

ECE 111 - Homework #12

Complex Numbers

Complex Numbers

1) Determine the rectangular or polar form of each complex number

Note: This is easier on an HP calculator. Matlab can work though

$$X = 3 - j6$$

```
>> X = 3 - j*6;  
>> r = abs(X)  
  
r = 6.7082  
  
>> q = angle(X) * 180/pi  
  
q = -63.4349
```

$$X = 6.7082 \angle -63.4349^\circ$$

HP Prime

place in RPN mode

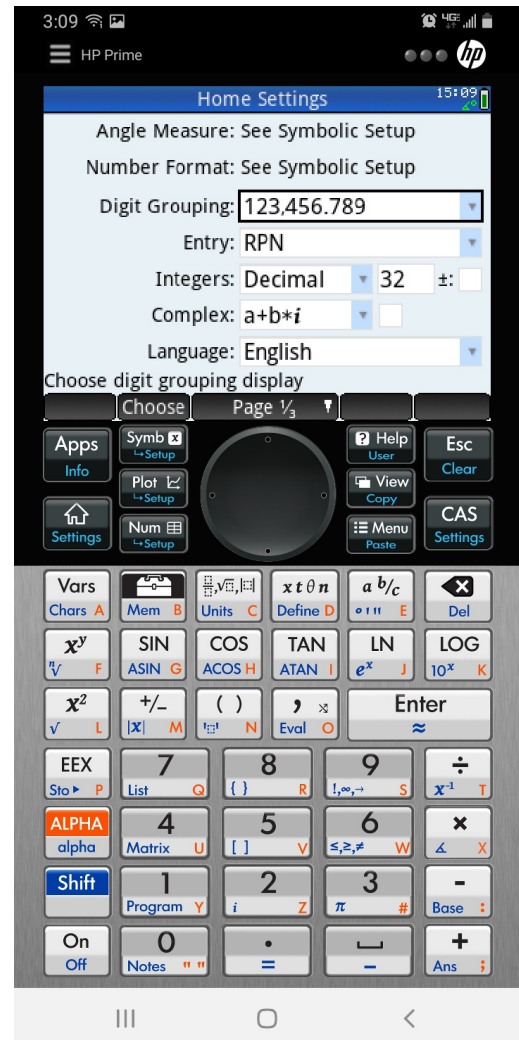
Settings
RPN
Home

3
i
6
+/-
enter
∠

$$X = -7 - j2$$

```
>> X = -7 - j*2;  
>> r = abs(X)  
  
r = 7.2801  
  
>> q = angle(X) * 180/pi  
  
q = -164.0546
```

$$X = 7.2801 \angle -164.0546^\circ$$



$$A = 8\angle 147^\circ$$

Option #1

```
>> A = 8 * exp(j*147*pi/180)

A = -6.7094 + 4.3571i
```

Option #2

```
>> Ax = 8 * cos(147*pi/180)

Ax = -6.7094

>> Ay = 8 * sin(147*pi/180)

Ay = 4.3571
```

HP Prime

```
8
 $\angle$ 
147
enter
 $\angle$ 
```



$$B = 12\angle -85^\circ$$

Option 1:

```
>> Bx = 12*cos(-85*pi/180)

Bx = 1.0459

>> By = 12*sin(-85*pi/180)

By = -11.9543
```

Option #2

```
>> B = 12 * exp(-j*85*pi/180)

B = 1.0459 -11.9543i
```

2) Determine y as a complex number

$$y = (6 + j8) + (-7 + j8) + (3 - j7)$$

Option 1:

```
>> y1 = 6 + j*8;  
>> y2 = -7 + j*8;  
>> y3 = 3 - j*7;  
>> y = y1 + y2 + y3  
  
y = 2.0000 + 9.0000i
```

Option 2:

```
>> y = (6+j*8) + (-7+j*8) + (3-j*7)  
  
y = 2.0000 + 9.0000i
```

>>

On an HP Prime (RPN mode)

```
6i8  
enter  
-8i8  
+  
3i-7  
+
```



$$y = (4\angle 84^\circ) + (7\angle -62^\circ) + (9\angle 85^\circ)$$

Option 1:

```
>> y1 = 4*exp(j*84*pi/180);
>> y2 = 7*exp(-j*62*pi/180);
>> y3 = 9*exp(j*85*pi/180);
>> y = y1 + y2 + y3

y = 4.4888 + 6.7632i
```

