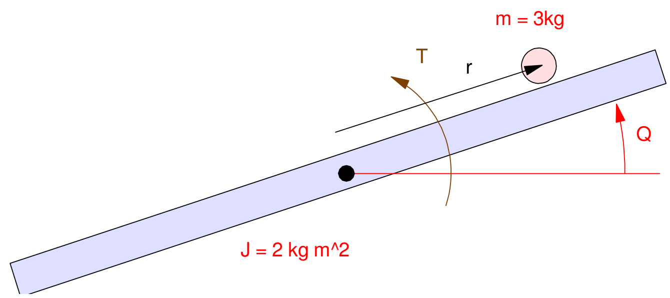
ECE 463/663 - Homework #7

Servo Compensators. Due Monday, March 11th
Please submit as a hard copy, email to jacob.glower@ndsu.edu, or submit on BlackBoard



The dynamics of a Ball and Beam System (homework set #4) with a disturbance are

$$s\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -7 & 0 & 0 \\ -5.88 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.2 \end{bmatrix} T + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.2 \end{bmatrix} d$$

Full-State Feedback with Constant Disturbances

1) For the nonlinear simulation, use the feedback control law you computed in homework #6

- With R = 1 and the mass of the ball = 3.0kg (same result you got for homework #6), and
- With R = 1 and the mass of the ball decreased to 2.5kg

(i.e. a constant disturbance on the system due to a different mass of the ball)

Step 1: Find feedback gains to stabilze the system. Input the dynamics (A, B, C, D matricies)

```
>> A = [0,0,1,0;0,0,0,1;0,-7,0,0;-5.88,0,0,0]

         0         0    1.0000         0
         0         0         0    1.0000
         0   -7.0000         0         0
   -5.8800         0         0         0

>> B = [0;0;0;0.2];
>> C = [1,0,0,0];
>> D = 0;
```

Find Kx and Kr to stabilize the system

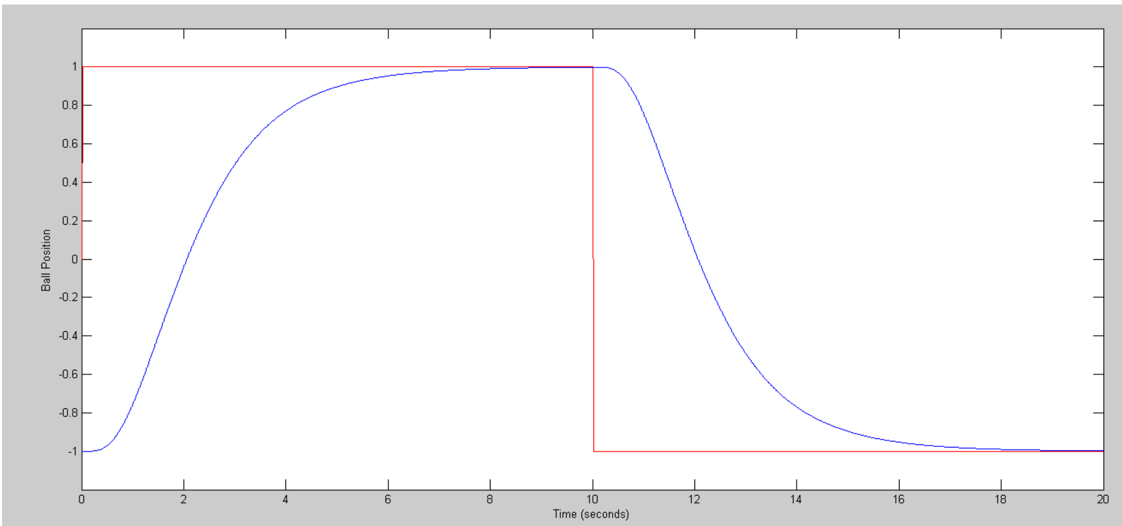
```
>> Kx = ppl(A, B, [-0.8,-2,-3,-4])

Kx =   -43.1143   166.0000   -32.0000    49.0000

>> DC = -C*inv(A-B*Kx)*B
```

```
DC =   -0.0729

>> Kr = 1/DC

Kr =  -13.7143

>> ylim([-1.2,1.2])
```

