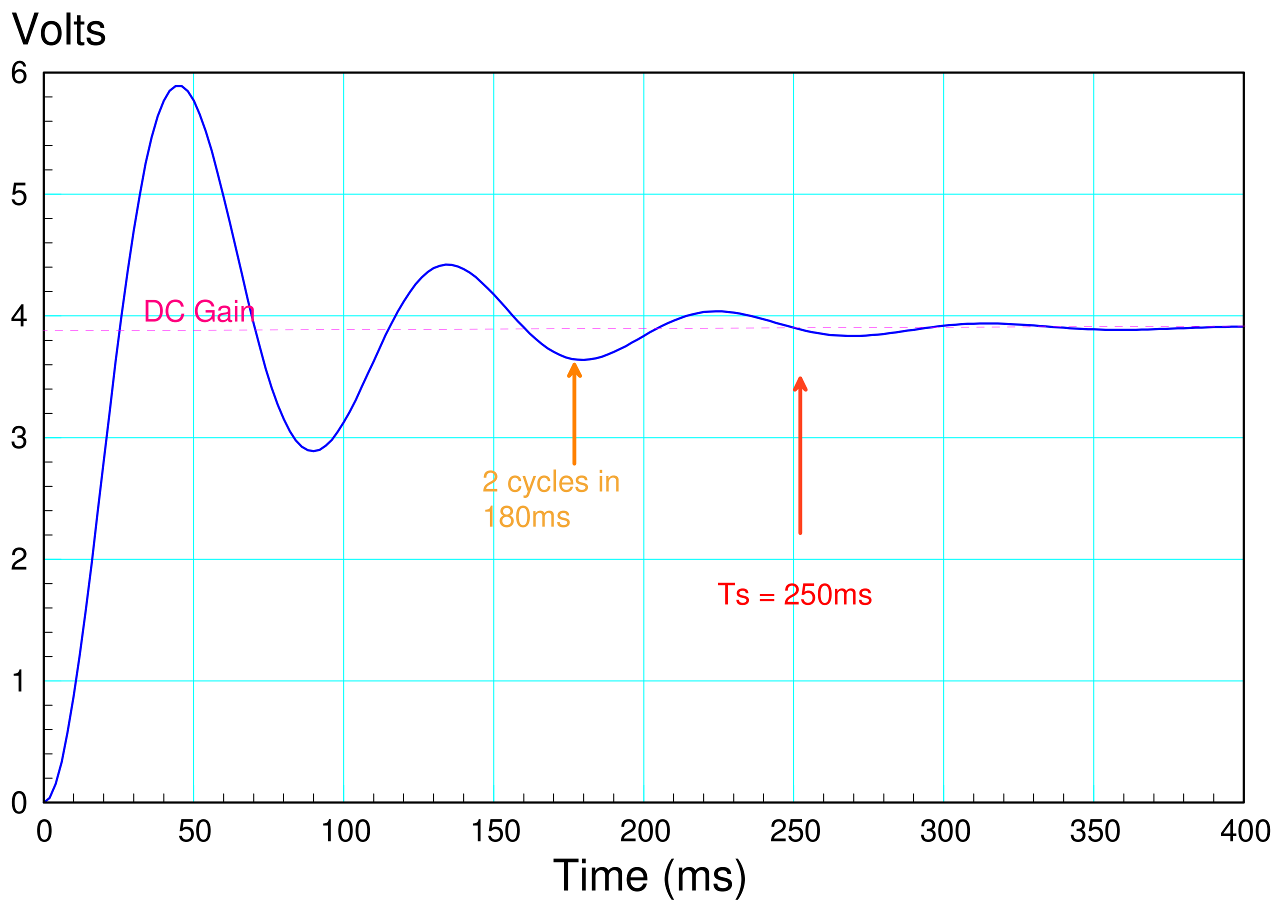
ECE 463/663: Test #1.   Name _______________

Spring 2024. Calculators allowed. Individual Effort

1) Find the transfer funciton for a system with the following step response



The 2% settling time is about 250ms

$$real(pole) \approx \frac{4}{0.25} = 16$$

The frequency of oscillation (imaginary part of pole)

$$imag(pole) \approx \left(\frac{2 \text{ cycles}}{180ms}\right) 2\pi = 69.8 \frac{rad}{\sec}$$

DC gain is 3.9 (ish)

$$G(s) \approx \left(\frac{19{,}999}{(s+16+j69.8)(s+16-j69.8)}\right)$$

(The numerator sets the DC gain to 3.9)

2) Determine a 2nd-order system which has approximately the same step response as the following system

$$Y = \left(\frac{50{,}000(s+2)(s+30)}{(s+3+j5)(s+3-j5)(s+22)(s+35)(s+40)}\right)X$$

