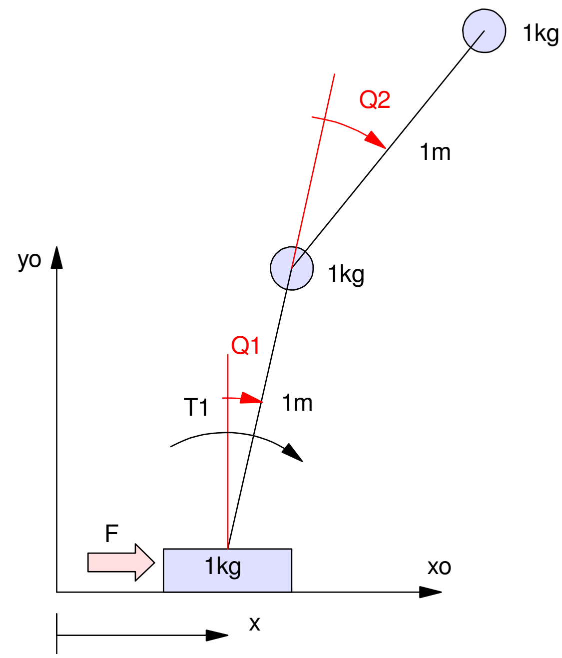
ECE 463/663 - Test #3: Name ________________

Due midnight Sunday, May 8th. Individual Effort Only (no working in groups)

The linearized dynamics for

- A cart
- With two pendulums, and
- Two inputs (F and T1) are:



$$s\begin{bmatrix} x \\ \theta_1 \\ \theta_2 \\ \dot{x} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -2g & 0 & 0 & 0 & 0 \\ 0 & 3g & -g & 0 & 0 & 0 \\ 0 & -3g & 3g & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta_1 \\ \theta_2 \\ \dot{x} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & -1 \\ -1 & 2 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} F \\ T_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} d$$

Design a feedback control law using LQR or LQG/LTR or VSS techniques (your pick) which

- Uses both inputs (F and T1),
- Results in a 2% settling time between 6 to 12 seconds
- Less than 10% overshoot for a step input, and
- An ability to track a constant set point

Turn in for your exam

- A block diagram of your plant and controller
- Matlab code used to determine your control law,
- The resulting control law
- A step response with respect Ref = 1, d = 1 for the linear model (above),
- A step response for the nonlinear simulation (Cart2 / Cart2Display / Cart2Dynamics) with your control law, and
- The main calling routine (Cart2.m) you used to generate this step response.

C Level (max 80 points)

Assume

- No noise
- All states are measured
- A constant set point, and
- A constant disturbance (d = 1)

B Level (max 90 points)

Assume

- No noise
- Only positions and angles are measured $\{x, \theta_1, \theta_2\}$
- A constant set point, and
- No disturbance (d = 0)

A Level (max 100 points)

Assume

- No noise
- Only positions and angles are measured $\{x, \theta_1, \theta_2\}$
- A constant set point, and
- An input disturbance (d = 1)

Code (B-Level)

Step 1: Form the augmented system (plant & servo)

