
Modern Control

ECE 463 - Modern Control

Jake Glower - Lecture #0

Please visit [Bison Academy](#) for corresponding lecture notes, homework sets, and solutions



Feedback

Feedback is very useful

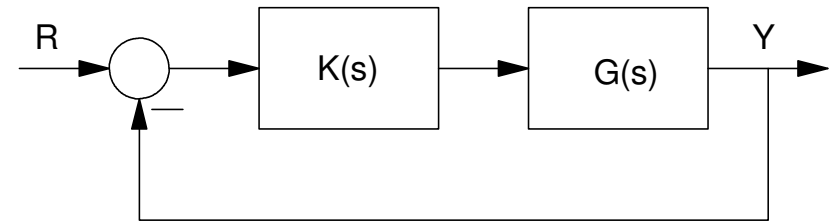
- Instead of specifying the input, you specify the desired output
- It can improve the response of a system
- It can turn an unstable system into a stable system (closed-loop)



ECE 461: Classic Control

One solution: Use output feedback

- $G(s)$ is the plant
- $K(s)$ is a pre-filter, making a poorly behaving system behave like a well-behaved system



Tools for finding $K(s)$

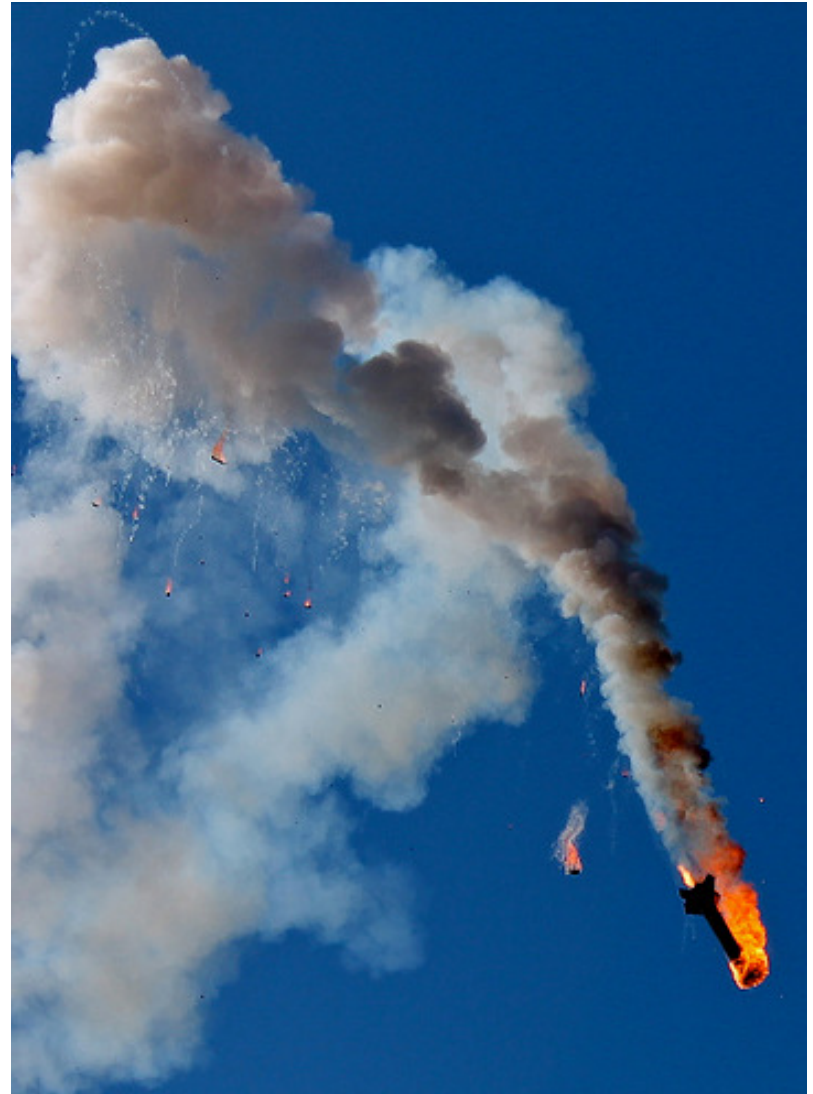
- Root Locus
 - Nichols Charts
 - Bode Plots
-

Sometimes, classic control works very well

- Controlling DC motors
- Controlling temperature
- Controlling water flow

Sometimes, Classic Control doesn't

- Rockets
- Systems with multiple unstable poles



Sputnik & Modern Control

<https://theinvisibleagent.files.wordpress.com/2011/09/sputnik3.jpg>

While NASA was struggling to get a rocket into space, the Soviet Union placed a satellite into orbit in 1957

This was a major shock to the U.S.

