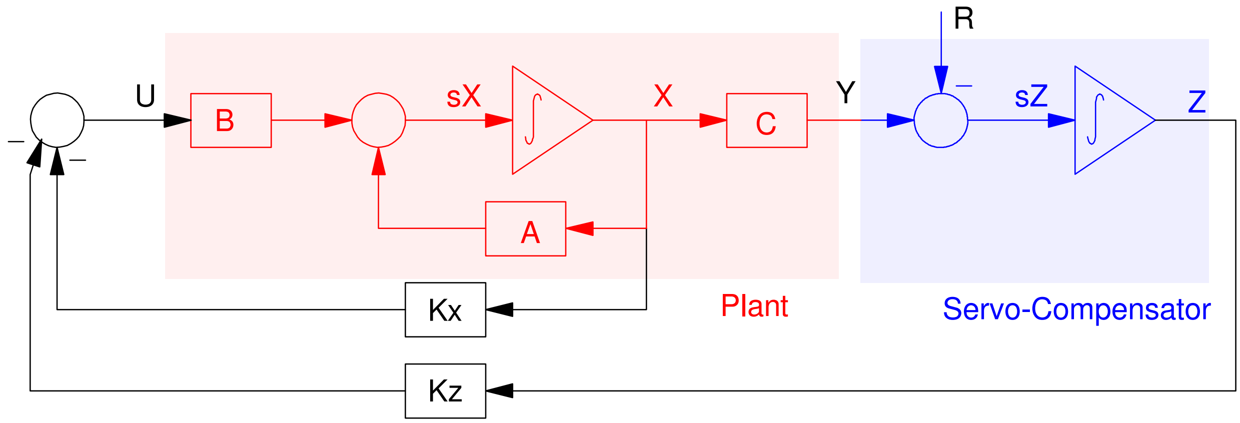
ECE 463/663 - Homework #11

LQG Control with Servo Compensators. Due Monday, April 15th



Cart and Pendulum (HW #4): For the cart and pendulum system of homework #4

$$s\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -2.45 & 0 & 0 \\ 0 & 9.42 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.25 \\ -0.1923 \end{bmatrix} F$$

Use LQG methods to design a full-state feedback control law of the form

$$F = U = -K_z Z - K_x X$$

$$\dot{Z} = (x - R)$$

for the cart and pendulum system from homework #4 using LQG control so that

- You track constant setpoints,
- You reject constant disturbances,
- The 2% settling time is 8 seconds, and
- There is less than 10% overshoot for a step input.

1) Give the control law (Kx and Kz) and explain how you chose Q and R

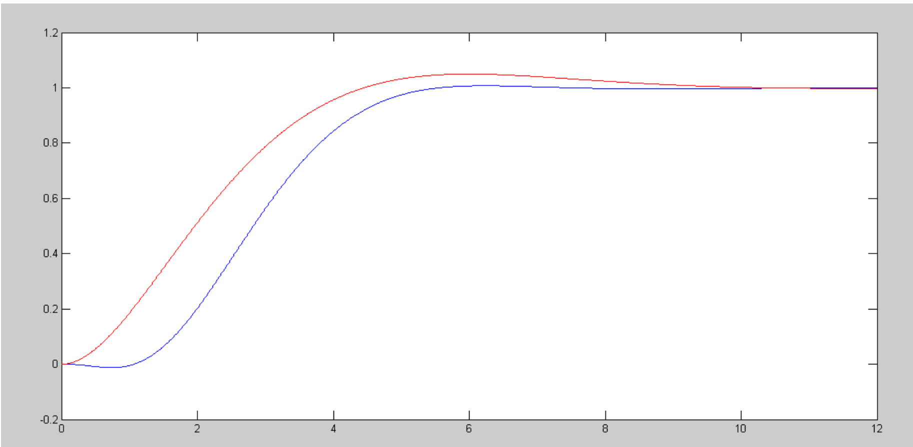
```
>> Kx = K5(1:4)

Kx =  -15.9909 -203.1194  -20.6044  -70.3334

>> Kz = K5(5)

Kz =   -5.4772
```

