} U + \begin{bmatrix} 0 \\ -B_z \end{bmatrix} R$$

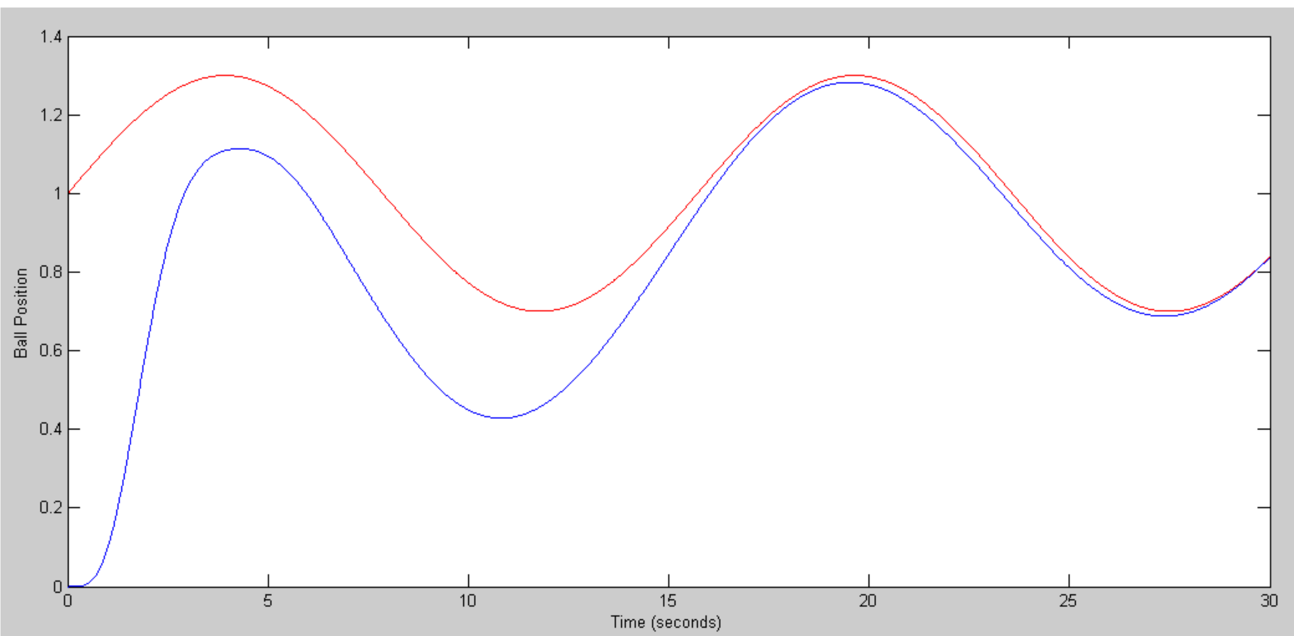
Use pole placement to stabilize the system

```
>> A = [0,0,1,0;0,0,0,1;0,-7,0,0;-7.84,0,0,0]

         0         0    1.0000         0
         0         0         0    1.0000
         0   -7.0000         0         0
   -7.8400         0         0         0

>> B = [0;0;0;0.4]

         0
         0
         0
    0.4000

>> C = [1,0,0,0];
>> Az = [0,0,0;0,0,0.4;0,-0.4,0]

         0         0         0
         0         0    0.4000
         0   -0.4000         0

>> eig(Az)

        0 + 0.4000i
        0 - 0.4000i
        0

>> Bz = [1;1;1];
```

