ECE 111 - Homework #3

Math 105: Trigonometry.

Polar to Rectangular Conversions

1) Determine the final position of A: (x,y)

$$A = (22\angle 77^0) + (17\angle -38^0) + (12\angle 19^0)$$

In Matlab

```
>> A1x = 22 * cos(77*pi/180);
>> A1y = 22 * sin(77*pi/180);
>> A2x = 17 * cos(-38*pi/180);
>> A2y = 17 * sin(-38*pi/180);
>> A3x = 12 * cos(19*pi/180);
>> A3y = 12 * sin(19*pi/180);
>> Ax = A1x + A2x + A3x

Ax =  29.691328914071104

>> Ay = A1y + A2y + A3y

Ay =  14.876714198224864
```

On an HP42 calculator

```
modes
polar
22
enter
77
complex
17
enter
-38
complex
+
12
enter
19
complex
+
modes
rect
```

2) Determine final position of B: (x,y)

$$B = (5\angle 2^0) + (22\angle 28^0) + (20\angle 55^0)$$

In Matlab

```
>> B1x = 5 * cos(2*pi/180);
>> B1y = 5 * sin(2*pi/180);
>> B2x = 22 * cos(28*pi/180);
>> B2y = 22 * sin(28*pi/180);
>> B3x = 20 * cos(55*pi/180);
>> B3y = 20 * sin(55*pi/180);
>> Bx = B1x + B2x + B3x

Bx =   35.893329905012791

>> By = B1y + B2y + B3y

By =   26.885912750581937
```

With a HP42

```
modes
polar
5
enter
2
complex
22
enter
28
complex
+
20
enter
55
complex
+
modes
rect
```

3) Where is B relative to A

- In (x,y) coordinates
- In polar coordinates

i.e. What is B - A?

In Matlab

```
>> Cx = Bx - Ax

Cx =    6.202000990941688

>> Cy = By - Ay

Cy =  12.009198552357073
```

Point B relative to A is

- 6.202 meters away in the x direction, and
- 12.009 meters away in the y direction

On an HP42

```
x <> y
-
mode
polar
```

Point B is

- 13.516 meters away (distance), and
- At an angle of 62.68 degrees

Plotting Polar Functions

4) Plot the following functions in Matlab for $0 < \theta < 6\pi$

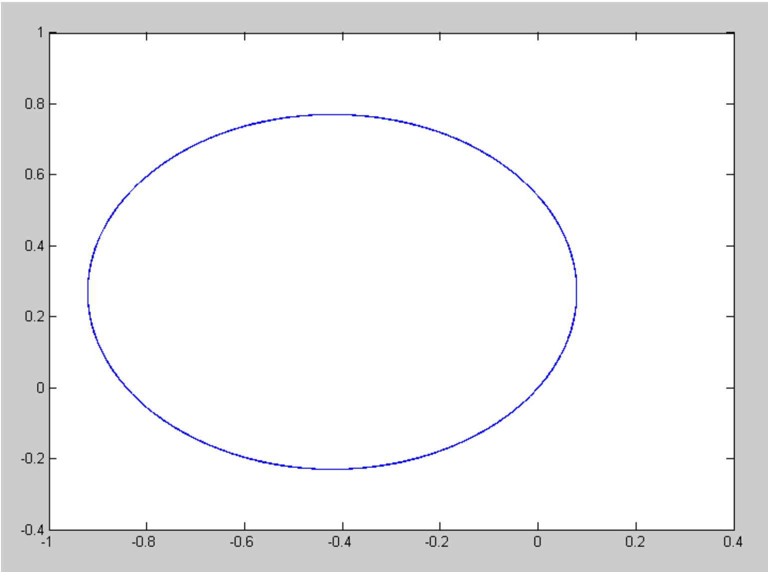
- Note: plot() plots in cartesian coordinates. Each function needs to be converted from polar to rectangular.

a) $r = \sin(\theta - 1)$

```
>> q = [0:0.01:6*pi]';
>> r = sin(q - 1);
>> x = r .* cos(q);
>> y = r .* sin(q);
>> plot(x,y)
>>
```