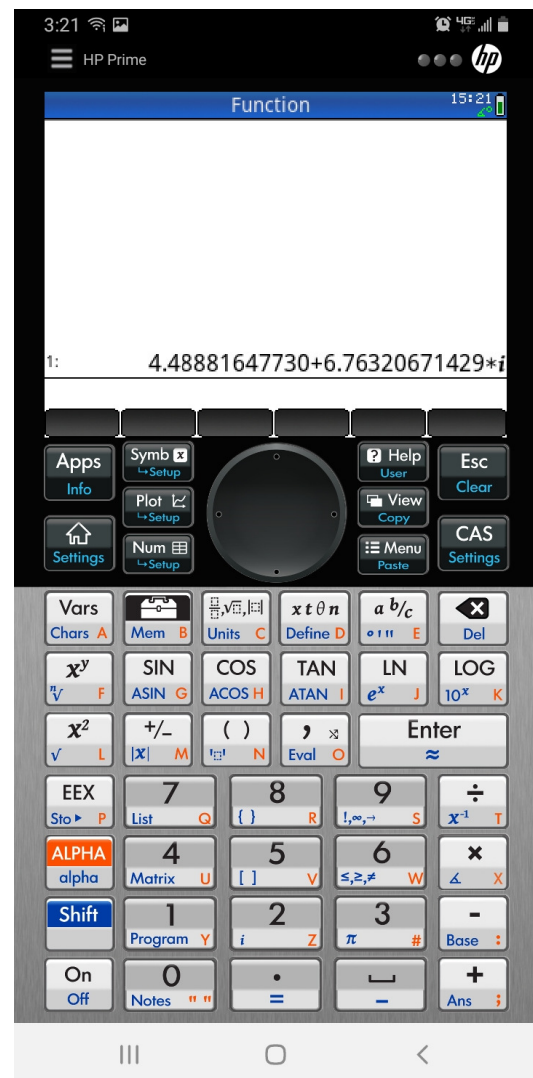
Option 2:

```
>> y = 4*exp(j*84*pi/180) + 7*exp(-j*62*pi/180) + 9*exp(j*85*pi/180)

y = 4.4888 + 6.7632i
```

On an HP Prime (RPN mode)

```
4∠84
enter
7∠-62
+
9∠85
+
```



3) Determine y as a complex number

$$y = \left(\frac{(6+j8)(-7+j8)}{(3-j7)} \right)$$

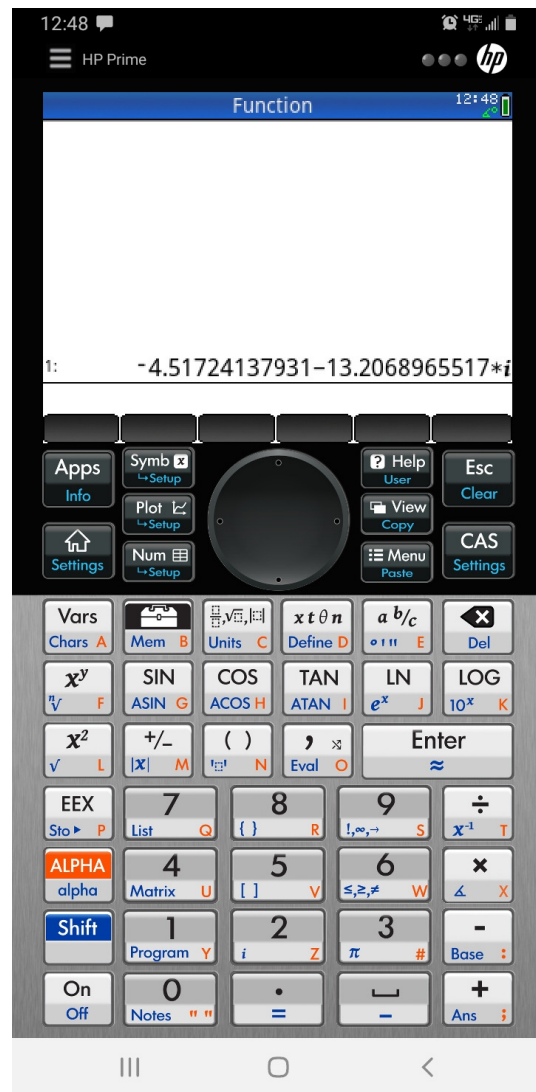
In Matlab

```
>> y = (6+j*8)*(-7+j*8) / (3-j*7)
```

```
y = -4.5172 -13.2069i
```