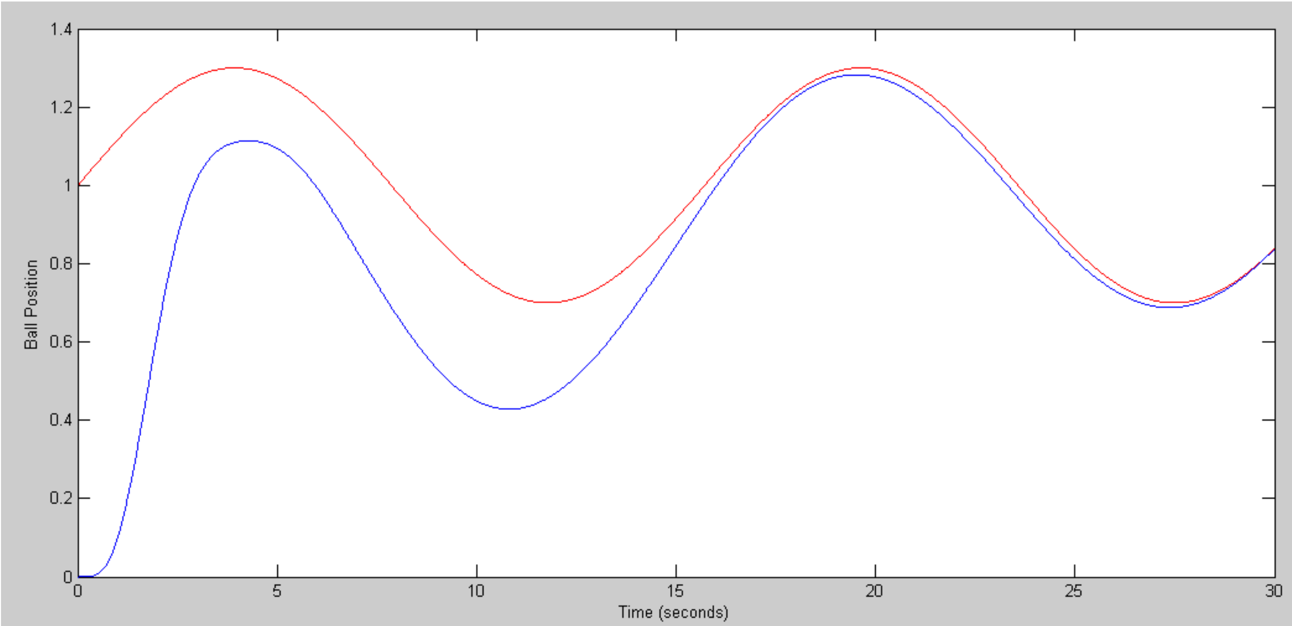
```
>> A7 = [A, zeros(4,3) ; Bz*C, Az]

         0         0    1.0000         0         0         0         0
         0         0         0    1.0000         0         0         0
         0   -7.0000         0         0         0         0         0
   -7.8400         0         0         0         0         0         0
    1.0000         0         0         0         0         0         0
    1.0000         0         0         0         0         0    0.4000
    1.0000         0         0         0         0   -0.4000         0

>> B7u = [B; 0*Bz]

         0
         0
         0
    0.4000
         0
         0
         0

>> B7r = [0*B; -Bz]

     0
     0
     0
     0
    -1
    -1
    -1

>> C7 = [1,0,0,0,0,0,0];
>> D7 = 0;
>> X0 = zeros(7,1);

>> K7 = ppl(A7, B7u, [-0.5, -0.5+j*0.4,-0.5-j*0.4, -1, -2, -3, -4])

K7 =  -64.1643  126.8750  -39.2732   28.7500  -10.9821  -12.6149   -1.7889

>> Kx = K7(1:4)

Kx =  -64.1643  126.8750  -39.2732   28.7500

>> Kz = K7(5:7)

Kz =  -10.9821  -12.6149   -1.7889

>> t = [0:0.01:20]';
>> R = 1 + 0.3*sin(0.4*t);
>> y = step3(A7-B7u*K7, B7r, C7, D7, t, X0, R);
>> plot(t,y,'b',t,R,'m--')
>> xlabel('Time (seconds)');
```



Response of Linear System



Response of Nonlinear System: m = 2.0kg



Nonlienar Response with m = 1.9kg

B Level (max 90 points)

Design a feedback control law for the ball and beam system assuming

- Only position and angle are measured, (observer is required)
- A constant & sinusoidal set point (R(t) = 1 + 0.3 sin(0.4t)), and
- No disturbance (d(t) = 0)

Validate your feedback control law on the linear system

Validate your feedback control law on the nonlinear system

- With the ball having a mass of 2.0kg (nominal case)

The augmented system is

$$\begin{bmatrix} sX \\ sZ \\ sX_e \end{bmatrix} = \begin{bmatrix} A & -BK_z & -BK_x \\ B_zC & A_z & 0 \\ HC & -BK_z & A-HC-BK_x \end{bmatrix} \begin{bmatrix} X \\ Z \\ X_e \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ B \end{bmatrix} U + \begin{bmatrix} 0 \\ -B_z \\ 0 \end{bmatrix} R$$