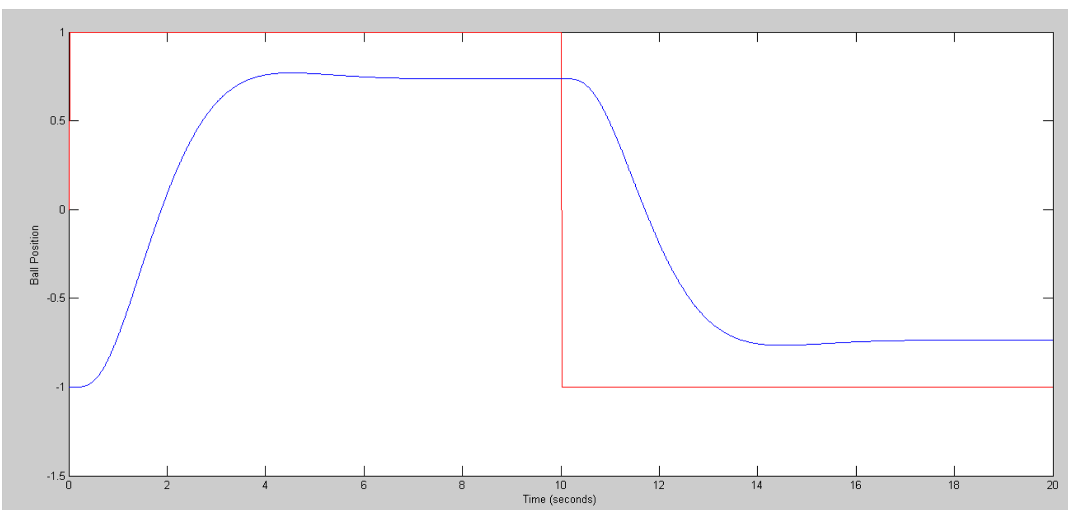
Use the feedback control law

U = Kr * Ref - Kx * X;

Results:

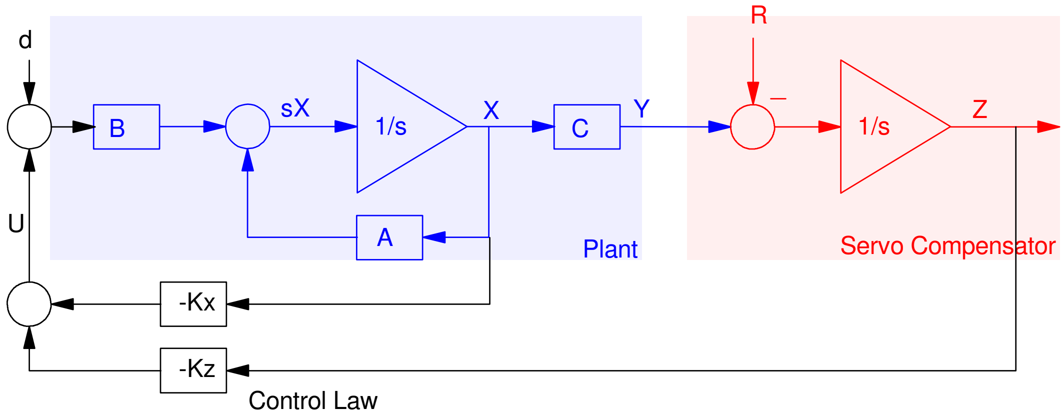


Step Response with m = 3.0kg (nominal case)



Step Response with m = 2.5kg (pertubation)

Servo Compensators with Constant Set-Points



2) Assume a constant disturbance and/or a constant set point. Design a feedback control law that results in

- The ability to track a constant set point (R = constant)
- The ability to reject a constant disturbance (d = constant),
- A 2% settling time of 8 seconds, and
- No overshoot for a step input.

Create the augmented system (plant + servo compensator)

```
>> A5 = [A, zeros(4,1) ; C, 0]

         0         0    1.0000         0         0
         0         0         0    1.0000         0
         0   -7.0000         0         0         0
   -5.8800         0         0         0         0
    1.0000         0         0         0         0

>> B5u = [B;0]

         0
         0
         0
    0.2000
         0

>> B5r = [0;0;0;0;-1]

     0
     0
     0
     0
    -1

>> C5 = [C,0]

     1     0     0     0     0
```