- Russia was ahead of the U.S.

This led to researchers looking at Soviet technical journals to learn how the Soviets stabilize their rockets

- The techniques used is termed "Modern Control"



Modern Control (ECE 463/663)

- A matrix approach to designing feedback control systems

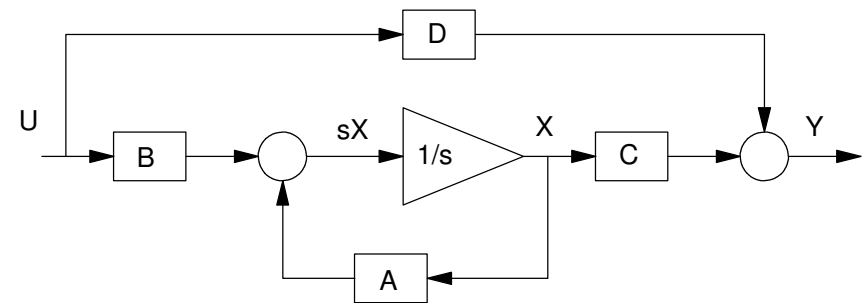
Let X define the energy in the system

- $X = [\text{position, velocity, ...}]$
- potential energy, kinetic energy, ...

If you specify how the energy moves around, you've specified the dynamics

$$sX = AX + BU$$

$$Y = CD + DU$$



Modern vs. Classical Control

The transfer function from U to Y is

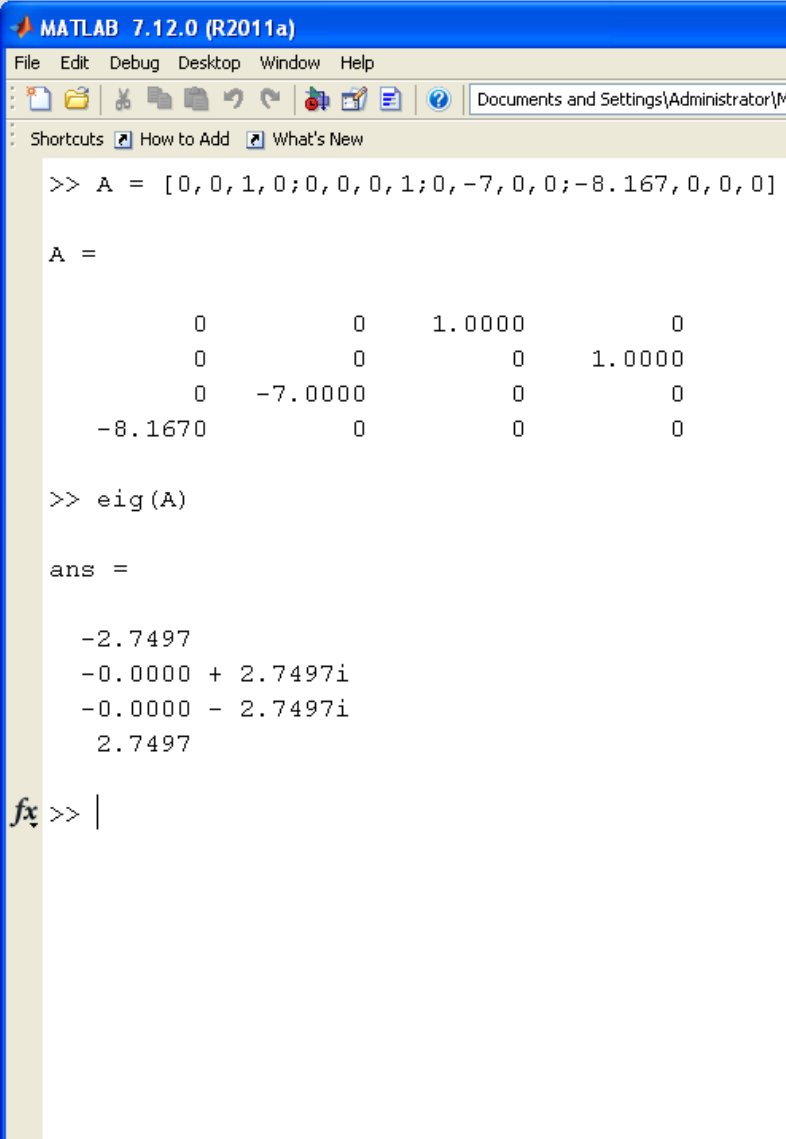
$$Y = \left(C(sI - A)^{-1}B + D \right) U$$