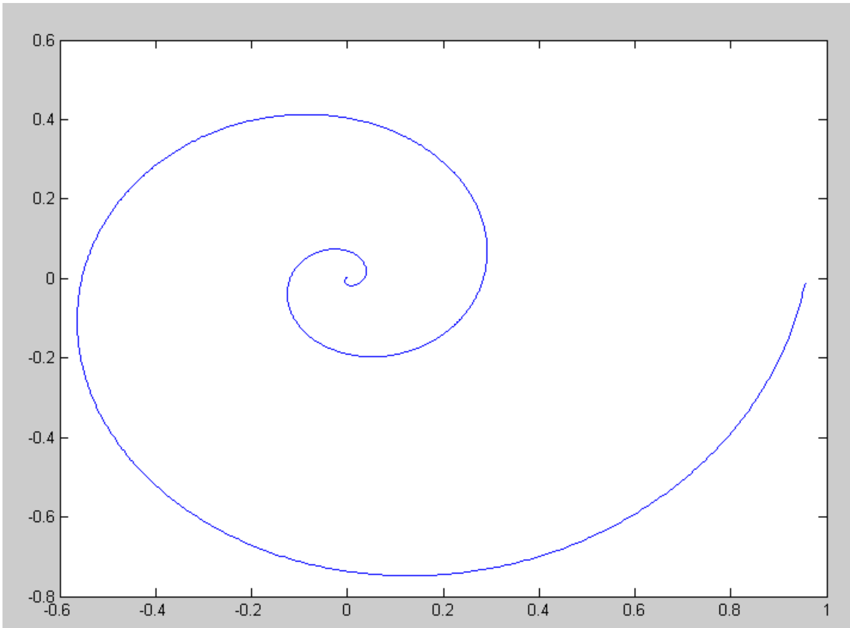
Big surprise - you get a circle. Trig functions are all about circles.

Note: Matlab's plot() function only works in rectangular corrdinates (x, y). You likewise have to convert your polar function to rectangular form to plot it in Matlab.



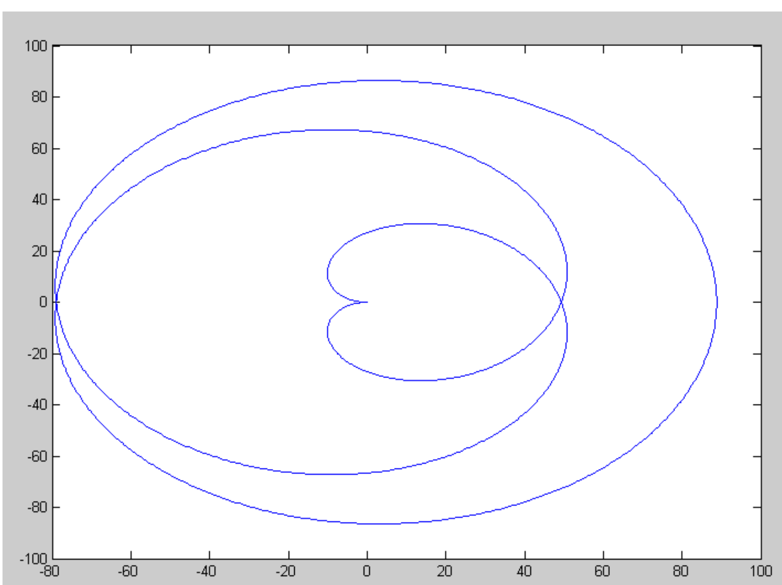
b) $r = \theta^3/7000$

```
>> q = [0:0.01:6*pi]';
>> r = q.^3 / 7000;
>> x = r .* cos(q);
>> y = r .* sin(q);
>> plot(x,y)
```



c) $r = \theta(\theta - 6\pi)$

```
>> q = [0:0.01:6*pi]';
>> r = q .* (q - 6*pi);
>> x = r .* cos(q);
>> y = r .* sin(q);
>> plot(x,y)
```



Robot Tip Position (Forward Kinematics)

A 2D robot has three arms with lengths of {0.5, 0.6, 0.7} meters. The final tip positionis

$$x_1 = 0.5\cos(\theta_1) \qquad y_1 = 0.5\sin(\theta_1)$$

$$x_2 = x_1 + 0.6\cos(\theta_1+\theta_2) \qquad y_2 = y_1 + 0.6\sin(\theta_1+\theta_2)$$

$$x_3 = x_2 + 0.7\cos(\theta_1+\theta_2+\theta_3) \qquad y_3 = y_2 + 0.7\sin(\theta_1+\theta_2+\theta_3)$$

Start with a Matlab funciton which

- Is passed the angles in degrees
- Computes the joint positions
- Plots the robot position, and
- Returns the tip position: (x3, y3)

```
function [x3,y3] = RRR(Z)
   q1 = Z(1) * pi/180;
   q2 = Z(2) * pi/180;
   q3 = Z(3) * pi/180;

   x1 = 0.5 * cos(q1);
   y1 = 0.5 * sin(q1);

   x2 = x1 + 0.6 * cos(q1 + q2);
   y2 = y1 + 0.6 * sin(q1 + q2);

   x3 = x2 + 0.7 * cos(q1 + q2 + q3);
   y3 = y2 + 0.7 * sin(q1 + q2 + q3);

   plot([0,x1,x2,x3],[0,y1,y2,y3],'b.-');

end
```