Step 1: Find the observer gains

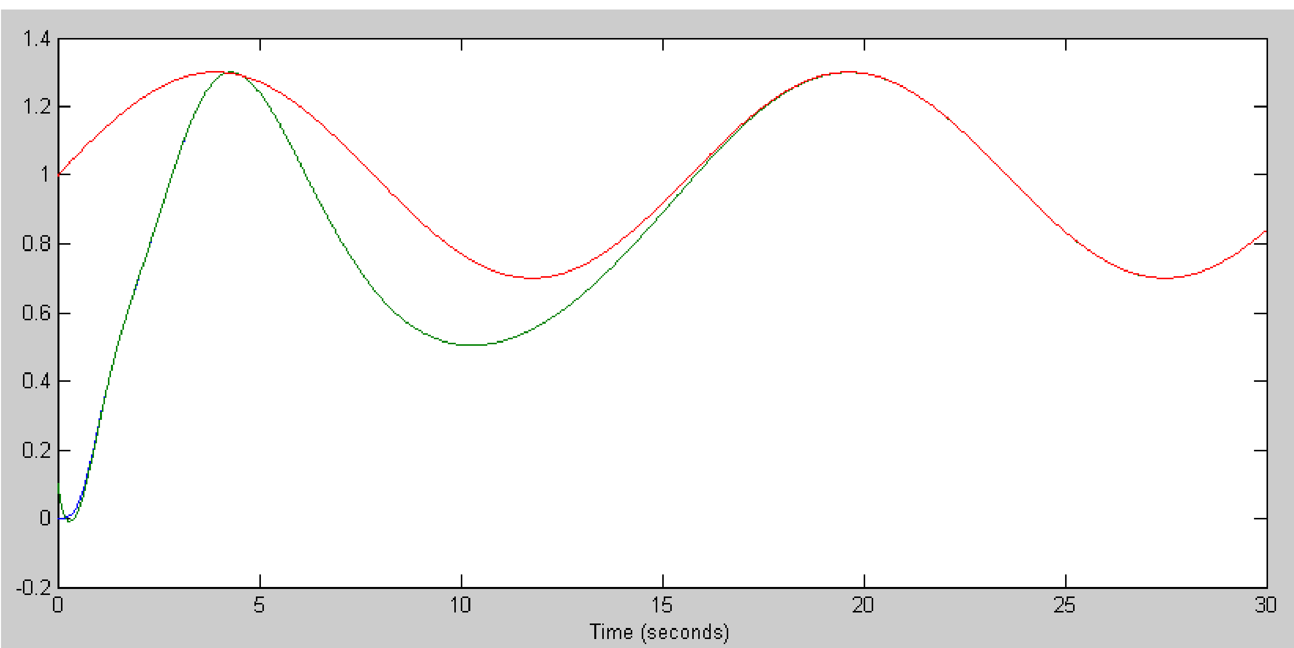
```
>> Hq = [0;4;0;4];
>> Cr = [1,0,0,0];
>> Cq = [0,1,0,0];
>> Hr = ppl( (A-Hq*Cq)', Cr', [-2,-3,-4,-5] )'

   10.0000
   -0.8571
   27.0000
   -9.5543

>> H = [Hr,Hq]

   10.0000         0
   -0.8571    4.0000
   27.0000         0
   -9.5543    4.0000

>> C = [Cr;Cq]

     1     0     0     0
     0     1     0     0

>> D = zeros(2,1);
```

Step 2: Form the augmented 11x11 system

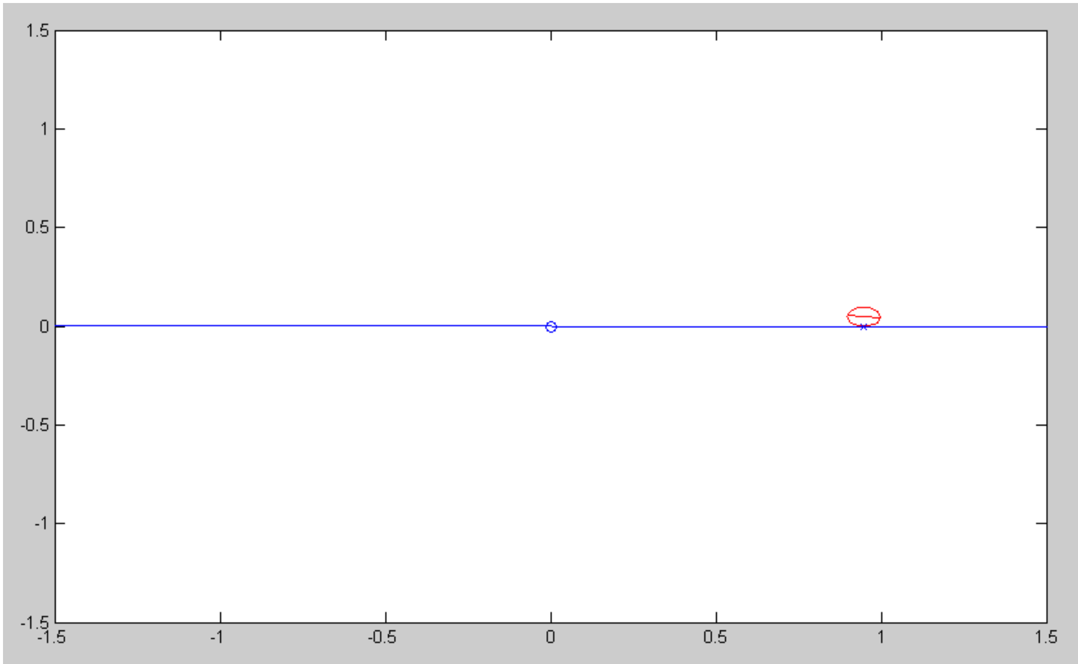
```
>> A11 = [A,-B*Kz,-B*Kx ; Bz*Cr,Az, zeros(3,4) ; H*C, -B*Kz,A-H*C-B*Kx]

A11 =

     0         0    1.0000         0         0         0         0         0         0         0         0
     0         0         0    1.0000         0         0         0         0         0         0         0
     0        -7         0         0         0         0         0         0         0         0         0
  -7.840       0         0         0    4.3928    5.0460    0.7156   25.6657  -50.7500   15.7093  -11.500
     1         0         0         0         0         0         0         0         0         0         0
     1         0         0         0         0         0    0.4000         0         0         0         0
     1         0         0         0         0   -0.4000         0         0         0         0         0
    10         0         0         0         0         0         0  -10.0000         0    1.0000         0
   -0.8571     4         0         0         0         0         0    0.8571   -4.0000         0    1.0000
   27.0000     0         0         0         0         0         0  -27.0000   -7.0000         0         0
   -9.5543     4         0         0    4.3928    5.0460    0.7156   27.3800  -54.7500   15.7093  -11.5000

>> B11u = [B;0*Bz;B];
>> B11r = [0*B;-Bz;0*B];
>> C11 = [1,0,0,0,0,0,0,0,0,0,0;0,0,0,0,0,0,0,1,0,0,0]

     1     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     1     0     0     0

>> D11 = [0;0];
```

