

# ECE 463/663 - Homework #12

LQR Observers. Due Monday, April 22nd  
Please submit as a hard copy, email to jacob.glower@ndsu.edu, or submit on BlackBoard

## Kalman Filters

**Cart and Pendulum (HW #4):** The dynamics for a cart and pendulum system with sensor and input noise is as follows

$$s \begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -2.45 & 0 & 0 \\ 0 & 9.42 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.25 \\ -0.1923 \end{bmatrix} (F + \eta_u)$$

$$y_1 = x + n_x$$

$$y_2 = \theta + n_\theta$$

where there is Gaussian noise at the input and output

$$n_u \sim \mathcal{N}(0, 0.5^2) \quad \text{mean zero, standard deviation } 0.5$$

$$n_x \sim \mathcal{N}(0, 0.1^2) \quad \text{mean zero, standard deviation } 0.1$$

$$n_\theta \sim \mathcal{N}(0, 0.05^2) \quad \text{mean zero, standard deviation } 0.05$$

1) Use a servo-compensator to force the DC gain to one (i.e. use the servo compensator from homework set #10).

Plot the step response

- Without noise (same as homework set #10)
- With noise

2) Design a full-order observer using pole-placement to place the observer poles at  $\{-3, -3, -3, -3\}$

- Simulate the response of the cart with noise added at the input and output.
- Plot the states of the plant and the observer with noise,.

3) Design a Kalman filter (i.e. a full-order observer with a specific Q and R)

- Simulate the response of the cart with noise added at the input and output.
- Plot the states of the plant and the observer with noise,.