Q and R fround by trial and error

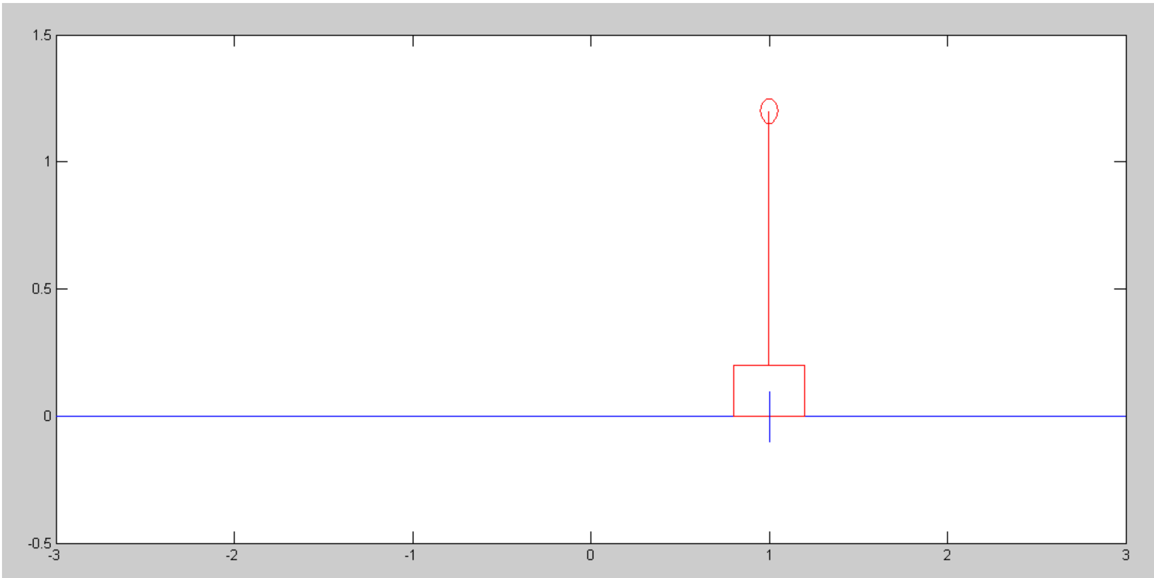
- Start with Q weighting Z: Q= diag([0,0,0,0,1]), R = 1
- Increase Q until it's a little fast
- Increase the weighting on X: Q = diag([30,0,0,0,30]), R = 1