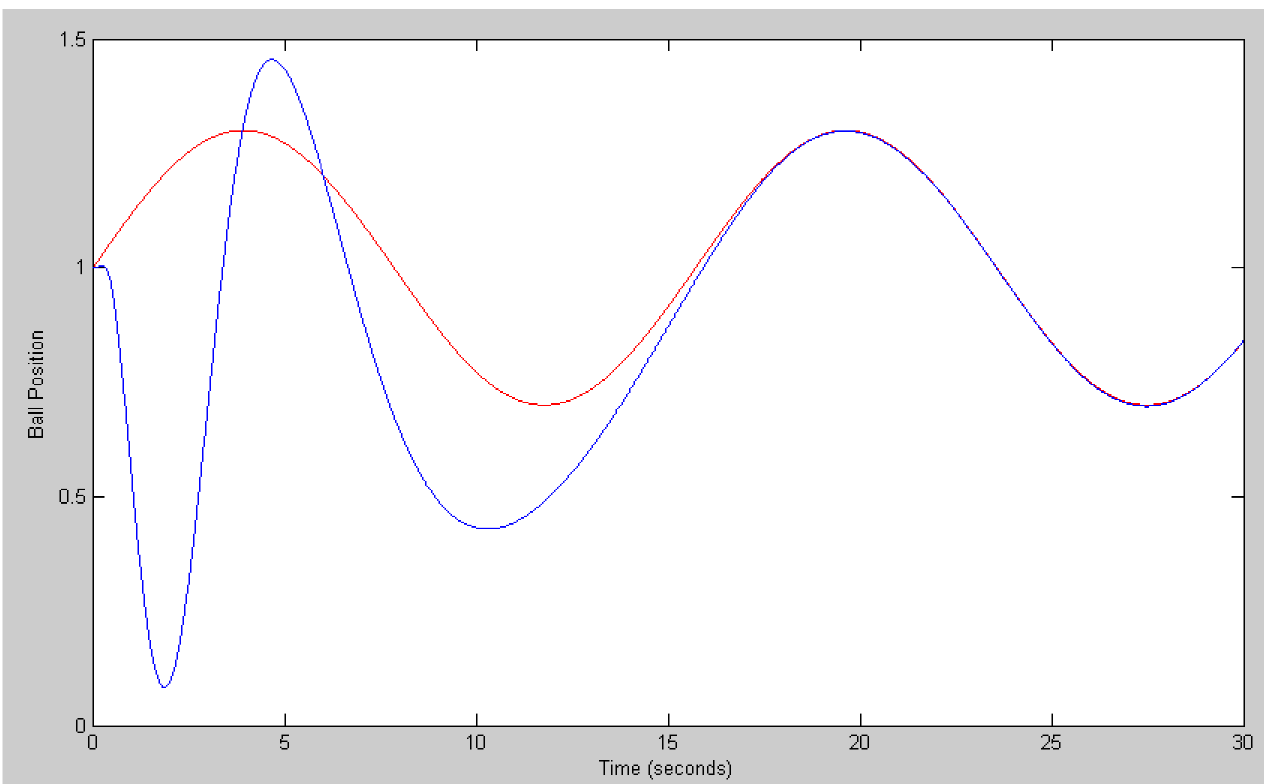
Step 3: Take the step response of the linear system

```
>> X0 = [0,0,0,0,0,0,0,0.1,0.1,0.1,0.1]';
>> t = [0:0.01:30]';
>> R = 1 + 0.3*sin(0.4*t);
>> y = step3(A11, B11r, C11, D11, t, X0, R);
>> plot(t,y,t,R)
>> xlabel('Time (seconds)');
```



Now try it on the nonlinar system:





Nonlinear System Response: Observer states used for t > 10

A Level (max 100 points)

Design a feedback control law for the ball and beam system assuming

- Only position and angle are measured, (observer is requried)
- A constant & sinusoidal set point (R(t) = 1 + 0.3 sin(0.4t)), and
- A constant disturbance (d(t) = 1)

Validate your feedback control law on the linear system

Validate your feedback control law on the nonlinear system

- With the ball having a mass of 2.0kg (nominal case)
- With the ball having a mass of 1.9kg (constant disturbance)

