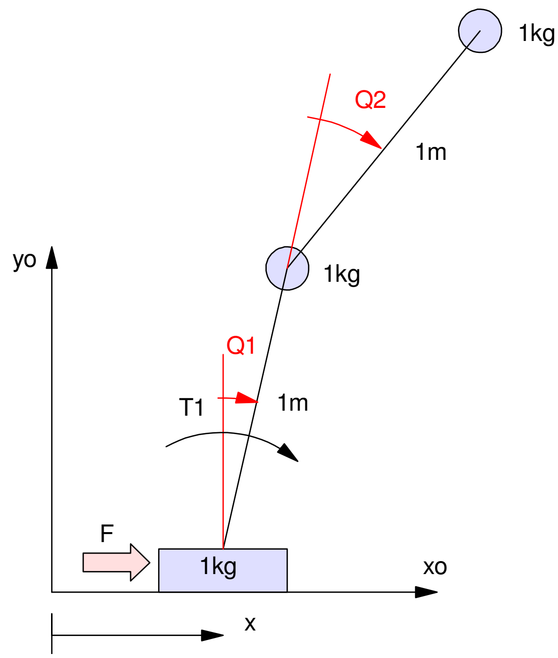
ECE 463/663 - Test #3: Name ________________

Due midnight Sunday, May 8th. Individual Effort Only (no working in groups)

The linearized dynamics for

- A cart
- With two pendulums, and
- Two inputs (F and T1) are:



$$s\begin{bmatrix} x \\ \theta_1 \\ \theta_2 \\ \dot{x} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -2g & 0 & 0 & 0 & 0 \\ 0 & 3g & -g & 0 & 0 & 0 \\ 0 & -3g & 3g & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta_1 \\ \theta_2 \\ \dot{x} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & -1 \\ -1 & 2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} F \\ T_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} d$$

Design a feedback control law using LQR or LQG/LTR or VSS techniques (your pick) which

- Uses both inputs (F and T1),
- Results in a 2% settling time between 6 to 12 seconds
- Less than 10% overshoot for a step input, and
- An ability to track a constant set point

Turn in for your exam

- A block diagram of your plant and controller
- Matlab code used to determine your control law,
- The resulting control law
- A step response with respect Ref = 1, d = 1 for the linear model (above),
- A step response for the nonlinear simulation (Cart2 / Cart2Display / Cart2Dynamics) with your control law, and
- The main calling routine (Cart2.m) you used to generate this step response.

C Level (max 80 points)

Assume

- No noise
- All states are measured
- A constant set point, and
- A constant disturbance (d = 1)

B Level (max 90 points)

Assume

- No noise
- Only positions and angles are measured $\{x, \theta_1, \theta_2\}$
- A constant set point, and
- No disturbance (d = 0)

A Level (max 100 points)

Assume

- No noise
- Only positions and angles are measured $\{x, \theta_1, \theta_2\}$
- A constant set point, and
- An input disturbance (d = 1)

Startig Code (uses pole placement for Kx and sets K(torque1) = 0)

```
% ECE 463/663 Final Exam
% Cart with two pendulums

Ref = 1;
dt = 0.01;
t = 0;
n = 0;
y = [];
X = [-1,0,0,0,0,0]';
Z = 0;

g = 9.8;
a1 = [0,0,0,1,0,0]
a2 = [0,0,0,0,1,0];
a3 = [0,0,0,0,0,1];
a4 = [0,-2*g,0,0,0,0];
a5 = [0,3*g,-g,0,0,0];
a6 = [0,-3*g,3*g,0,0,0];
A = [a1;a2;a3;a4;a5;a6];
Bf = [0;0;0;1;-1;1];  % force input
Bt = [0;0;0;-1;2;-3]; % T1 input
B = [Bf,Bt];
C = [1,0,0,0,0,0;0,1,0,0,0,0;0,0,1,0,0,0];
D = zeros(3,2);

% pole placement to find force
Kf = ppl(A, Bf, [-1,-2,-3,-4,-5,-6]);
% set T1 = 0 (Kx = 0 for the torque input)
Kt = zeros(1,6);
%Kx is a 6x2 matrix (6 states, 2 inputs)
Kx = [Kf ; Kt];  % change to using a servo compensator and LQR to find Kx

DC = -C*inv(A - B*Kx)*B;
Kr = [1/DC(1,1); 0];

Ae = A;
Be = B;
Ce = C;
Xe = X;

while(t < 20)
 Ref = sign(sin(0.314*t));
 U = Kr*Ref - Kx*X;

 %dX = Cart2Dynamics(X, F, T1)
 dX = Cart2Dynamics(X, U(1), U(2));
 dZ = X(1) - Ref;  % servo compensator (not used in the initial code)
 dXe = 0;  % observer (not used in the initial code)

 X = X + dX * dt;
 Z = Z + dZ * dt;
 Xe = X + [0.1;0;0;0;0;0];  % cheating for now - observer = plant + 0.1

 t = t + dt;
 n = mod(n+1, 5);
 if(n == 0)
    Cart2Display(X, Xe, Ref);
 end
 y = [y ; X(1), X(1), X(3), Ref];
end

hold off;
t = [1:length(y)]' * dt;
plot(t,y);
```