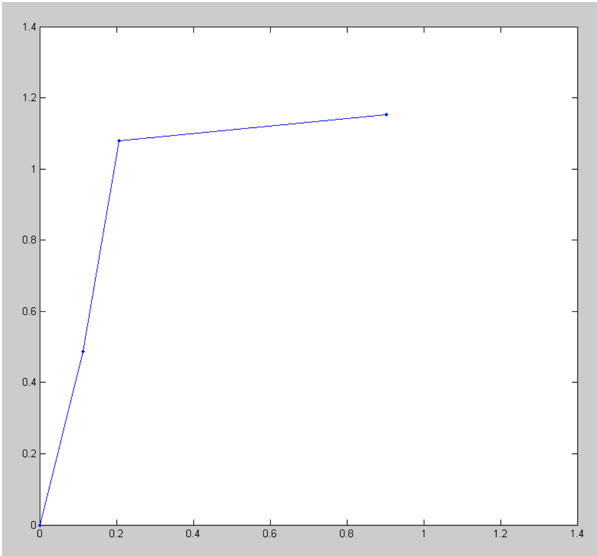
5) Plot the tip position (x3, y3) for

$$\theta_1 = 77^0 \quad \theta_2 = 4^0 \quad \theta_3 = -75^0$$

```
>> [x3,y3] = RRR([77,4,-75])
>> [x3,y3] = RRR([77,4,-75])

x3 =    0.9025

y3 =    1.1530
```



Call this with

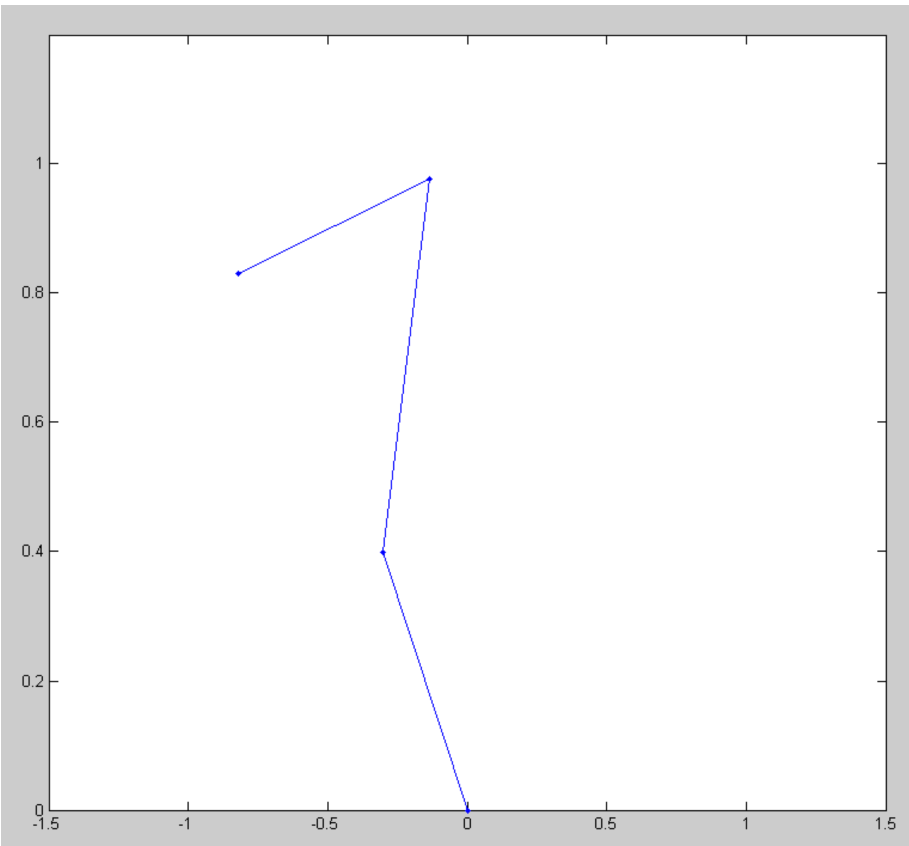
6) Plot the tip position (x3, y3) for

$$\theta_1 = 127^0 \qquad \theta_2 = -53^0 \qquad \theta_3 = 118^0$$

```
>> [x3,y3] = RRR([127,-53,118])
x3 =   -0.8202


y3 =    0.8305

>>
```



Robot Tip Position (Inverse Kinematics & fminsearch())

7) Write a Matlab function which

- Is passed the angles $(\theta_1, \theta_2, \theta_3)$,
- Computes the tip position, and
- Returns the distance from the tip position and point (x = 1.2, y = 1.2)

```
function [e] = Prob7(Z);

[x3, y3] = RRR(Z);

e = (x3 - 1.2)^2 + (y3 - 1.2)^2;
pause(0.1);
end
```

8) Use the fminsearch() to determine the joint angles which place the robot at (x3 = 1.2, y3 = 1.2)

```
>> [Z,e] = fminsearch('Prob7',[40,50,60])
```

```
           Q1        Q2        Q3
Z =     14.3928   35.1188   12.7213

e =   7.5455e-014
```

Checking the answer: the tip is at (1.2000, 1.2000) as desired (note: there are multiple solutions)

```
>> [x3,y3] = RRR(Z)

x3 =    1.2000

y3 =    1.2000
```