For the observer, create an augmented system (add a constant disturbance)

$$\begin{bmatrix} sX_e \\ sd_e \end{bmatrix} = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix} \begin{bmatrix} X_e \\ d_e \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} U$$

$$y = \begin{bmatrix} y_r \\ y_q \end{bmatrix} = \begin{bmatrix} C_r & 0 \\ C_q & 0 \end{bmatrix} \begin{bmatrix} X_e \\ d_e \end{bmatrix}$$

Step 1: Find the observer gains, H. Start with the augmented system:

```
>> A5 = [A, B ; zeros(1,4), 0]

         0         0    1.0000         0         0
         0         0         0    1.0000         0
         0   -7.0000         0         0         0
   -7.8400         0         0         0    0.4000
         0         0         0         0         0
```

Hq is given. Find Hr using pole placement.

```
>> C5r = [1,0,0,0,0];
>> C5q = [0,1,0,0,0];
>> Hq = [0;4;0;4;0];
>> Hr = ppl( (A5-Hq*C5q)', C5r', [-3,-3.2, -3.4, -3.6, -3.8] )'

   13.0000
  -14.4857
   59.4000
  -68.3598
 -159.4697

>> H = [Hr,Hq]

   13.0000         0
  -14.4857    4.0000
   59.4000         0
  -68.3598    4.0000
 -159.4697         0
```

Step 2: Form the augmented system

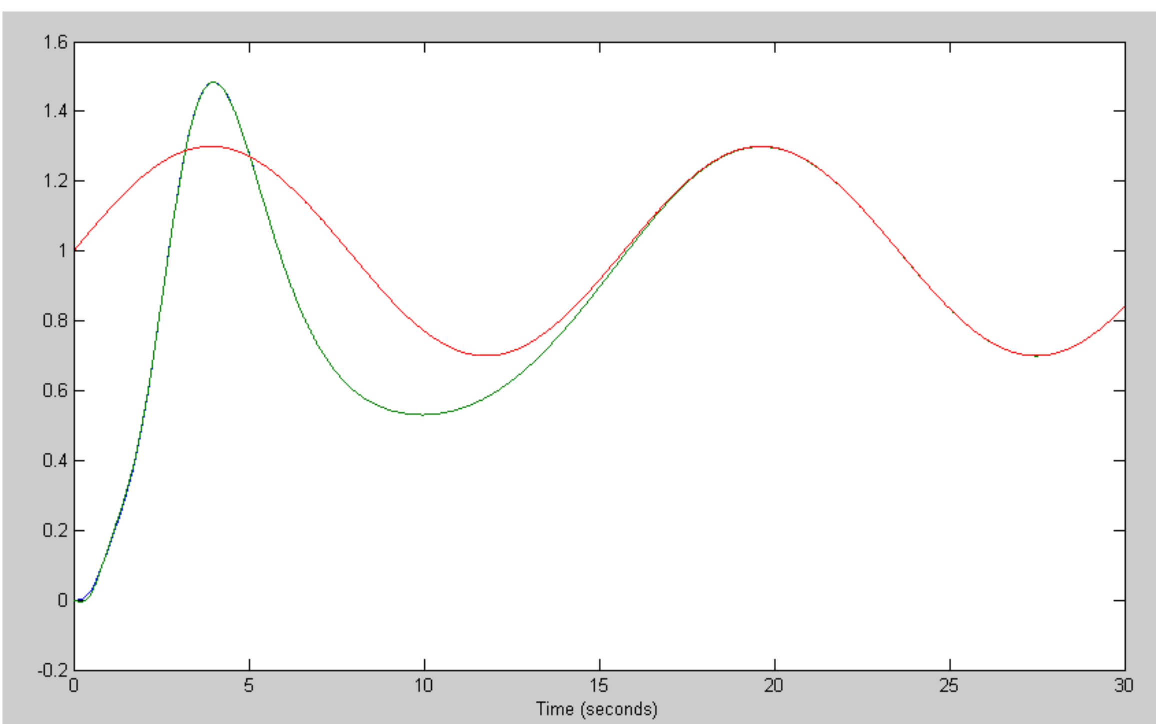
$$\begin{bmatrix} sX \\ sZ \\ sX_e \end{bmatrix} = \begin{bmatrix} A & -BK_z & -BK_x \\ B_zC_r & A_z & 0 \\ HC & -B_eK_z & A_e - HC_e - B_eK_x \end{bmatrix} \begin{bmatrix} X \\ Z \\ X_e \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} d + \begin{bmatrix} 0 \\ -B_z \\ 0 \end{bmatrix} R$$

```
>> A12 = [A, -B*Kz,-B*Kx ; Bz*Cr, Az, zeros(3,5) ; H*C, -Be*Kz, Ae-H*Ce-Be*Kx];
>> B12r = [0*B;-Bz;0*Be];
>> B12d = [B ; zeros(8,1)];
>> C12 = [1,0,0,0,0,0,0,0,0,0,0,0;0,0,0,0,0,0,0,1,0,0,0,0];
>> D12 = [0;0];
```