$$G(s) = C(sI - A)^{-1}B + D$$

The eigenvalues of A are all important

- Eigenvalues = Poles
- Negative definite = stable
- Positive real = unstable
- Complex = oscillatory

If you can shift the eigenvalues of A , you can change how the system behaves



```
MATLAB 7.12.0 (R2011a)
File Edit Debug Desktop Window Help
Shortcuts How to Add What's New
>> A = [0, 0, 1, 0; 0, 0, 0, 1; 0, -7, 0, 0; -8.167, 0, 0, 0]
A =
     0     0     1.0000     0
     0     0         0     1.0000
     0    -7.0000     0     0
    -8.1670     0     0     0
>> eig(A)
ans =
    -2.7497
   -0.0000 + 2.7497i
   -0.0000 - 2.7497i
     2.7497
fx >> |
```

Full State Feedback

A is an $N \times N$ matrix

- It has N eigenvalues
- It has N poles

If you define U as

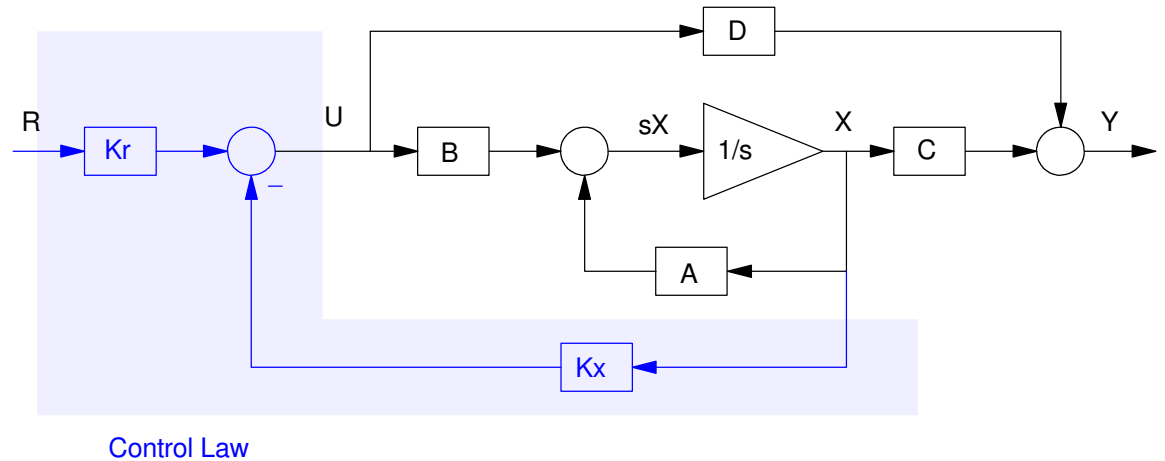
$$U = -K_x X + K_r R$$

the dynamics become

$$sX = (A - BK_x)X + BK_r R$$

$$Y = CX + DU$$

With K_x , you have N degrees of freedom to place the N closed-loop eigenvalues.



How do you determine the system dynamics?

- Week 1-5 (Test #1)
- LaGrangian formulation

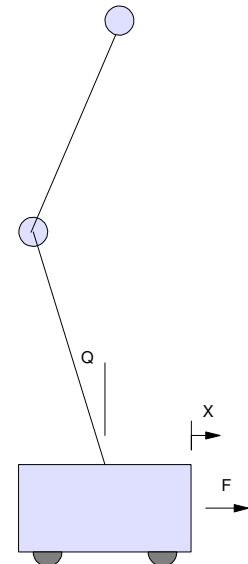
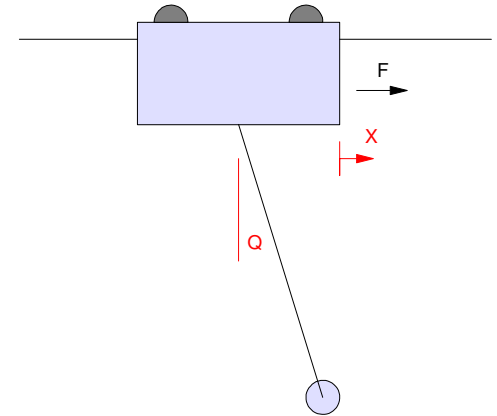
Define the energy in the system