Keep

- The dominant pole (s + 3+j5)
- It's complex conjugate (s + 3 - j5), and
- Zeros within similar magnitude (s+2)

$$Y \approx \left(\frac{k(s+2)}{(s+3+j5)(s+3-j5)}\right)X$$

Pick k to match the DC gain

$$\left(\frac{50{,}000(s+2)(s+30)}{(s+3+j5)(s+3-j5)(s+22)(s+35)(s+40)}\right)_{s=0} = 2.8648$$

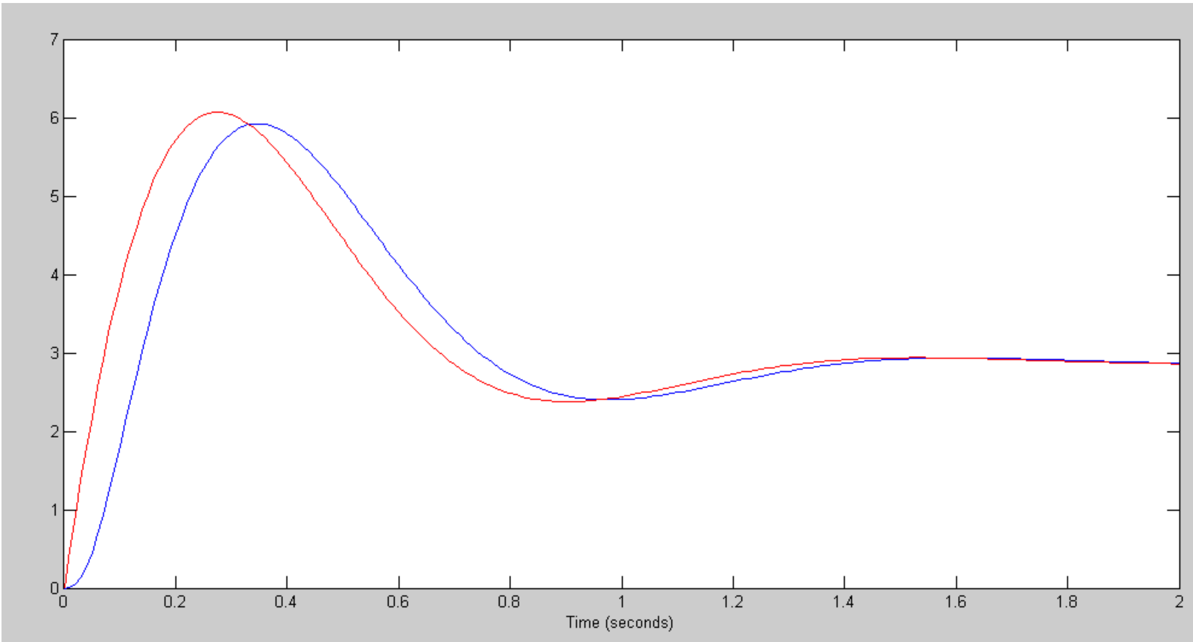
$$\left(\frac{k(s+2)}{(s+3+j5)(s+3-j5)}\right)_{s=0} = 2.8648$$

k = 48.7013

so

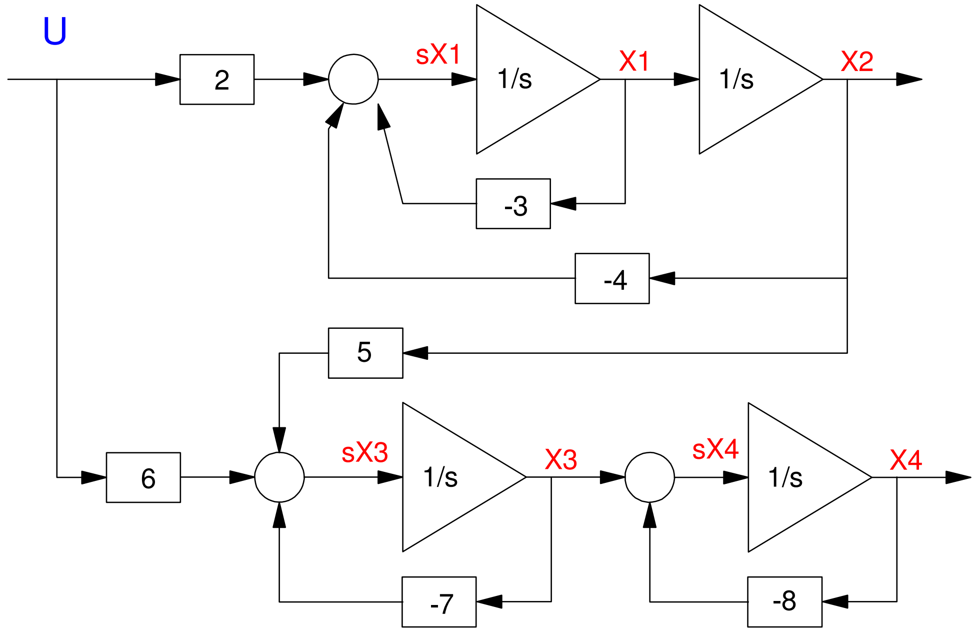
$$Y \approx \left(\frac{48.70(s+2)}{(s+3+j5)(s+3-j5)}\right)X$$

(not asked for) In Matlab, the two systems look like...