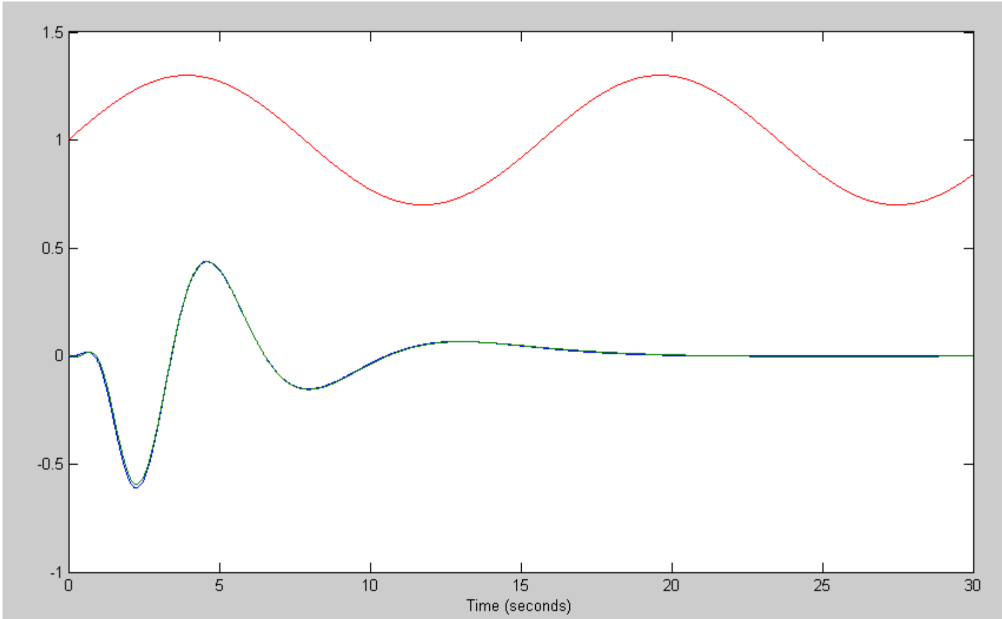
Take the step response of the linear system

- With respect to the set point, R,
- With respect to the disturabance, d

```
>> X0 = zeros(12,1);
>> X0(9) = 0.1;
>> t = [0:0.01:30]';
>> R = 1 + 0.3*sin(0.4*t);

>> y = step3(A12, B12r, C12, D12, t, X0, R);
>> plot(t,y,t,R)
>> xlabel('Time (seconds)');

>> y = step3(A12, B12d, C12, D12, t, X0, R);
>> plot(t,y,t,R)
>> xlabel('Time (seconds)');
```