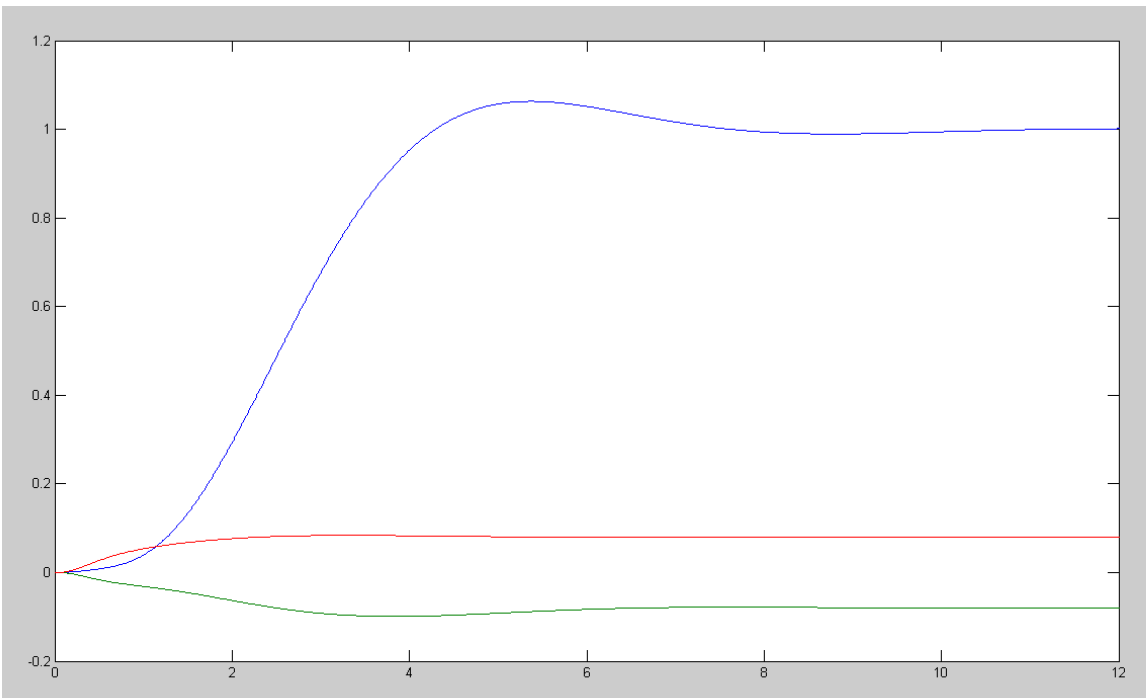
```
g = 9.8;
a1 = [0,0,0,1,0,0]
a2 = [0,0,0,0,1,0];
a3 = [0,0,0,0,0,1];
a4 = [0,-2*g,0,0,0,0];
a5 = [0,3*g,-g,0,0,0];
a6 = [0,-3*g,3*g,0,0,0];
A = [a1;a2;a3;a4;a5;a6];
Bf = [0;0;0;1;-1;1];  % force input
Bt = [0;0;0;-1;2;-3]; % T1 input
B = [Bf,Bt];
C = [1,0,0,0,0,0;0,1,0,0,0,0;0,0,1,0,0,0];
D = zeros(3,2);

Cx = [1,0,0,0,0,0];
A7 = [A, zeros(6,1) ; Cx, 0]

         0         0         0    1.0000         0         0         0
         0         0         0         0    1.0000         0         0
         0         0         0         0         0    1.0000         0
         0  -19.6000         0         0         0         0         0
         0   29.4000   -9.8000         0         0         0         0
         0  -29.4000   29.4000         0         0         0         0
    1.0000         0         0         0         0         0         0

>> B7 = [B ; 0,0]

     0     0
     0     0
     0     0
     1    -1
    -1     2
     1    -3
     0     0
```

Step 2: Find the feedback gains so that the 2% settling time is 6..12 seconds

```
>> K7 = lqr(A7, B7, diag([0,0,0,0,0,0,1e1]),diag([1,1]))

   -2.3839  -72.2161  -52.7278   -4.3232  -25.5362  -15.3923   -0.3370
   -7.4648  -70.7767  -92.4656   -9.3013  -33.8052  -25.0764   -3.1443

>> eig(A7 - B7*K7)

  -6.8390
  -6.7806
  -3.5743
  -3.4738
  -0.5452 + 0.8810i
  -0.5452 - 0.8810i
  -0.9828
```

Step 3: Plot the step response of the linear system

```
>> C7 = [C, zeros(3,1)]

     1     0     0     0     0     0     0
     0     1     0     0     0     0     0
     0     0     1     0     0     0     0

>> X0 = zeros(7,1);
>> t = [0:0.01:20]';
>> R = 0*t + 1;

>> B7r = [0;0;0;0;0;0;-1];
>> D7 = zeros(3,1);
>> y = step3(A7-B7*K7, B7r, C7, D7, t, X0, R);
>> plot(t,y)
>> t = [0:0.01:12]';
>> xlim([0,12])
```



Now add a full-order observer

```
>> H = lqr(A', C', 100*diag([1,1,1,1,1,1]), diag([1,1,1]) )'

   11.5971   -1.5425    0.4349
   -1.5425   14.1722   -2.4202
    0.4349   -2.4202   15.1904
   18.5308  -29.1678    7.0162
  -11.6352   54.5434  -26.9124
    8.3679  -44.8222   68.3981

>> eig(A - H*C)

  -13.6085
  -11.1097
   -9.3640
   -4.1695
   -1.7058
   -1.0022
```