On an HP Prime

```
6i8  
enter  
-7i8  
*  
3i-7  
/  
enter
```



$$y = \left(\left(\frac{6+j8}{-7+j8} \right) + \left(\frac{9+j6}{-j7} \right) \right) \left(\frac{-2+j6}{8+j9} \right)$$

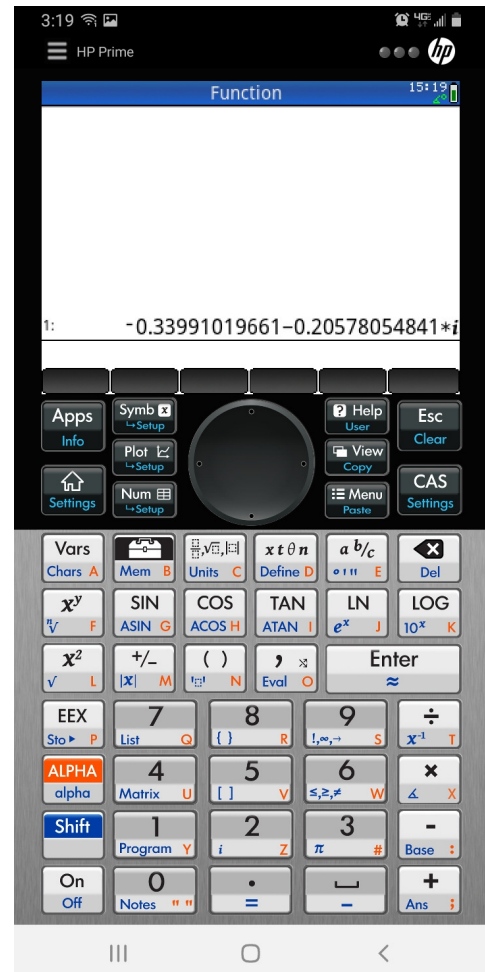
In Matlab

```
>> y = ( (6+j*8)/(-7+j*8) + (9+j*6)/(-j*7) ) * (-2+j*6)/(8+j*9)

y = -0.3399 - 0.2058i
```

With a HP Prime (in RPN mode)

```
6i8
enter
-8i8
/
9i6
enter
0i-7
/
+
-2i6
*
8i9
/
```



4) Determine y as a complex number

- $y = e^{(3+j7)}$
- $y = \ln(-9 + j9)$
- $y = (4 + j5)^{(5-j2)}$

In Matlab

```
>> y = exp( 3 + j*7 )  
  
y = 15.1425 +13.1959i  
  
>> y = log(-9 + j*9)  
  
y = 2.5438 + 2.3562i  
  
>> y = (4 + j*5) ^ (5 - j*2)  
  
y = 4.6528e+004 +4.4820e+004i
```