Cart2Dynamics (Sp23)

```
function [ dX ] = Cart2Dynamics( X, F, T1 )
%cart dynamics (Sp23 final)
% cart = 1kg
% ball = 1kg
% length = 1m
% X = [x, q, dx, dq]
x = X(1);
q1 = X(2);
q2 = X(3);
dx = X(4);
dq1 = X(5);
dq2 = X(6);
g = -9.8;

c1 = cos(q1);
s1 = sin(q1);
c2 = cos(q2);
s2 = sin(q2);
c12 = cos(q1 + q2);
s12 = sin(q1 + q2);

M = [3,         2*c1+c12,  c12;
     2*c1+c12,  3+2*c2,    1+c2;
     c12,       1+c2,      1];


C = [2*s1*dq1*dq1 - s12*(dq1+dq2)^2;
    2*s1*dx*dq1 + s2*(dq1+dq2)*dq2 + s2*dq1*dq2 - 2*s1*dx*dq1;
    s2*dq1*dq2 - s2*dq1*(dq1+dq2)];

G = [0 ;  2*s1 + s12 ; s12];

ddX = inv(M)*(C - g*G + [F ; T1; 0] );

dX = [dx; dq1; dq2; ddX];

end
```

Cart2Display

```
function [] = Cart2Display(X, Xe, Ref)
% Observer
x1 = Xe(1);
y1 = 0.2;
q1 = Xe(2);
q2 = Xe(3);
% cart
X1 = [-0.2,0.2,0.2,-0.2,-0.2] + x1;
Y1 = [0,0,0.2,0.2,0];
x2 = x1 + sin(q1);
y2 = y1 + cos(q1);
x3 = x2 + sin(q1+q2);
y3 = y2 + cos(q1+q2);
X2 = [x1, x2,x3];
Y2 = [y1, y2,y3];
X3 = [-3,3];
Y3 = [0,0];
% ball
q = [0:0.1:1]' * 2*pi;
xb = 0.05*cos(q);
yb = 0.05*sin(q);

hold off
plot(X1,Y1,'m',X2,Y2,'m',xb+x2, yb+y2, 'm', xb+x3, yb+y3, 'm')
hold on

% Plant
x1 = X(1);
y1 = 0.2;
q1 = X(2);
q2 = X(3);
% cart
X1 = [-0.2,0.2,0.2,-0.2,-0.2] + x1;
Y1 = [0,0,0.2,0.2,0];
x2 = x1 + sin(q1);
y2 = y1 + cos(q1);
x3 = x2 + sin(q1+q2);
y3 = y2 + cos(q1+q2);
X2 = [x1, x2,x3];
Y2 = [y1, y2,y3];
X3 = [-3,3];
Y3 = [0,0];
% ball
q = [0:0.1:1]' * 2*pi;
xb = 0.05*cos(q);
yb = 0.05*sin(q);

plot(X1,Y1,'r',X2,Y2,'r',X3,Y3,'b',xb+x2, yb+y2, 'r', xb+x3, yb+y3, 'r',[Ref,
Ref],[-0.1,0.1],'b')
ylim([-0.5,2.5]);
pause(0.01);
end
```