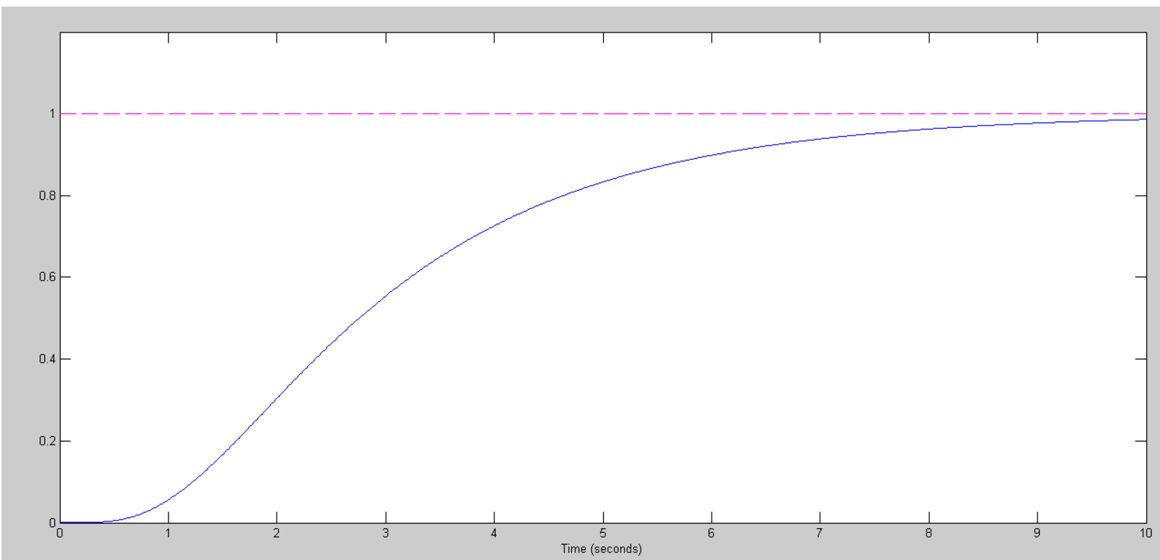
Find the feedback gains, Kx and Kz:

```
>> K5 = ppl(A5, B5u, [-0.5,-2,-3,-4,-5])

K5 = -170.1143  390.0000 -135.3571   72.5000  -42.8571

>> Kx = K5(1:4)

Kx = -170.1143  390.0000 -135.3571   72.5000

>> Kz = K5(5)

Kz =  -42.8571

>>
```

3) For the linear system, plot the step response
   - With respect to a step change in R, and
   - With respect to a step change in d

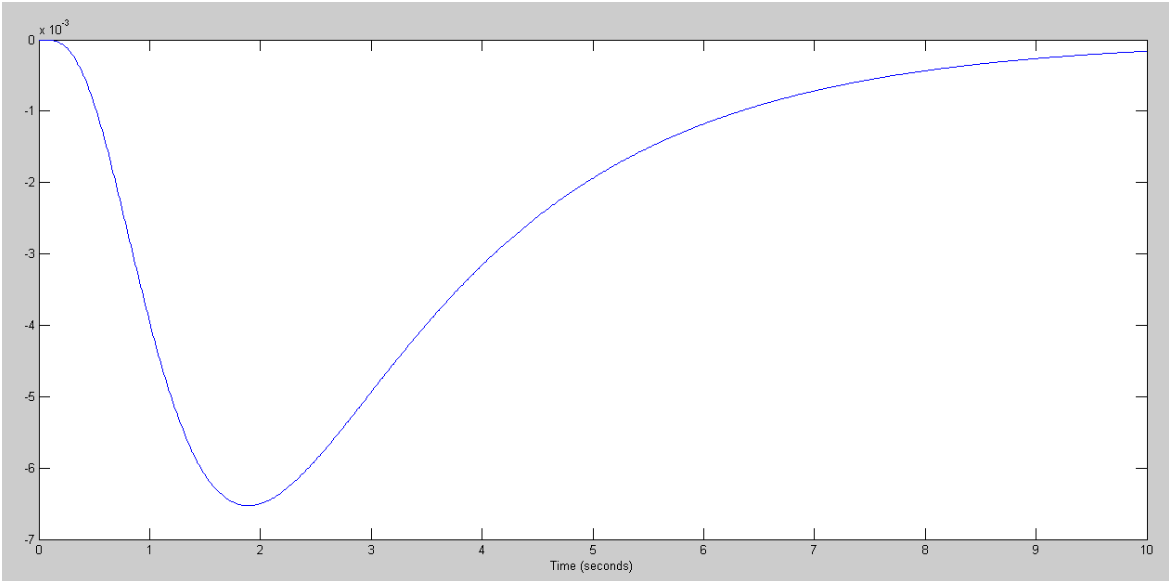
Step Respnse with respect to R:

```
>> t = [0:0.01:10]';
>> G5 = ss(A5 - B5u*K5, B5r, C5, D5);
>> y = step(G5,t);
>> plot(t,y,t,0*y+1,'m--')
>> ylim([0,1.2])
>> xlabel('Time (seconds)');
```