$$L = KE - PE$$

Define how the energy moves about the system

Linearize to determine the state-space model

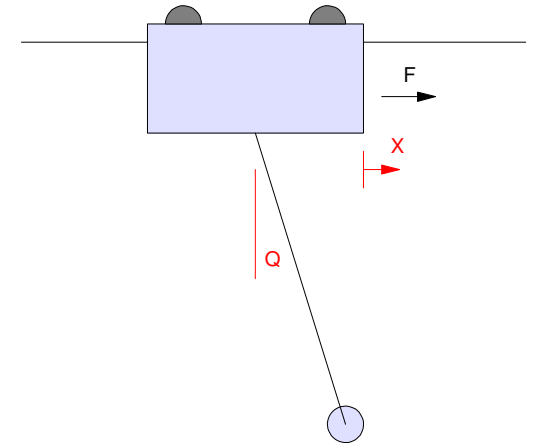
Works for both linear and nonlinear systems



Systems we look at in ECE 463

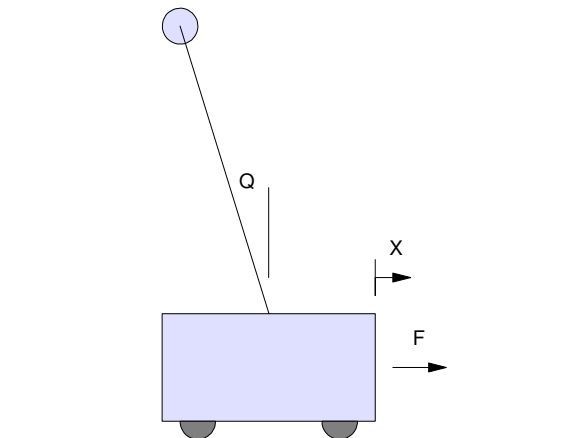
Gantry System

$$x = \left(\frac{2(s+j3)(s-j3)}{s^2(s+j4)(s-j4)} \right) F$$



Cart and Pendulum

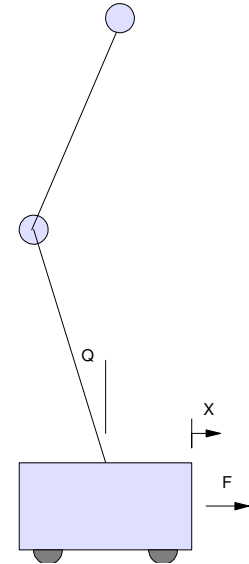
$$x = \left(\frac{2(s+3)(s-3)}{s^2(s+4)(s-4)} \right) F$$



Systems we look at in ECE 463

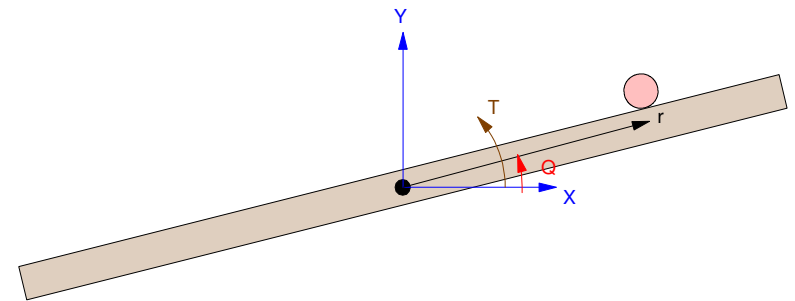
Double Pendulum

$$x = \left(\frac{(s+3.5 \pm j1.5)(s-3.5 \pm j1.5)}{s^2(s+4.5 \pm j1.5)(s-4.5 \pm j1.5)} \right) F$$



Ball and Beam

$$r = \left(\frac{10}{(s+3)(s-3)(s+j3)(s-j3)} \right) T$$



Problem: How do you determine K_x ?

