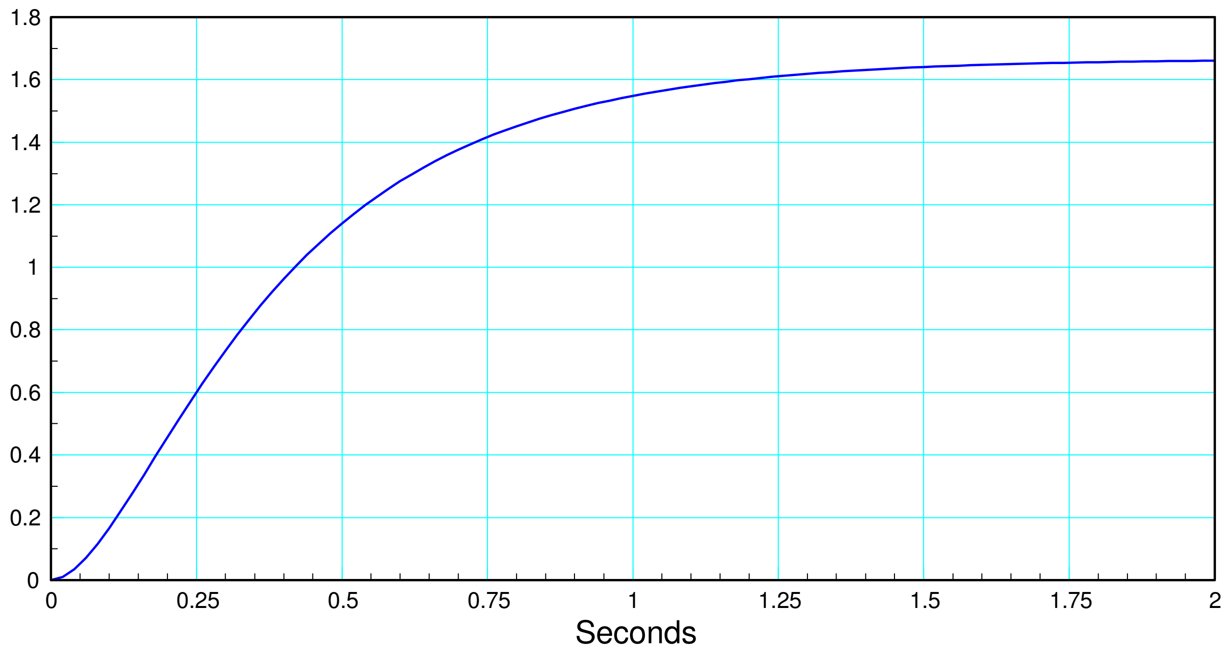
ECE 463/663 - Homework #1

LaPlace Transforms and Dominant Poles. Due Wednesday, Jan 17th
Please submit as a hard copy or submit on BlackBoard

1) Name That System! Give the transfer function for a system with the following step response.



This looks like a 1st-order system (no oscillations)

$$G(s) \approx \left(\frac{a}{s+b}\right)$$

Ts = 1.5 seconds (approx)

$$b \approx \frac{4}{1.5s} = 2.667$$

DC gain = 1.65 (approx)

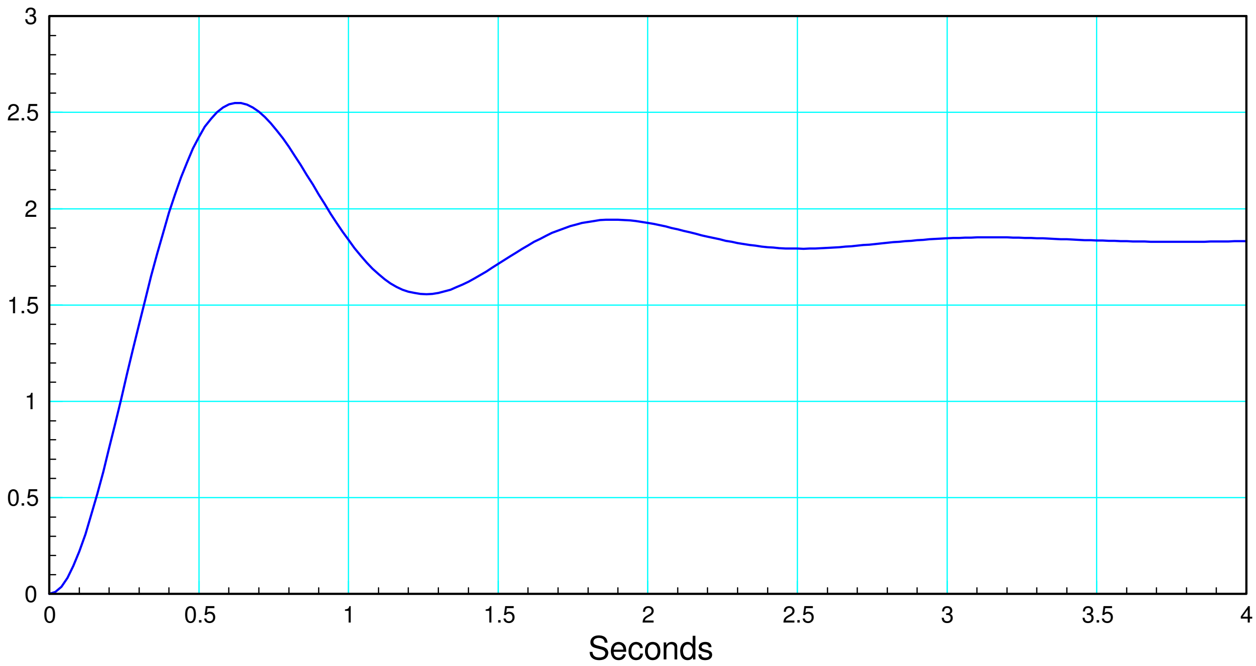
$$\left(\frac{a}{s+b}\right)_{s=0} = 1.65$$