Now form the total, 13th-order system

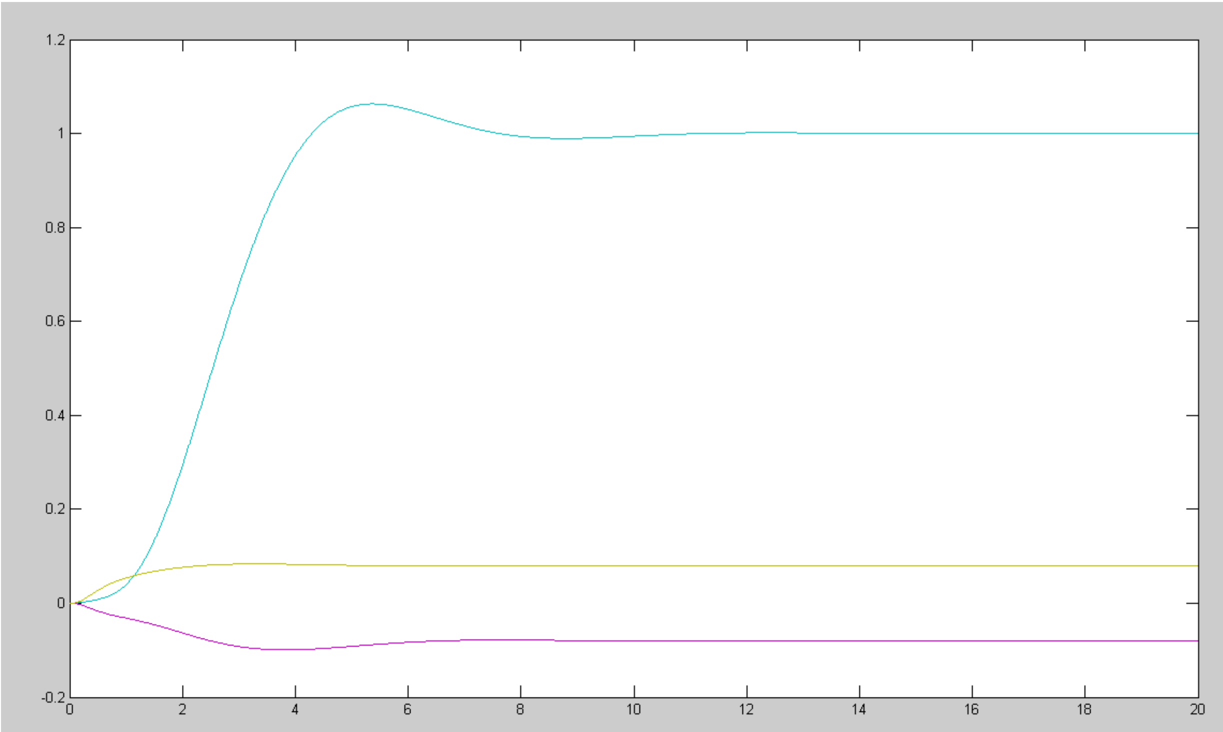
$$\begin{bmatrix} sX \\ sZ \\ sX_e \end{bmatrix} = \begin{bmatrix} A & -BK_z & -BK_x \\ C_x & 0 & 0 \\ HC & -BK_z & A-HC-BK_x \end{bmatrix} \begin{bmatrix} X \\ Z \\ X_e \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} R$$

```
>> A13 = [A, -B*Kz, -B*Kx ; Cx, 0, zeros(1,6) ; H*C, -B*Kz, A-H*C-B*Kx];
>> eig(A13)

 -13.6085
 -11.1097
  -9.3640
  -6.8390
  -6.7806
  -4.1695
  -3.5743
  -3.4738
  -0.5452 + 0.8810i
  -0.5452 - 0.8810i
  -1.7058
  -0.9828
  -1.0022
```