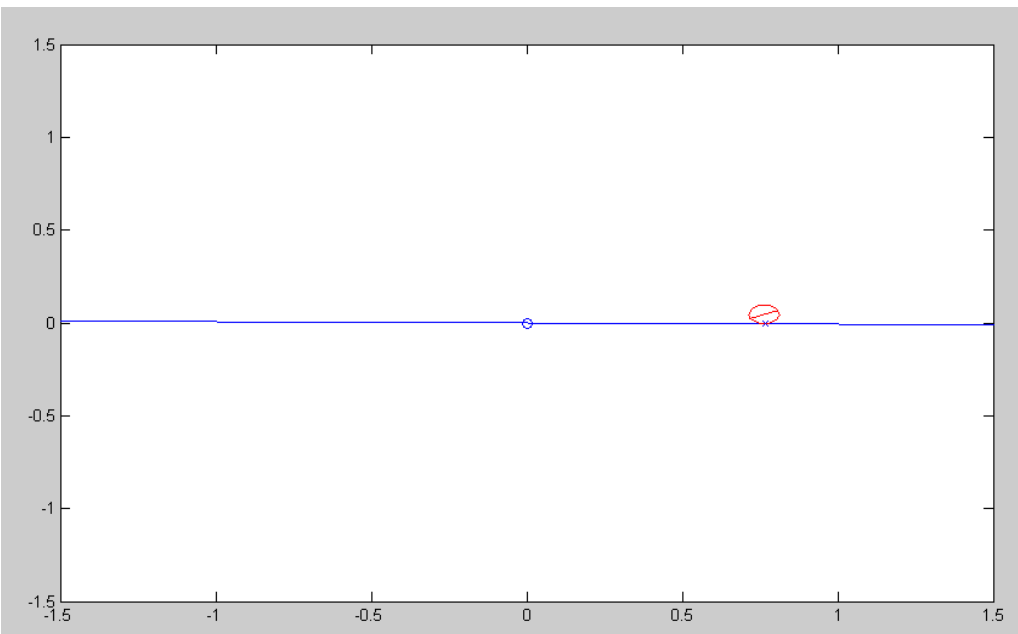


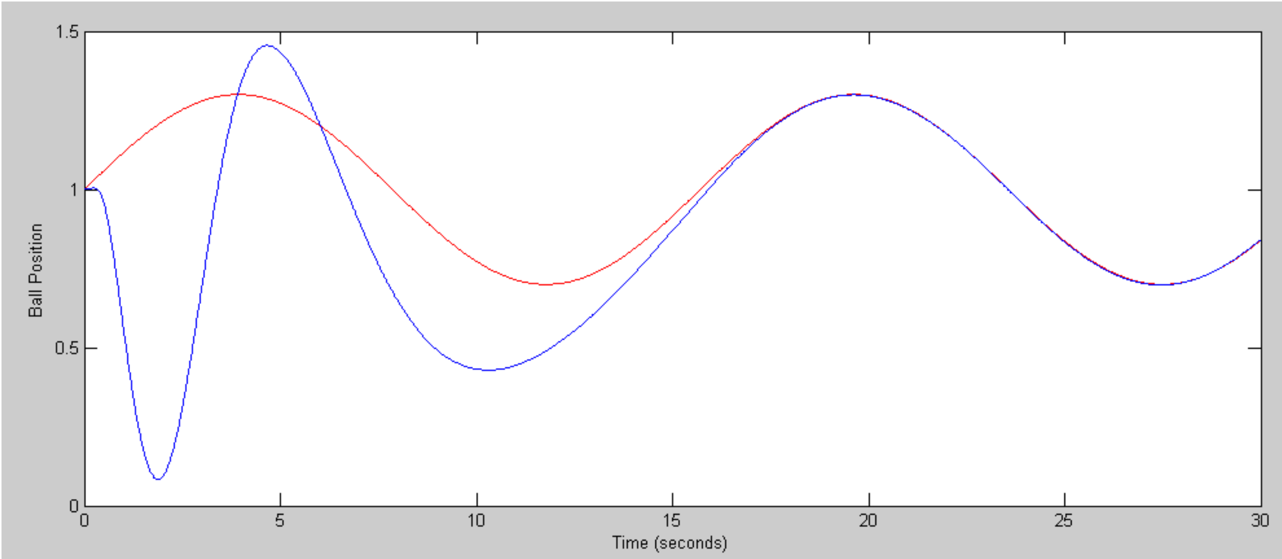
Tracks a constant and a sinusoidal set point



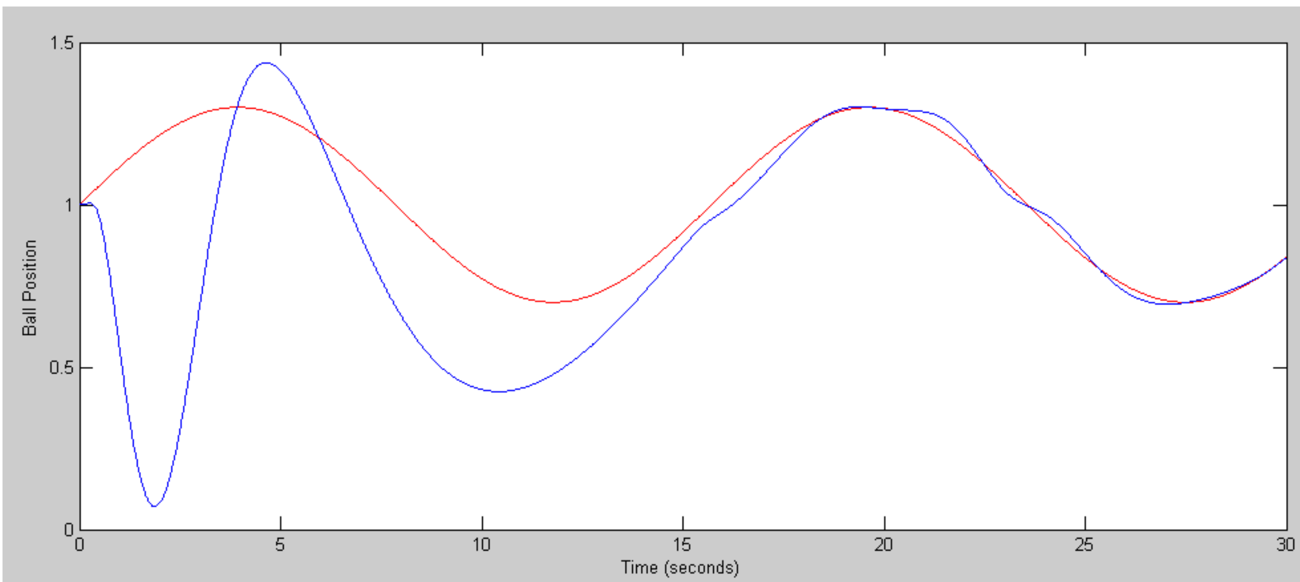
Rejects a constant and a sinusoidal disturbance