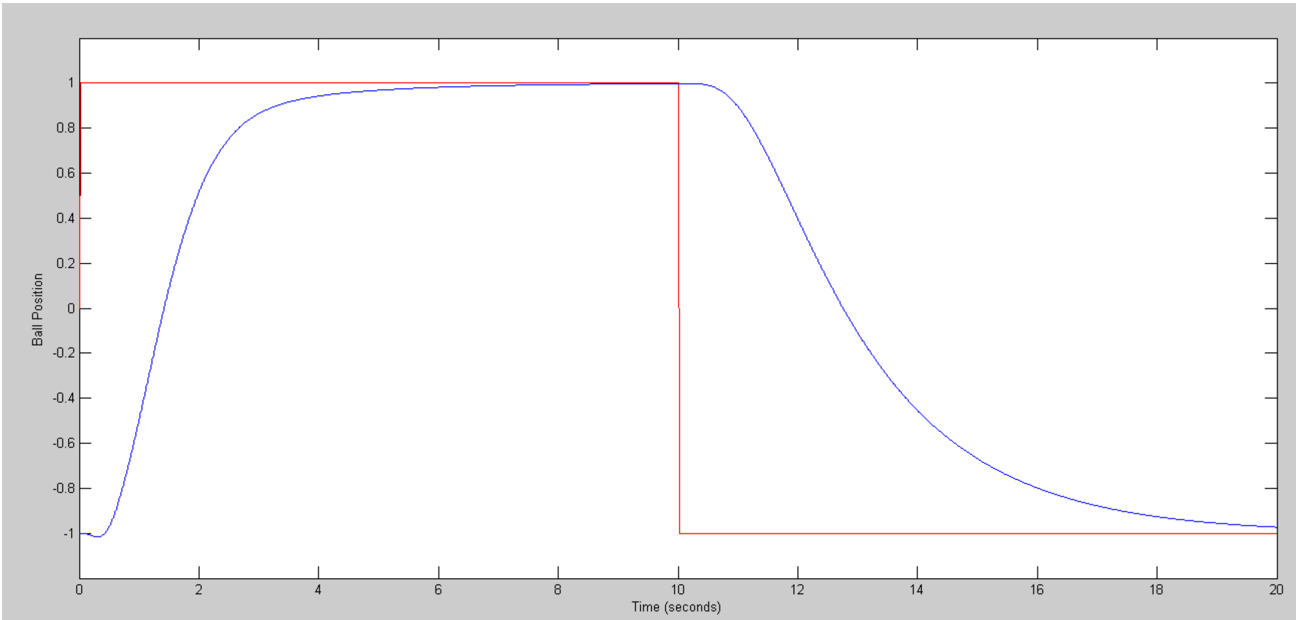
Step Response with respect to d:

```
>> G5 = ss(A5 - B5u*K5, B5u, C5, D5);
>> y = step(G5,t);
>> plot(t,y)
>> xlabel('Time (seconds)');
```