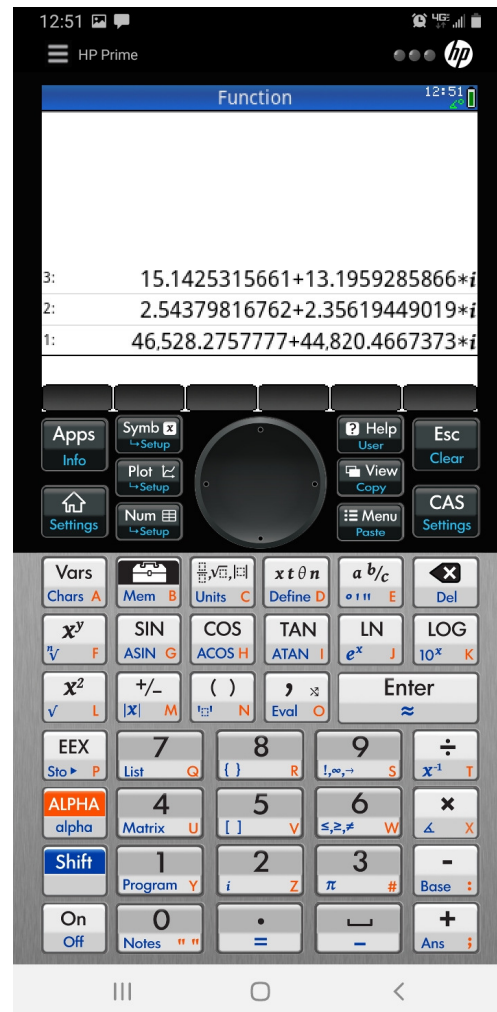
Note: Matlab has no problem working with complex numbers

- HP calculators don't either

With an HP Prime

```
3i7  
e^x  
  
-9i9  
ln  
  
4i4  
enter  
5-i2  
y^x
```

note: y^x is mislabeled as x^y



Partial Fractions with Complex Numbers

5) Determine the partial fraction expansion

$$\left(\frac{3(x-7)(x+4)}{(x+9+j4)(x+9-j4)(x+9)} \right) = \left(\frac{a}{x+9+j4} \right) + \left(\frac{b}{x+9-j4} \right) + \left(\frac{c}{x+9} \right)$$

Using the cover-up method:

```
>> x = -9 - j*4;  
>> a = 3*(x-7)*(x+4) / ( (x+9-j*4)*(x+9) )  
  
a = -6.0000 - 7.8750i  
  
>> x = -9 + j*4;  
>> b = 3*(x-7)*(x+4) / ( (x+9+j*4)*(x+9) )  
  
b = -6.0000 + 7.8750i  
  
>> x = -9;  
>> c = 3*(x-7)*(x+4) / ( (x+9+j*4)*(x+9-j*4) )  
  
c = 15  
  
>>
```

so the answer is

$$\left(\frac{3(x-7)(x+4)}{(x+9+j4)(x+9-j4)(x+9)} \right) = \left(\frac{-6-j7.8750}{x+9+j4} \right) + \left(\frac{-6+j7.8750}{x+9-j4} \right) + \left(\frac{15}{x+9} \right)$$

6) Determine the partial fraction expansion

$$\left(\frac{8(x+j6)(x-j6)}{x(x+4)(x+9+j)(x+9-j)} \right) = \left(\frac{a}{x+0} \right) + \left(\frac{b}{x+4} \right) + \left(\frac{c}{x+9+j} \right) + \left(\frac{d}{x+9-j} \right)$$

Using the cover-up method

```
>> x = 0;
>> a = 8*(x+j*6)*(x-j*6) / ( (x+4)*(x+9+j)*(x+9-j) )

a =    0.8780

>> x = -4;
>> b = 8*(x+j*6)*(x-j*6) / ( (x)*(x+9+j)*(x+9-j) )

b =    -4

>> x = -9 - j;
>> c = 8*(x+j*6)*(x-j*6) / ( (x)*(x+4)*(x+9-j) )

c =    1.5610 +10.0488i

>> x = -9 + j;
>> d = 8*(x+j*6)*(x-j*6) / ( (x)*(x+4)*(x+9+j) )

d =    1.5610 -10.0488i

>>
```

so

$$\left(\frac{8(x+j6)(x-j6)}{x(x+4)(x+9+j)(x+9-j)} \right) = \left(\frac{0.8780}{x+0} \right) + \left(\frac{-4}{x+4} \right) + \left(\frac{1.5610+j10.0488}{x+9+j} \right) + \left(\frac{1.5610-j10.0488}{x+9-j} \right)$$