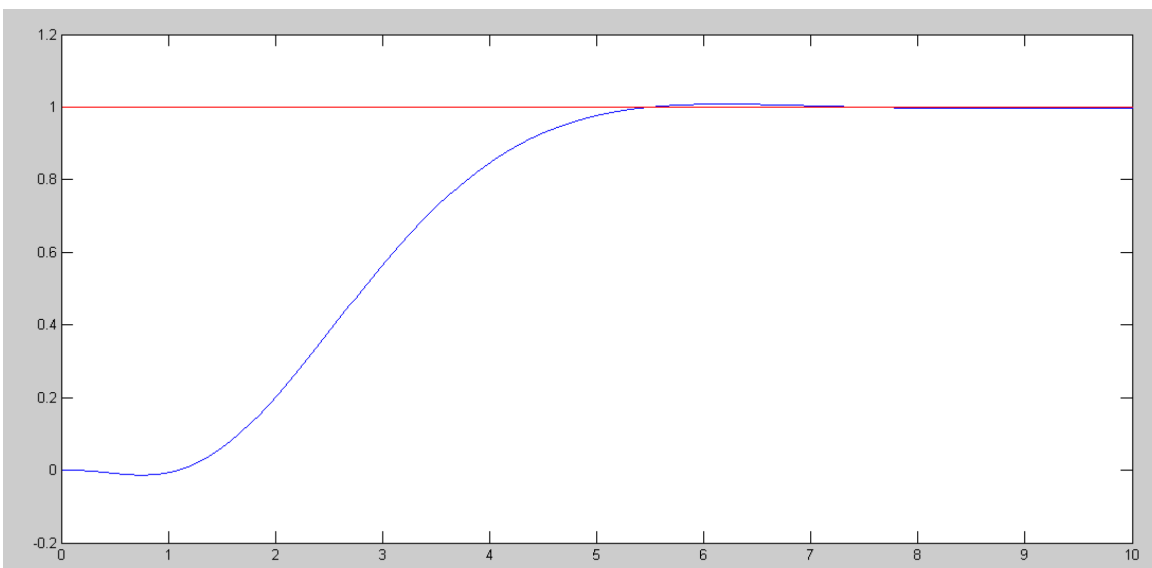
2) Plot the step response of the linear system

```
A = [0,0,1,0;0,0,0,1;0,-2.45,0,0;0,9.42,0,0];
B = [0;0;0.25;-0.1923];
C = [1,0,0,0];

A5 = [A, zeros(4,1) ; C, 0];
B5 = [B; 0];
C5 = [C, 0];
B5r = [zeros(4,1); -1];

Gd = tf(0.5249, [1,1,0.5249]);
t = [0:0.01:12]';
Yd = step(Gd, t);

Q = diag([30,0,0,0,30]);
R = 1;

K5 = lqr(A5, B5, Q, R);
Gcl = ss(A5-B5*K5, B5r, C5, 0);
Y = step(Gcl, t);

plot(t,Y,'b',t,Yd,'r');
eig(A5 - B5*K5)
```



3) Check your design with the nonlinear simulation of the cart and pendulum system.





Code:

```
% Cart and Pendulum ( Sp21 version)
% m1 = 1.0kg
% m2 = 4.0kg
% L = 1.0m

X = [0; 0 ; 0 ; 0];
Ref = 1;
dt = 0.01;
t = 0;
Kx = [-15.9909 -203.1194  -20.6044  -70.3334];
Kz = -5.4772;
Z = 0;

y = [];
while(t < 10)
 Ref = 1;
 U = - Kx*X - Kz*Z;

 dX = CartDynamics(X, U);
 dZ = X(1) - Ref;

 X = X + dX * dt;
 Z = Z + dZ * dt;
 t = t + dt;

 CartDisplay(X, X, Ref);
 y = [y ; X(1), Ref];
end

clf
t = [1:length(y)]' * dt;
plot(t,y(:,1),'b',t,y(:,2),'r');
```

Ball and Beam (HW #4): For the ball and beam system of homework #4

$$s\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -7 & 0 & 0 \\ -5.88 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.2 \end{bmatrix} T$$

Use LQG methods to design a full-state feedback control law of the form

$$T = U = -K_z Z - K_x X$$

$$\dot{Z} = (x - R)$$

for the ball and beam system from homework #6 using LQG control so that

- You track constant setpoints,
- You reject constant disturbances,
- The 2% settling time is 8 seconds, and
- There is less than 5% overshoot for a step input.

4) Give the control law (Kx and Kx) and explain how you chose Q and R

```
Kx =  -87.7063  136.5570  -47.9119   36.9536

>> Kz = K5(5)

Kz =  -26.4575
```