Finally, test on the nonlienar system. m = 2.0kg

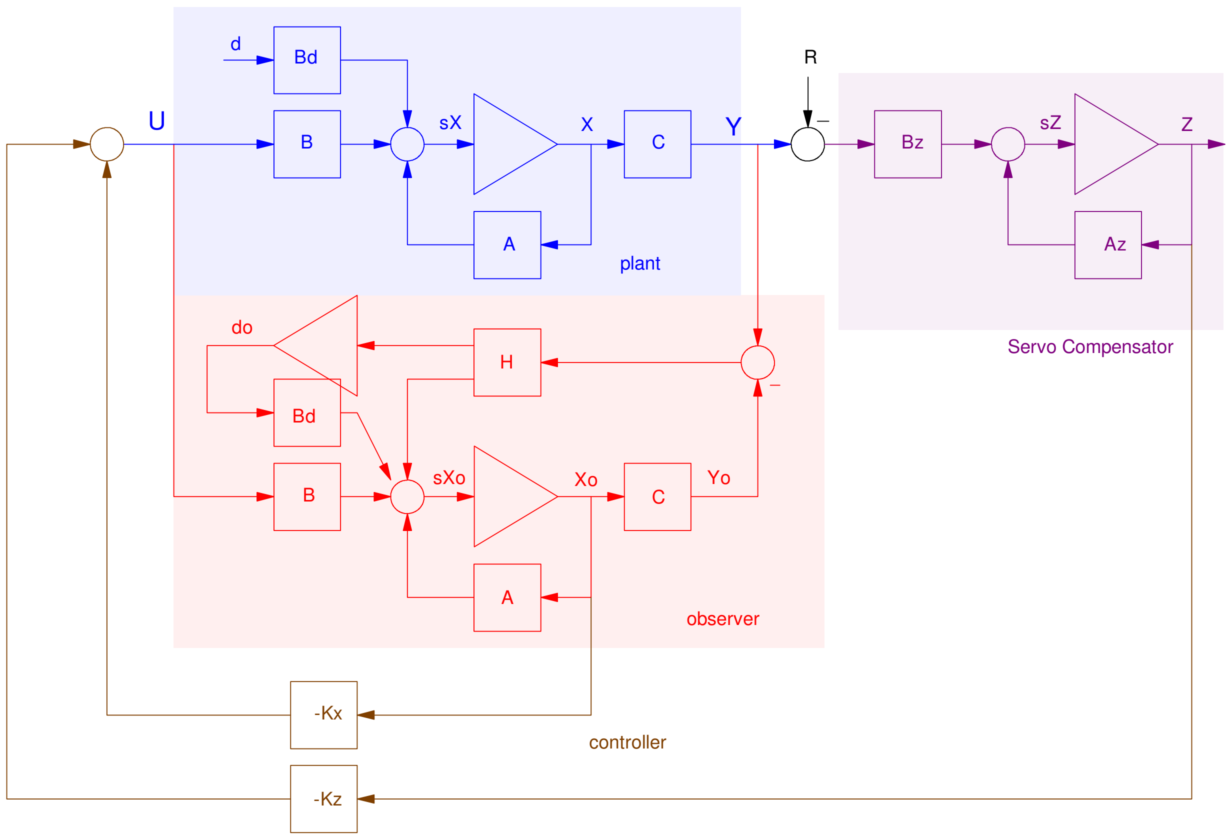




Nonlinear Response with m = 2.0kg



Nonlienar Response with m = 1.9kg



Block diagram for the Plant, Servo Compensator, Disturbance, Observer, and Full-State Feedback

Final Code

```
% Ball & Beam System

X = [1, 0, 0, 0]';
dt = 0.01;
t = 0;
Kx = [ -64.1643  126.8750  -39.2732   28.7500];
Kz = [-10.9821  -12.6149   -1.7889];

% Servo Compensator
Z = zeros(3,1);
Az = [0,0,0;0,0,0.4;0,-0.4,0];
Bz = [1;1;1];
n = 0;
y = [];

% Full-Order Observer
A = [0,0,1,0;0,0,0,1;0,-7,0,0;-7.84,0,0,0];
B = [0;0;0;0.4];
Cr = [1,0,0,0];
Cq = [0,1,0,0];
C = [Cr;Cq];
Hq = [0;4;0;4;0];

A5 = [A, B ; zeros(1,5)];
B5 = [B;0];
C5r = [Cr, 0];
C5q = [Cq, 0];

Hr = ppl((A5-Hq*C5q)',C5r',[-3,-3.2, -3.4, -3.6, -3.8])';
H5 = [Hr, Hq];
C5 = [C5r ; C5q];

Xe = [X ; 0];

while(t < 30)
 Ref = 1 + 0.3*sin(0.4*t);
 if(t<15)
    U = -Kz*Z - Kx*X;
 else
    U = -Kz*Z - [Kx, 0]*Xe;
 end
 dX = BeamDynamics(X, U);
 dZ = Az*Z + Bz*(X(1) - Ref);
 dXe = A5*Xe + B5*U + H5*(C*X - C5*Xe);

 X = X + dX * dt;
 Z = Z + dZ * dt;
 Xe = Xe + dXe * dt;

 t = t + dt;

 y = [y ; Ref, X(1), Xe(1)];
 n = mod(n+1,5);
 if(n == 0)
    BeamDisplay3(X, Xe, Ref);
 end
 end

t = [1:length(y)]' * dt;


plot(t,y(:,1),'r',t,y(:,2),'b');
xlabel('Time (seconds)');
ylabel('Ball Position');
```