

ECE 463/663 - Homework #10

LQG Control. Due Monday, April 8th
Please submit as a hard copy, email to jacob.glower@ndsu.edu, or submit on BlackBoard

LQG Control

1) Cart & Pendulum (HW #4 & HW#6):

$$s \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -2.45 & 0 & 0 \\ 0 & 9.42 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.25 \\ -0.1923 \end{bmatrix} F$$

Design a full-state feedback control law of the form

$$F = U = K_r R - K_x X$$

for the cart and pendulum system from homework #4 using LQG control so that

- The DC gain is 1.00
- The 2% settling time is 8 seconds, and
- There is less than 5% overshoot for a step input.

Compare your results with homework #6

- Where are the closed-loop poles with pole placement and with LQG control?
- Are the feedback gains larger or smaller with LQG control?
- Which one works better?

2) Cart & Pendulum with Multiple Inputs

Assume a torque on the beam is also allowed

$$s \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -2.45 & 0 & 0 \\ 0 & 9.42 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.25 \\ -0.1923 \end{bmatrix} F + \begin{bmatrix} 0 \\ 0 \\ -0.1923 \\ 0.7396 \end{bmatrix} T$$

Design a full-state feedback control law of the form

$$\begin{bmatrix} F \\ T \end{bmatrix} = K_r R - K_x X$$

where K_x is a 2x4 matrix using LQG techniques so that

- The DC gain from R to x is 1.00
- The 2% settling time is 8 seconds, and
- There is less than 5% overshoot for a step input.

3) Ball and Beam (HW #4 & HW#6):

$$s \begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -7 & 0 & 0 \\ -5.88 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.2 \end{bmatrix} T$$

Design a full-state feedback control law of the form

$$T = U = K_r R - K_x X$$

for the ball and beam system from homework #4 using LQG control so that

- The DC gain is 1.00
- The 2% settling time is 8 seconds, and
- There is less than 5% overshoot for a step input.

Compare your results with homework #6

- Where are the closed-loop poles with pole placement and with LQG control?
- Are the feedback gains larger or smaller with LQG control?
- Which one works better?