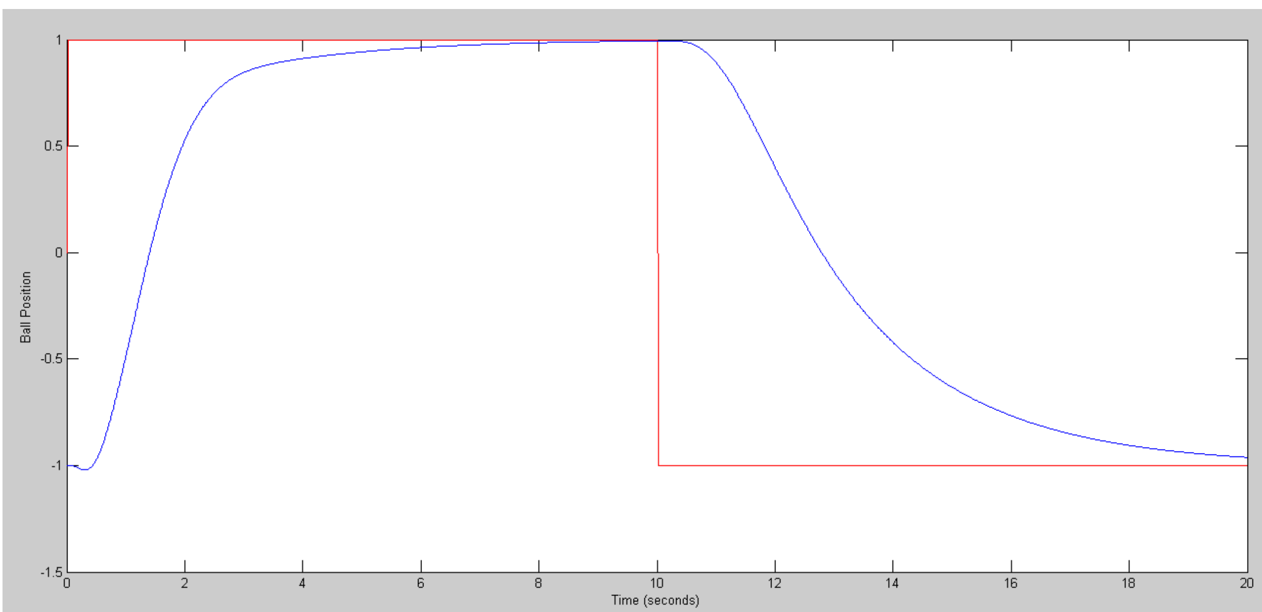

x 10^-3
0
-1
-2
-3
-4
-5
-6
-7
0
1
2
3
4
5
6
7
8
9
10
Time (seconds)

4) Implement your control law on the nonlinear ball and beam system

With R = 1 and the mass of the ball being 3.0kg, and



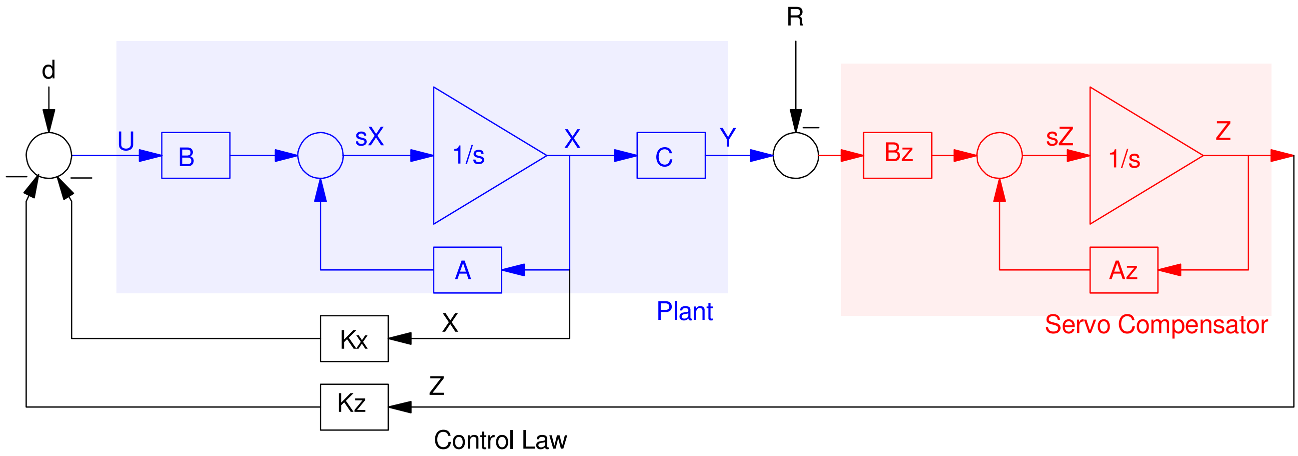
With R = 1 and the mass of the ball being 2.5kg



Code:

```
% Ball & Beam System

X = [-1, 0, 0, 0]';
dt = 0.01;
t = 0;
% Feedback Control & Servo Compensator
Az = 0;
Bz = 1;
Z = 0;

Kx = [ -170.1143  390.0000 -135.3571   72.5000];
Kz = [-42.8571];

n = 0;
y = [];

while(t < 20)
 Ref = sign(sin(2*pi*t/20));
 U = -Kz*Z - Kx*X;
 % U = Kr*Ref - Kx*X;
 dX = BeamDynamics(X, U);
 dZ = Az*Z + Bz*(X(1) - Ref);

 X = X + dX * dt;
 Z = Z + dZ * dt;
 t = t + dt;

 y = [y ; Ref, X(1)];
 n = mod(n+1,5);
 if(n == 0)
    BeamDisplay(X, Ref);
 end
 end

t = [1:length(y)]' * dt;


plot(t,y(:,1),'r',t,y(:,2),'b');
xlabel('Time (seconds)');
ylabel('Ball Position');
```