Stable - so A13 looks OK

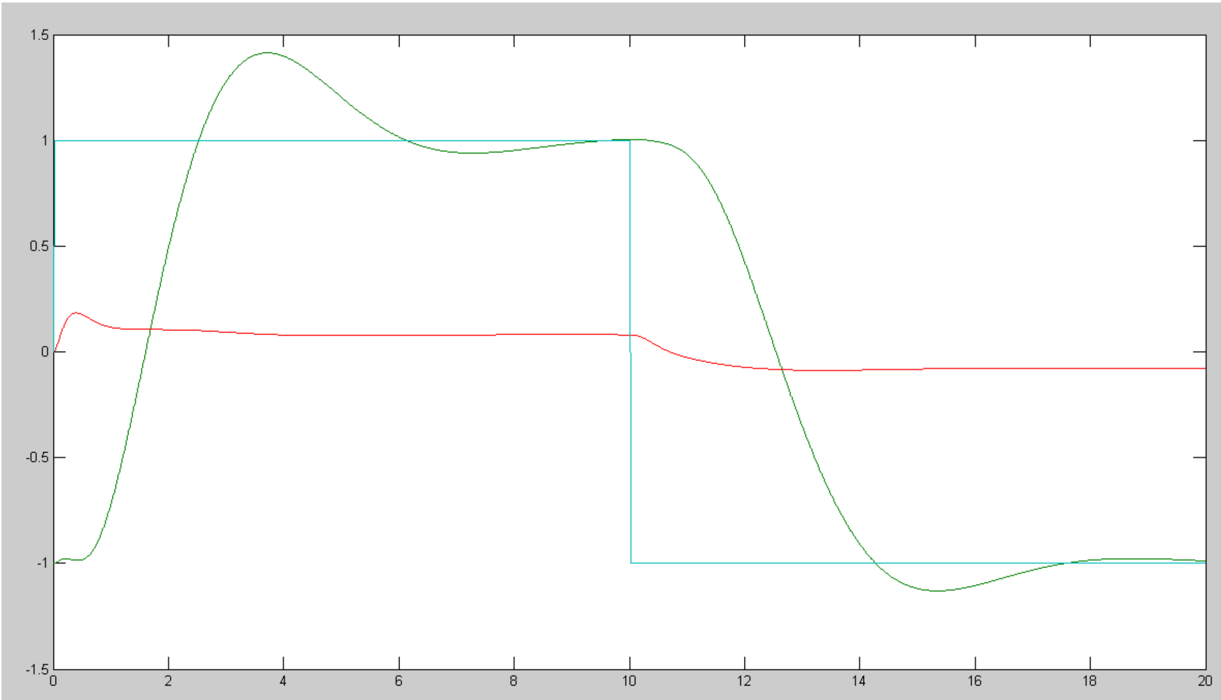
```
>> B13r = [zeros(6,1);-1;zeros(6,1)];
>> C13 = [C,zeros(3,1),0*C ; 0*C,zeros(3,1),C]

     1     0     0     0     0     0     0     0     0     0     0     0     0
     0     1     0     0     0     0     0     0     0     0     0     0     0
     0     0     1     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     1     0     0     0     0     0
     0     0     0     0     0     0     0     0     1     0     0     0     0
     0     0     0     0     0     0     0     0     0     1     0     0     0

>> D13 = zeros(6,1);
>> X0 = zeros(13,1);
>> R = 0*t+1;
```

```
>> y = step3(A13, B13r, C13, D13, t, X0, R);
>> plot(t,y)
>>
```