$$a = 4.40$$

so

$$G(s) \approx \left(\frac{4.400}{s+2.667}\right)$$

2) Name That System! Give the transfer function for a system with the following step response.



This is a second-order system (oscillations)

$$G(s) \approx \left(\frac{a}{(s+b+jc)(s+b-jc)} \right)$$

Ts = 2.5 seconds (approx)

$$b \approx \frac{4}{2.4} = 1.60$$

The frequency of oscillations (in rad/sec) is

$$c = \left(\frac{2 \text{ cycles}}{2.5\text{s}} \right) 2\pi = 5.02$$

DC gain is 1.8 (approx)

$$\left(\frac{a}{(s+1.6+j5.02)(s+1.6-j5.02)} \right)_{s=0} = 1.8$$

$$a = 50.08$$

and

$$G(s) \approx \left(\frac{50.08}{(s+1.6+j5.02)(s+1.6-j5.02)} \right)$$

Problem 3 - 6) Assume

$$Y = \left(\frac{40(s+6)}{(s+2)(s+8)(s+9)}\right)X$$

3) What is the differential equation relating X and Y?

Cross multiply and multiply out the polynomials

$$((s+2)(s+8)(s+9))Y = 40(s+6)X$$

$$(s^3 + 19s^2 + 106s + 144)Y = (40s + 240)X$$

Note that sY means 'the derivative of y(t)'

$$\boxed{y''' + 19y'' + 106y' + 144y = 40x' + 240x}$$

4) Determine y(t) assuming x(t) is a sinusoidal input:

$$x(t) = 4\cos(6t) + 2\sin(6t)$$

Use phasors

$$s = j6$$

$$X = 4 - j2$$

$$Y = \left(\frac{40(s+6)}{(s+2)(s+8)(s+9)}\right)_{s=j6} \cdot (4 - j2)$$

$$Y = -1.2308 - 1.8462i$$

meaning

$$\boxed{y(t) = -1.2308\cos(6t) + 1.8462\sin(6t)}$$

5) Determine y(t) assuming x(t) is a step input:

$$x(t) = u(t)$$

$$Y = \left(\frac{40(s+6)}{(s+2)(s+8)(s+9)}\right)X$$

$$Y = \left(\frac{40(s+6)}{(s+2)(s+8)(s+9)}\right)\left(\frac{1}{s}\right)$$

Use partial fractions

$$Y = \left(\frac{1.667}{s}\right) + \left(\frac{3.8095}{s+2}\right) + \left(\frac{13.333}{s+8}\right) + \left(\frac{-17.1429}{s+9}\right)$$

Take the inverse LaPlace transform

$$y(t) = (1.667 + 3.8095e^{-2t} + 13.333e^{-8t} - 17.1429e^{-9t})u(t)$$

6a) Determine a 1st-order approximation for this system

$$Y = \left(\frac{40(s+6)}{(s+2)(s+8)(s+9)}\right)X \approx \left(\frac{a}{s+b}\right)X$$

Keep the dominant pole (s+2)