Servo Compensators with Sinulsoidal Set-Points



5) Assume a 0.7 rad/sec disturbance and/or set point (R). Design a feedback control law that results in

- The ability to track a constant set point (R = sin(0.7t))
- The ability to reject a constant disturbance (d = sin(0.7t)),
- A 2% settling time of 12 seconds, and

Input the augmented system into Matlab

```
>> Az = [0,0.7 ; -0.7,0]

         0    0.7000
   -0.7000         0

>> Bz = [1;1]

     1
     1

>> A6 = [A, zeros(4,2) ; Bz*C, Az]

         0         0    1.0000         0         0         0
         0         0         0    1.0000         0         0
         0   -7.0000         0         0         0         0
   -5.8800         0         0         0         0         0
    1.0000         0         0         0         0    0.7000
    1.0000         0         0         0   -0.7000         0

>> B6u = [B ; 0 ; 0]
>> B6r = [zeros(4,1) ; -Bz]
>> C6 = [C, 0, 0];
>> D6 = 0;
```

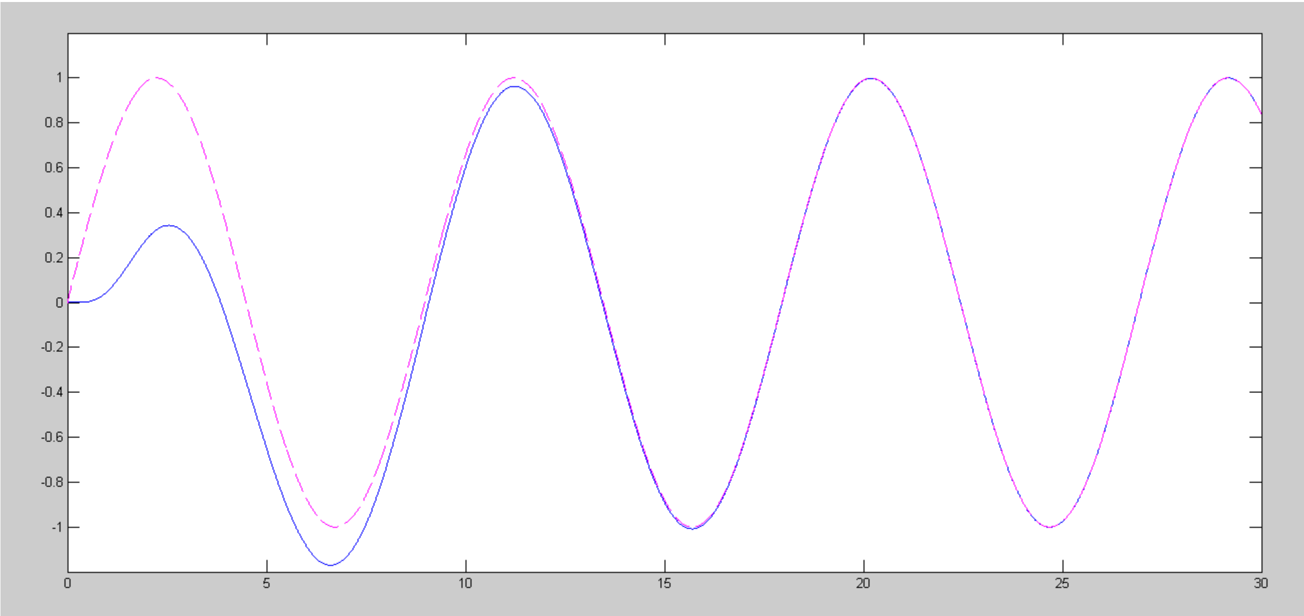
Find the feedback gains to place the poles for a 12 second settling time:

```
>> K6 = ppl(A6, B6u, [-4/12, -2, -3, -4, -5, -6])

K6 = -856.7977  805.8833 -444.0738  101.6667 -105.4903 -439.7707
```

```
>> Kx = K6(1:4)

Kx = -856.7977  805.8833 -444.0738  101.6667

>> Kz = K6(5:6)

Kz = -105.4903 -439.7707
```