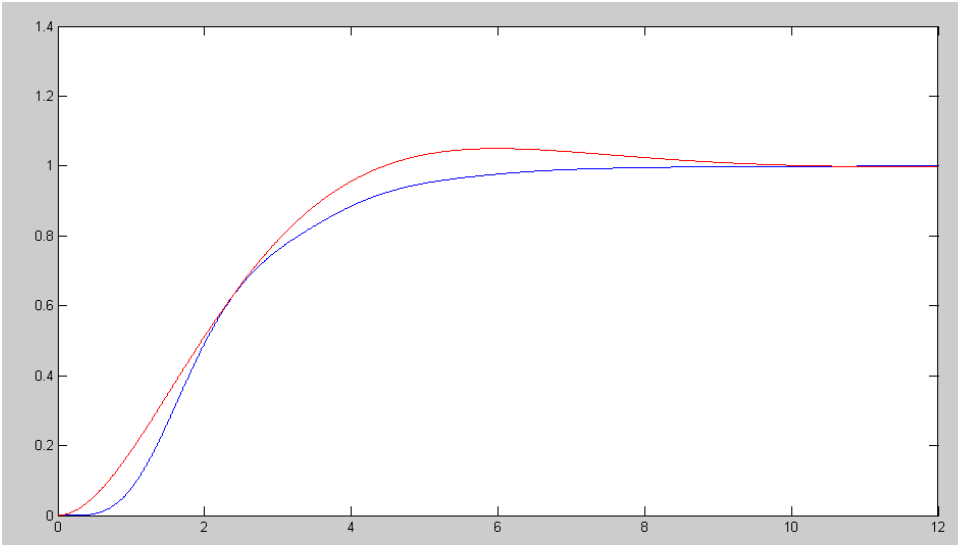
Procedure:

- Increase the weitghting on Z until the speed is about right
- Then see what happens if you increase the weighting on x, q, x', q'
- Turns out weighting x' works best for decreasing the oscillations
- Adding in some weight on x smooths out the response.

Final Q & R:

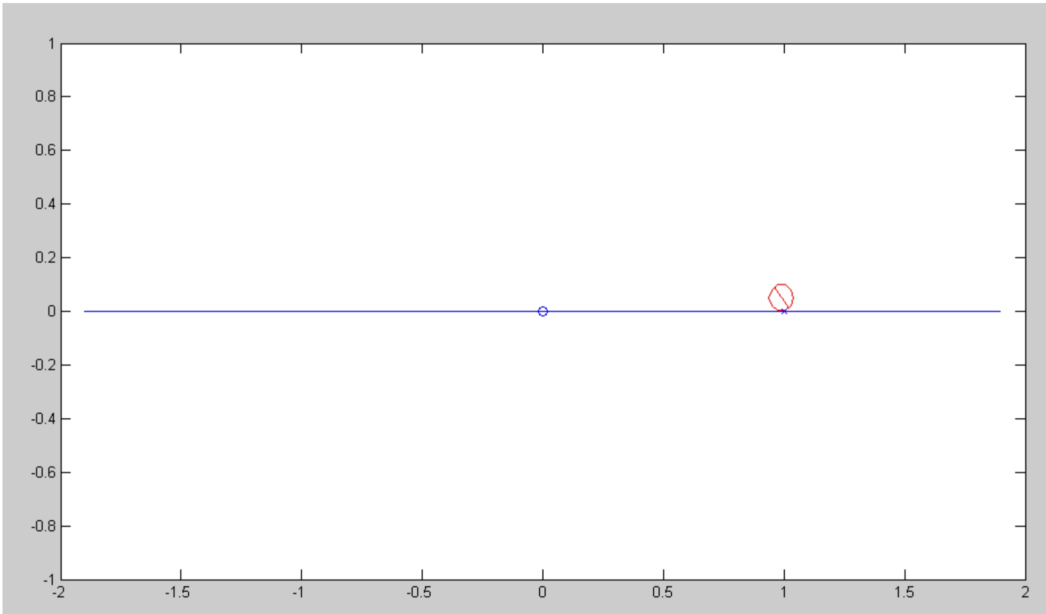
```
Q = diag([400,0,300,0,700]);
R = 1;
```