Match the DC gain

$$\left(\frac{40(s+6)}{(s+2)(s+8)(s+9)}\right)_{s=0} = 1.6667$$

$$\left(\frac{a}{s+2}\right)_{s=0} = 1.6667$$

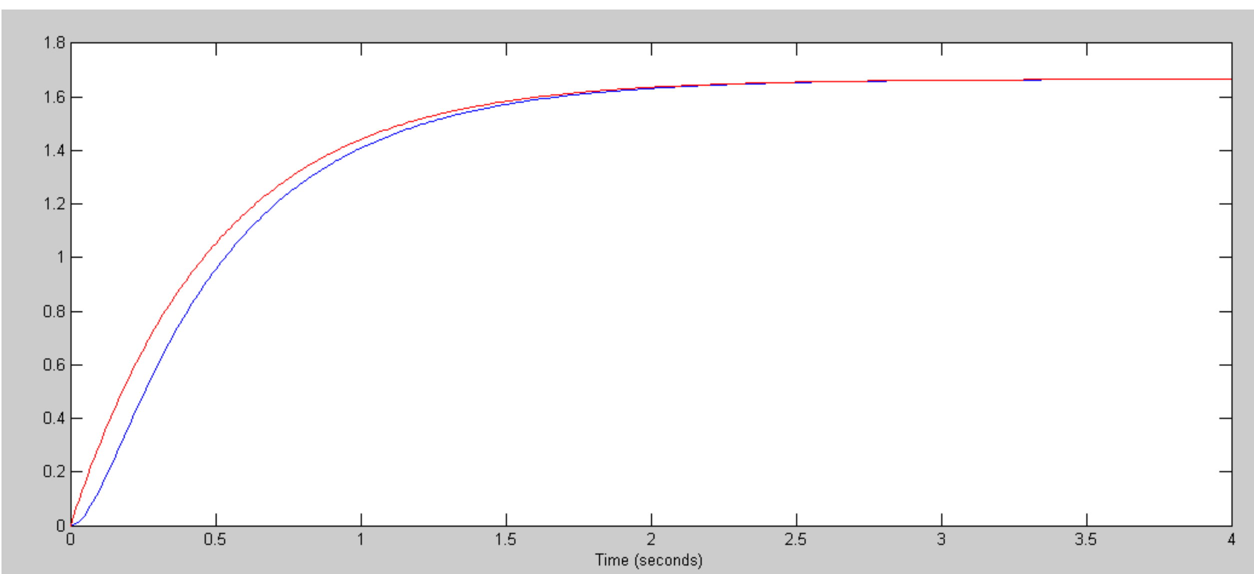
$$a = 3.33333$$

so

$$\left(\frac{40(s+6)}{(s+2)(s+8)(s+9)}\right)X \approx \left(\frac{3.333}{s+2}\right)$$

6b) Compare the step response of your 1st-order model to the actual 3rd-order system

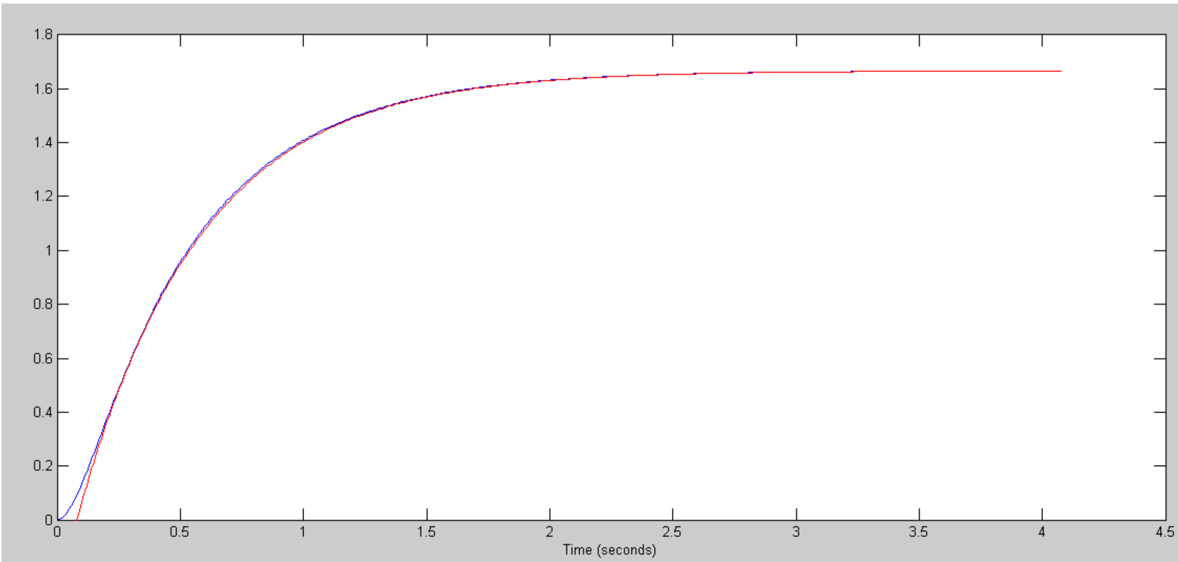
```
>> G3 = zpk(-6,[-2,-8,-9],40);
>> G1 = zpk([],-2,3.3333);
>> t = [0:0.01:4]';
>> y3 = step(G3,t);
>> y1 = step(G1,t);
>> plot(t,y3,'b',t,y1,'r')
>> xlabel('Time (seconds)')
>>
```



3rd-Order Model (blue) & 1st-Order Approximation (red)

Sidelight: Adding a delay makes for a better model. Adding an 80ms delay to the 1st-order model gives

```
>> plot(t,y3,'b',t+0.08,y1,'r')
>> xlabel('Time (seconds)')
```



3rd-Order Model (blue) & 1st-Order Approximation with an 80ms Delay (red)

so a better model would be

$$\left(\frac{40(s+6)}{(s+2)(s+8)(s+9)}\right)X \approx \left(\frac{3.333}{s+2}\right) \cdot e^{-0.08s}$$