

# ECE 463/663 - Homework #9

Calculus of Variations. Riccati Equation. Due Wednesday, April 3rd  
Please submit as a hard copy, email to [jacob.glower@nds.u.edu](mailto:jacob.glower@nds.u.edu), or submit on BlackBoard

## Soap Film

- 1) Calculate the shape of a soap film connecting two rings around the X axis:
  - $Y(0) = 6$
  - $Y(4) = 9$
- 2) Calculate the shape of a soap film connecting two rings around the X axis:
  - $Y(0) = 6$
  - $Y(2) = \text{free}$

## Hanging Chain

- 3) Calculate the shape of a hanging chain subject to the following constraints
  - Length of chain = 11 meters
  - Left Endpoint: (0,7)
  - Right Endpoint: (10,8)

## Ricatti Equation

- 4) Find the function,  $x(t)$ , which minimizes the following functional

$$J = \int_0^{10} (4x^2 + 2\dot{x}^2) dt$$

$$x(0) = 5$$

$$x(10) = 6$$

- 5) Find the function,  $x(t)$ , which minimizes the following functional

$$J = \int_0^{10} (4x^2 + 2u^2) dt$$

$$\dot{x} = -0.1x + u$$

$$x(0) = 5$$

$$x(10) = 6$$