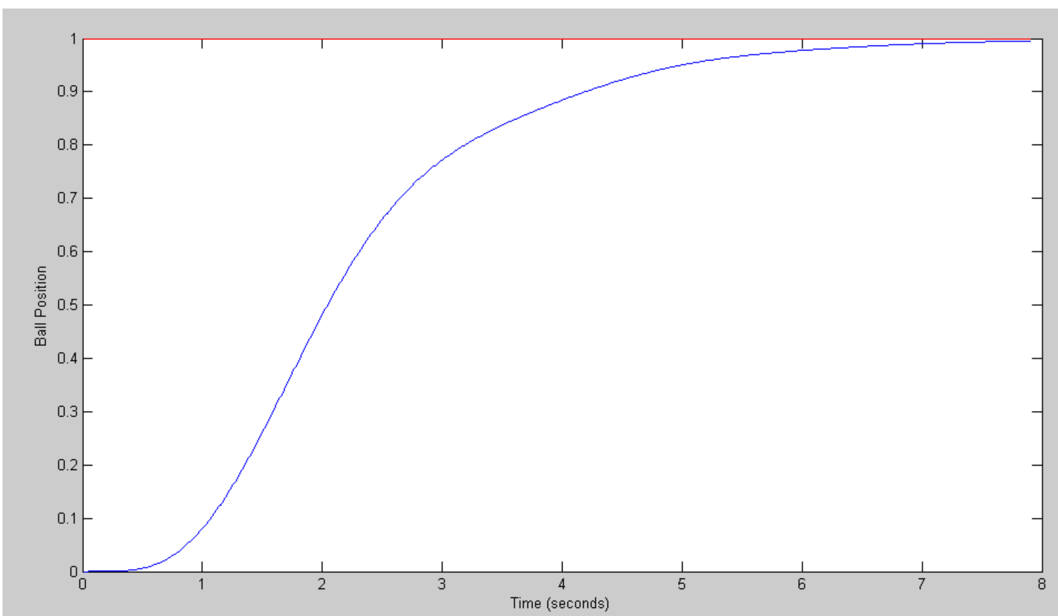
5) Plot the step response of the linear system




```
% Ball and Beam System
% Spring 2024

A = [0,0,1,0;0,0,0,1;0,-7,0,0;-5.88,0,0,0];
B = [0;0;0;0.2];
C = [1,0,0,0];

A5 = [A, zeros(4,1) ; C, 0];
B5 = [B; 0];
C5 = [C, 0];
B5r = [zeros(4,1); -1];

Gd = tf(0.5249, [1,1,0.5249]);
t = [0:0.01:12]';
Yd = step(Gd, t);

Q = diag([400,0,300,0,700]);
R = 1;

K5 = lqr(A5, B5, Q, R);
Gcl = ss(A5-B5*K5, B5r, C5, 0);
Y = step(Gcl, t);

plot(t,Y,'b',t,Yd,'r');
eig(A5 - B5*K5)
```

6) Check your design with the nonlinear simulation of the cart and pendulum system.





Code:

```
% Ball & Beam System
% m = 1kg
% J = 0.2 kg m^2

X = [0, 0, 0, 0]';
dt = 0.01;
t = 0;
Kx = [-92.2782  142.9900  -50.8232   37.8140];
Kz = [-26.4575];
Z = 0;
n = 0;
y = [];

while(t < 7.9)
 Ref = 1;
 U = -Kz*Z - Kx*X;
 dX = BeamDynamics(X, U);
 dZ = X(1) - Ref;

 X = X + dX * dt;
 Z = Z + dZ * dt;
 t = t + dt;

 y = [y ; Ref, X(1)];
 n = mod(n+1,2);
 if(n == 0)
    BeamDisplay(X, Ref);
 end
 end

t = [1:length(y)]' * dt;


plot(t,y(:,1),'r',t,y(:,2),'b');
xlabel('Time (seconds)');
ylabel('Ball Position');
```