Pole Placement

- Weeks 6 - 9 & Test #2
- Bass Gura
- K_x contains N terms
- $(A - B K_x)$ has N eigenvalues
- N equations and N unknowns

Good:

- Complete freedom
- Can place poles anywhere

Bad:

- Complete freedom
- Where *should* you place the poles

Optimal Control

- Weeks 10-14 & Test #3
- Based upon Calculus of Variations
- Define a cost function
- $J = f(\text{energy}) + g(\text{input}^2)$
- Determine K_x to minimize J

Good:

- Places all poles in their optimal spot

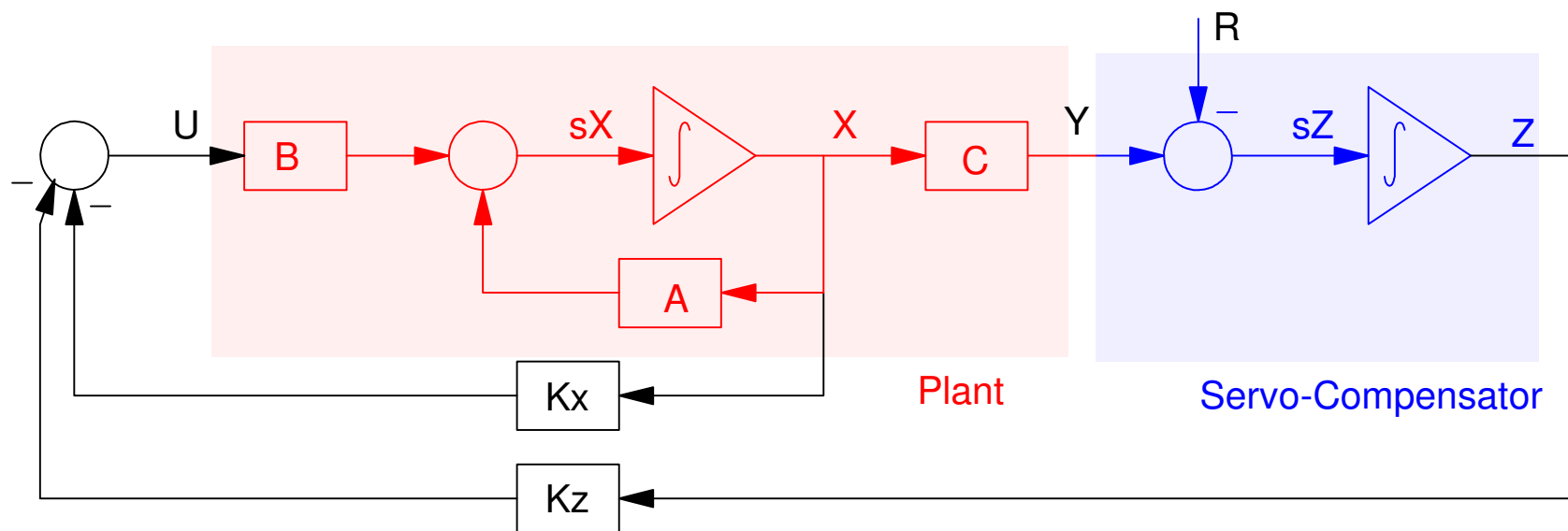
Bad:

- Cost function is arbitrary
 - I'd prefer "good control" over "optimal control"
 - It's a tool to find K_x
-

Problem: How to you track a constant set point?