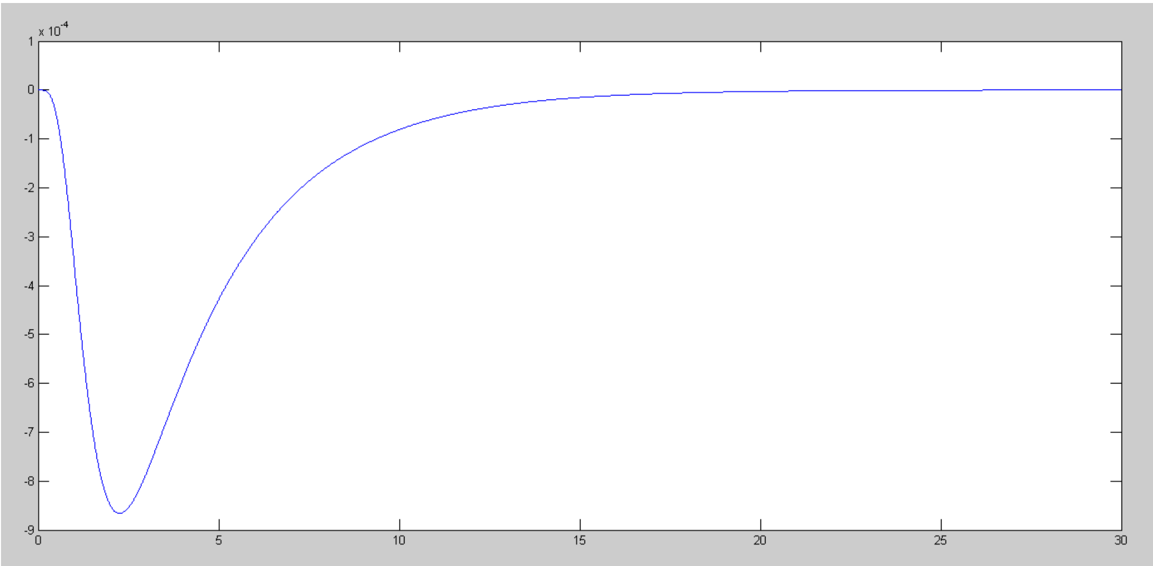
6) For the linear system, plot the response

```
>> t = [0:0.01:30]';
>> R = sin(0.7*t);

>> y = step3(A6-B6u*K6, B6r, C6, 0, t, X0, R);
>> plot(t,y,'b',t,R,'m--')
>> ylim([-1.2,1.2])
```



Step Response to R = sin(0.7t)

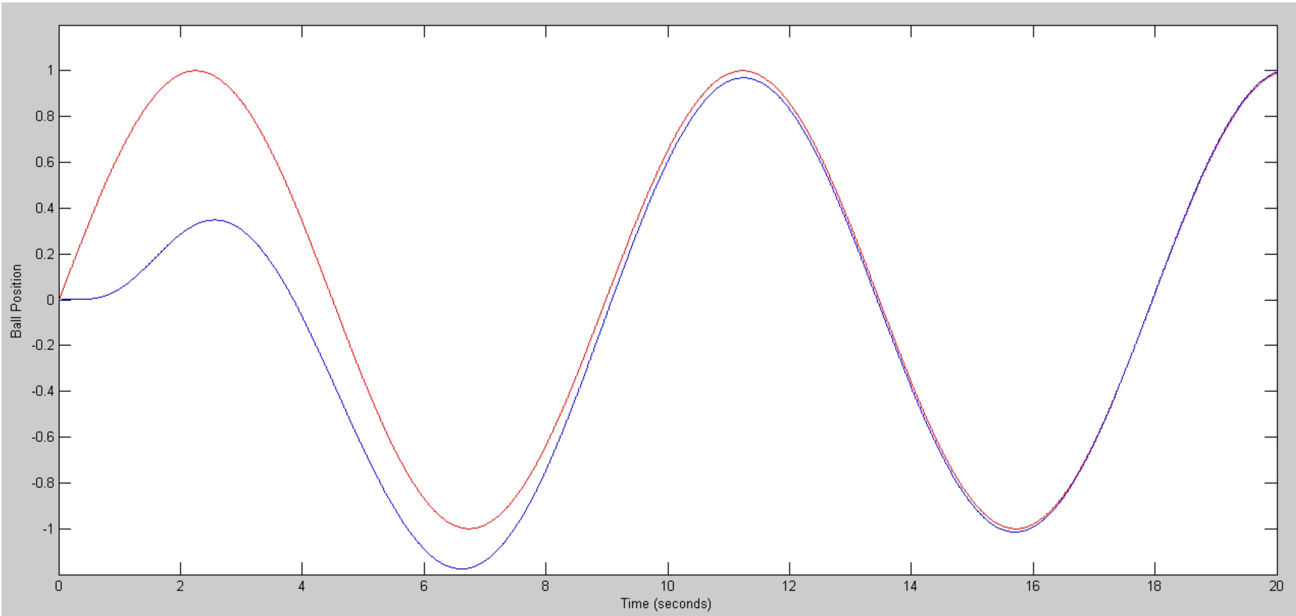
```
>> y = step3(A6-B6u*K6, B6u, C6, 0, t, X0, R);
>> plot(t,y,'b')
```