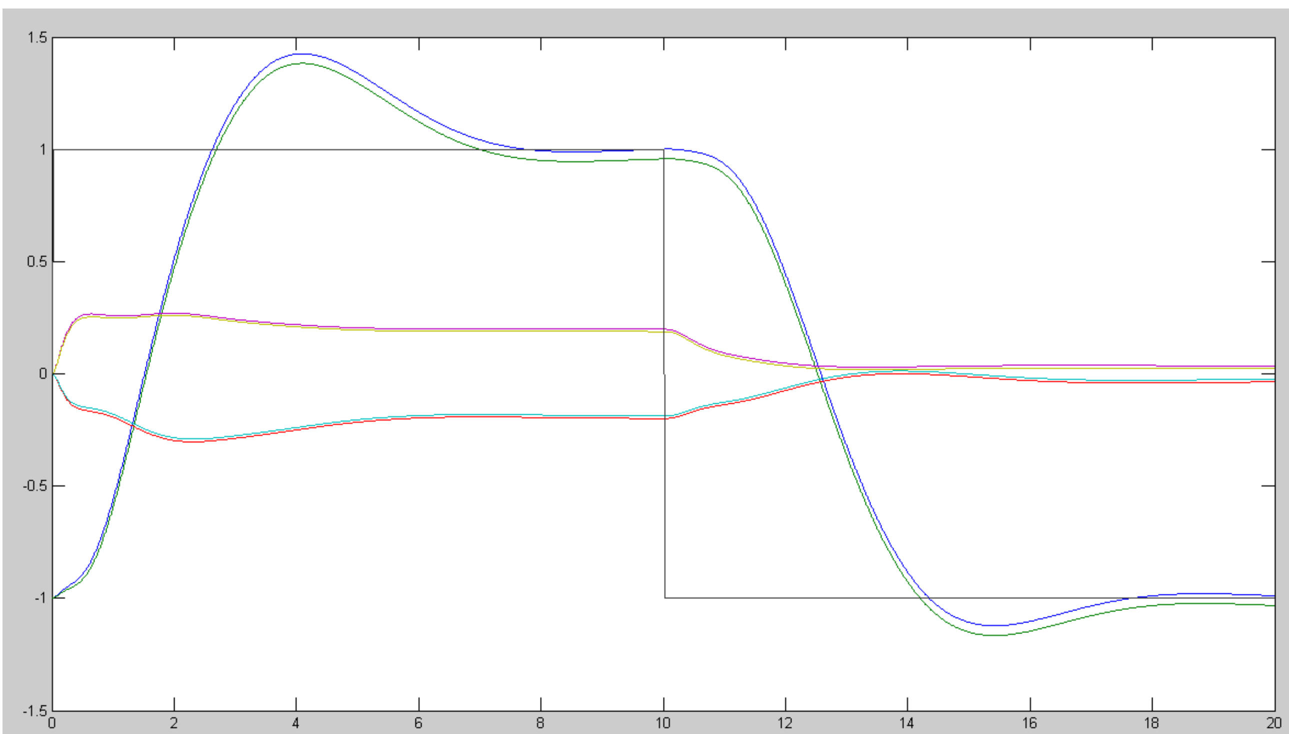


Now try the nonlinear model

```
% ECE 463/663 Test #3
% Cart with two pendulums

Ref = 1;
dt = 0.01;
t = 0;
n = 0;
y = [];
X = [-1,0,0,0,0,0]';
Z = 0;

g = 9.8;
a1 = [0,0,0,1,0,0]
a2 = [0,0,0,0,1,0];
a3 = [0,0,0,0,0,1];
a4 = [0,-2*g,0,0,0,0];
a5 = [0,3*g,-g,0,0,0];
a6 = [0,-3*g,3*g,0,0,0];
A = [a1;a2;a3;a4;a5;a6];
Bf = [0;0;0;1;-1;1];  % force input
Bt = [0;0;0;-1;2;-3]; % T1 input
B = [Bf,Bt];
C = [1,0,0,0,0,0;0,1,0,0,0,0;0,0,1,0,0,0];
D = zeros(3,2);

Ae = A;
Be = B;
Ce = C;
Xe = X;

while(t < 20)
 Ref = sign(sin(0.314*t));
 U = -Kz*Z - Kx*Xe;

 dX = Cart2Dynamics(X, U(1), U(2));
 dZ = X(1) - Ref;
 dXe = Ae*Xe + Be*U + H*(C*X - C*Xe);  % observer (not used in the initial code)

 X = X + dX * dt;
 Z = Z + dZ * dt;
 Xe = Xe + dXe * dt;

 t = t + dt;
 n = mod(n+1, 5);
 if(n == 0)
    Cart2Display(X, Xe, Ref);
 end
 y = [y ; X(1), Xe(1), X(2), Xe(2), X(3), Xe(3), Ref];
end

hold off;
t = [1:length(y)]' * dt;
plot(t,y);
```



Step response without an input disturbance



Step Response with an input disturbance (the model estimate is slightly off)