- ECE 461: Make it a type-1 system
- ECE 463: Add a servo compensator

Also allows you to track multiple sine waves

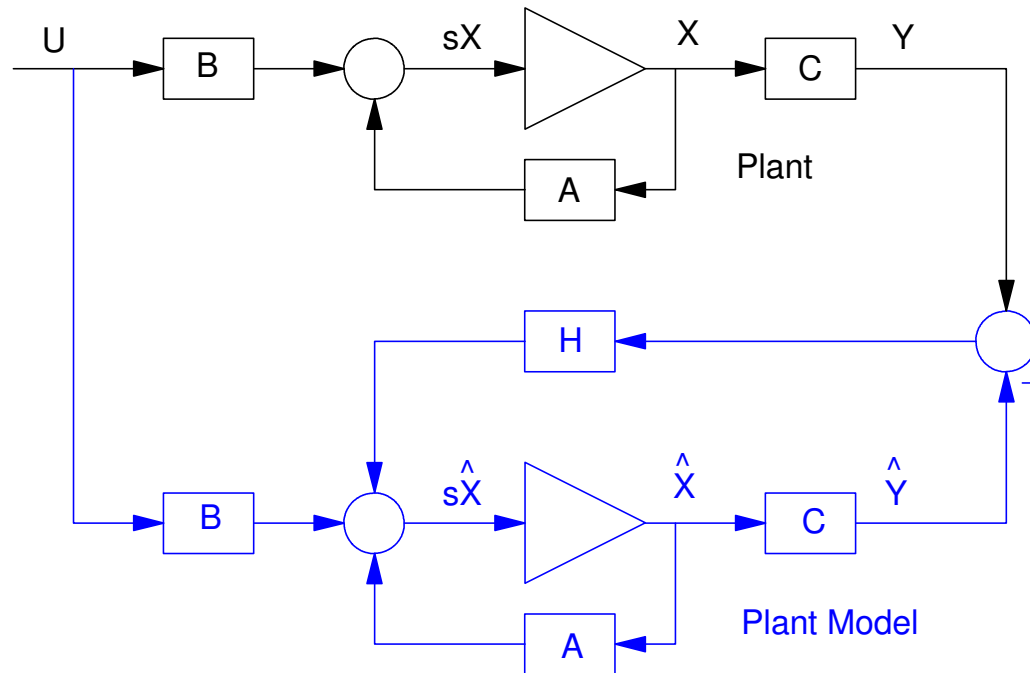


Problem: What if you can't measure *all* of the states?

- Kx contains N terms
- You need to know all N states in X

If you can't measure the states, estimate them from the input and output

- Full-Order Observer



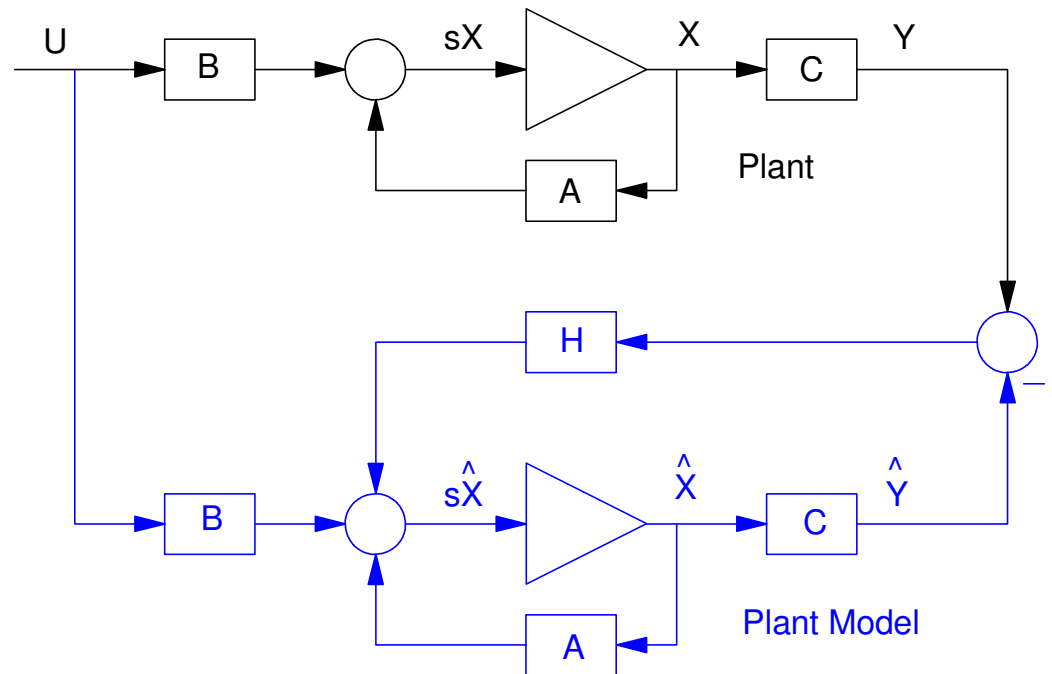
Problem: How do you choose H?