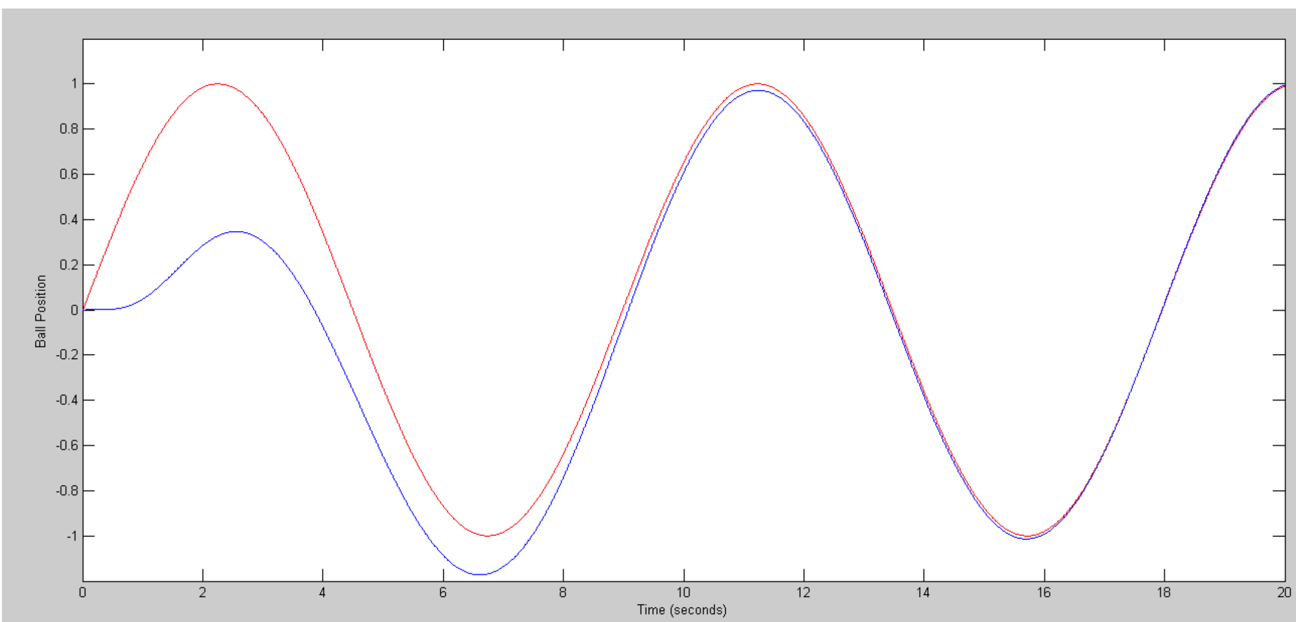
Step Response to d = sin(0.7*t)

7) Implement your control law on the nonlinear ball and beam system

- With R = sin(0.7t) and the mass of the ball being 3.0kg, and
- With R = sin(0.7t) and the mass of the ball being 2.5kg



Response with m = 3.0kg



Response with m = 2.5kg

Code:

```
% Ball & Beam System

X = [0, 0, 0, 0]';
dt = 0.01;
t = 0;
% Feedback Control & Servo Compensator
Az = [0,0.7 ; -0.7, 0];
Bz = [1;1];
Z = zeros(2,1);

Kx = [ -856.7977  805.8833 -444.0738  101.6667];
Kz = [-105.4903 -439.7707];

n = 0;
y = [];

while(t < 20)
 Ref = sin(0.7*t);
 U = -Kz*Z - Kx*X;
 % U = Kr*Ref - Kx*X;
 dX = BeamDynamics(X, U);
 dZ = Az*Z + Bz*(X(1) - Ref);

 X = X + dX * dt;
 Z = Z + dZ * dt;
 t = t + dt;

 y = [y ; Ref, X(1)];
 n = mod(n+1,5);
 if(n == 0)
    BeamDisplay(X, Ref);
 end
 end

t = [1:length(y)]' * dt;


plot(t,y(:,1),'r',t,y(:,2),'b');
xlabel('Time (seconds)');
ylabel('Ball Position');
```