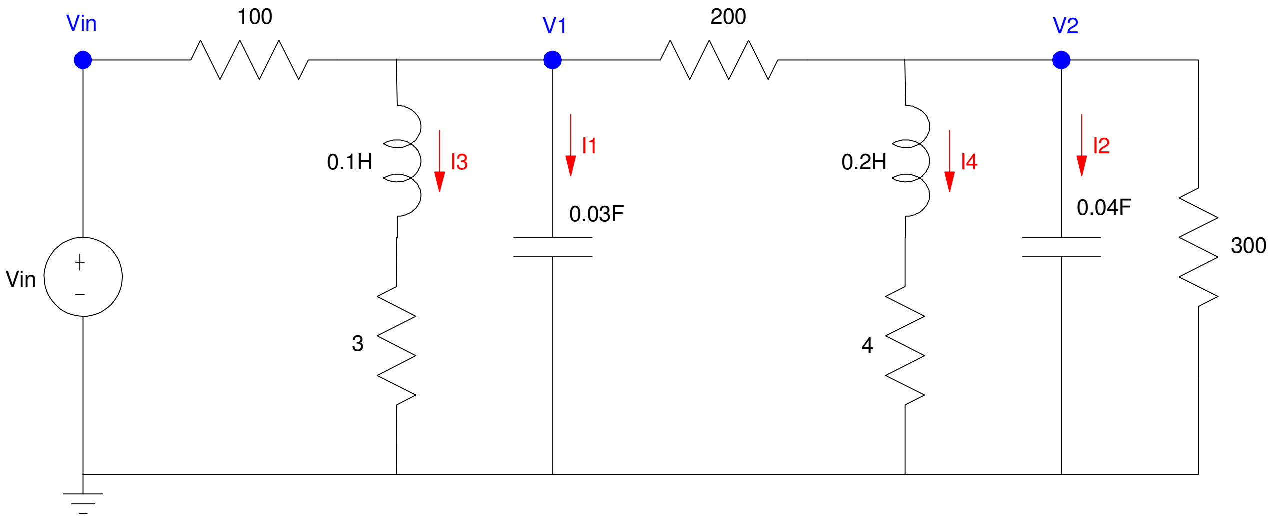
5th-Order Sytem (blue) & 2nd-Order Approximation (red)
(not required)

3) Give {A and B} for the the state-space model for the following system



$$\begin{bmatrix} sX1 \\ sX2 \\ sX3 \\ sX4 \end{bmatrix} = \begin{bmatrix} -3 & -4 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 5 & -7 & 0 \\ 0 & 0 & 1 & -8 \end{bmatrix} \begin{bmatrix} X1 \\ X2 \\ X3 \\ X4 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ 6 \\ 0 \end{bmatrix} U$$

4) Write four coupled differential equations to describe the following circuit. Assume the states are {V1, V2, I3, I4}. Note: For capacitors: $I = C\frac{dV}{dt}$, For inductors: $V = L\frac{dI}{dt}$



$$0.03sV_1 = \left(\frac{V_{in}-V_1}{100}\right) - I_3 - \left(\frac{V_1-V_2}{200}\right)$$

$$0.04sV_2 = \left(\frac{V_1-V_2}{200}\right) - I_4 - \left(\frac{V_2}{300}\right)$$

$$0.1sI_3 = V_1 - 3I_3$$

$$0.2sI_4 = V_2 - 4I_4$$

5) Assume the LaGrangian is:

$$L = 2x\cos(x)\dot{x}^2 + 3x\dot{x}\sin(\theta) + 7\cos(2\theta)\dot{\theta}^2$$

Determine

$$F = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \left(\frac{\partial L}{\partial x}\right)$$

$$F = \frac{d}{dt}(4x\cos(x)\dot{x} + 3x\sin(\theta))$$
$$-(2\cos(x)\dot{x}^2 - 2x\sin(x)\dot{x}^2 + 3\dot{x}\sin(\theta))$$

$$F = 4\cos(x)\dot{x}^2 - 4x\sin(x)\dot{x}^2 + 4x\cos(x)\ddot{x}$$
$$+3\dot{x}\sin(\theta) + 3x\cos(\theta)\dot{\theta}$$
$$-2\cos(x)\dot{x}^2 + 2x\sin(x)\dot{x}^2 - 3\dot{x}\sin(\theta)$$