Option #1: Pole Placement

- Bass Gura

Option #2: Optimal Control

- Produces a Kalman Filter
- Optimal H given sensor noise and input noise



ECE 463/663 Web Site

- www.BisonAcademy.com

Syllabus is posted

- Daily lecture topics

All lecture notes are posted

- pdf format

Videos of all lectures are linked

- YouTube videos

Homework Sets are posted

- Solutions posted after due date



ECE 463/663: Modern Control Syllabus - Spring 2021

[Syllabus - HW & Solutions - Resources - Comments](#)

Instructor: Jacob Glower

Contact Info: jacob_glower@yahoo.com (where to email your homework solutions)

Class Times: MWF 10am, In-Person and on Zoom

Please send homework solutions to Jacob_Glower@yahoo.com with header HW#: ECE 463
(or hardcopy or submit on blackboard)

	Date	Topic	Lecture (YouTube)	Code & Handouts	Homework Email solutions to Jacob_glower@yahoo.com Subject: ECE463/663 HW#
M	Jan 11	<i>Holiday</i>	0		HW #1
W	Jan 13	Syllabus MATLAB Review Slides #1	1 Matlab Review		
F	Jan 15	LaPlace Transforms Slides #2	2 LaPlace Xform	2 nd Order Approximations Step Responses	
M	Jan 18	<i>Holiday</i>			HW #2
W	Jan 20	State Space Slides #3	3 State Space		
F	Jan 22	Eigenvalues and Eigenvectors Slides #4	4 Eigenvalues	Heat	
M	Jan 25	Canonical Forms & Similarity Transforms Slides #5	5 Canonical Forms		HW #3
W	Jan 27	LaGrangian Dynamics Slides #6	6 LaGrangian Dynamics	Ball	
F	Jan 29	Cart and Pendulum System Slides #7	7	Gantry Gantry Display Gantry Dynamics · Cart Cart Display Cart Dynamics	
M	Feb 1	Ball and Beam System Slides #8	8 Ball and Beam	Beam BeamDisplay BeamDynamics	HW #4
W	Feb 3	Dynamics of a 2-Link Robot Slides #9	9 2-Link Robot	FreeFall 2-Link Dynamics	
F	Feb 5	Dynamics of a Double	10 Double	Gantry2 Gantry2Display	

Homework Sets & Solutions

Last 4 years posted

- Serves as a reference if you need help with the homework
- Systems and dynamics are different, but the procedure will be similar

Also has sample tests and solutions

BISON ACADEMY

Courses taught in the
Department of Electrical and Computer Engineering
North Dakota State University

ECE 463/663: Modern Control

Homework and Solutions

[Syllabus - HW & Solutions - Resources - Comments](#)

□

Spring 2019	Spring 2018	Spring 2017	Spring 2016
1: Dominant Poles Solution #1	1: Dominant Poles Solution #1	1: Dominant Poles Solution #1	1: LaPlace Transforms Solution #1
2: State Space Solution #2	2: State Space Solution #2	2: State Space Solution #2	2: Modeling in State Space Solution #2
3: Canonocal Forms Solution #3	3: Canonical Forms Solution #3	3: Canonical Forms Solution #3	3: Canonical Forms Solution #3
4: LaGrangian Dynamics Solution #4	4: LaGrangian Dynamics Solution #4	4: LaGrangian Dynamics Solution #4	4: LaGrangian Dynamics Solution #4
Test #1 Test#1 Solution	Test #1 Test #1 Solution	Test #1	5: Controllability and Observability Solution #5
5: Pole Placement Solution #5	5: Pole Placement Solution #5	5: Pole Placement Solution #5	6: Pole Placement Solution #6
6: Servo Compensators Solution #6	6: Servo Compensators Solution #6	6: Servo Compensators Solution #6	7: Servo Compensators Solution #7
7: Linear Observers Solution #7	7: Observers Solution #7	7: Linear Observers Solution #7	8: Observers Solution #8
Test #2 Solution A: Solution B	Test #2 Test #2 Solution	Test #2 Solutions: A Level □- B Level □- C Level	9: Calculus of Variation Solution #9
8: Calculus of Variations Solution #8	8: Calculus of Variations Solution #8	8: Calculus of Variations Solution #8	10: LQG Control Solution #10
9: Optimal Control Solution #9	9: Optimal Control Solution #9	9: LQG Control Solution #9	11: Kalman Filters

ECE 463: Modern Control

Fun class where we use Matlab extensively

- Simulating nonlinear dynamic systems
- Manipulating $N \times N$ matrices

Heavy use of matrix algebra

- We cover what you'll use in this class
- Math 129 is the main class we use

