ECE 111 - Homework #2

Newton's Method

Problems 1 & 2) Let x and y be related by:

$$y = x + \cos(x)$$

1) Use graphical methods solve for x when

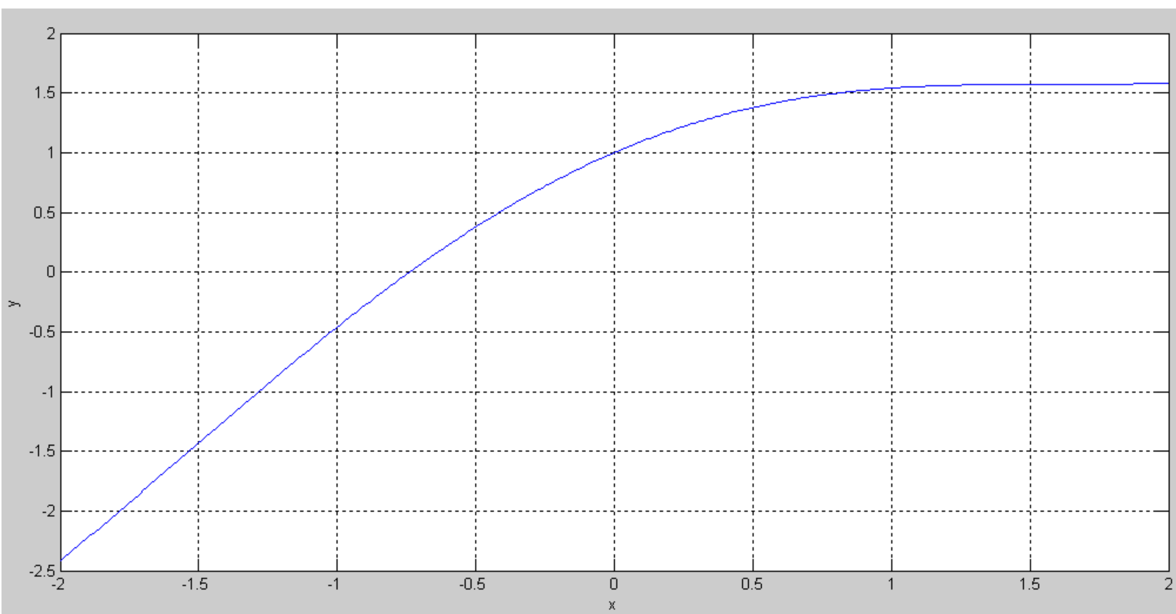
- y = -0.5
- y = +0.5

Matlab Code

```
>> x = [-2:0.01:2]';
>> y = x + cos(x);
>> plot(x,y);
>> grid on
>> xlabel('x');
>> ylabel('y');
```

From the graph

- y = -0.5, x =-1.1
- y = +0.5 x = -0.4



2) Find the solutions to problem #1 using Newton's method

Matlab Code for the function

```
function [e] = Prob1(x)
   y = x + cos(x);
   e = y + 0.5;
end
```

Matlab code for Newton's Method

```
x1 = 0;
y1 = Prob1(x1);
for n=1:10
    disp([n, x1, y1])
    pause(0.1);
    x2 = x1 + 0.01;
    y2 = Prob1(x2);
    x1 = x1 - (x2-x1)/(y2-y1) * y1;
    y1 = Prob1(x1);
end
```

Result: Find the solution to y = -0.5

n	x	error
1	0	1.500000000000000
2	-1.507537625312706	-0.944321105393970
3	-1.034825683032550	-0.024149869529197
4	-1.021822346884660	-0.000010100208080
5	-1.021816888602366	0.000000014310705
6	-1.021816896336066	-0.000000000020288
7	-1.021816896325103	0.000000000000029
8	-1.021816896325118	-0.000000000000000
9	-1.021816896325118	-0.000000000000000
10	**-1.021816896325118**	-0.000000000000000

Repeat for y = +0.5

n	x	error
1	0	0.500000000000000
2	-0.502512541770902	-0.126137325334089
3	-0.417125332823814	-0.002867991284901
4	-0.415077580788864	0.000007457158997
5	-0.415082912362423	-0.000000024442852
6	-0.415082894886813	0.000000000080075
7	-0.415082894944063	-0.000000000000262
8	-0.415082894943875	0.000000000000001
9	-0.415082894943876	0
10	**-0.415082894943876**	0

Note:

- These answers match up with problem #1, only with lots more decimal places
- Five iterations would have been enough

Problems 3 & 4) Let x and y be related by

$$y = \cos(2x)$$

$$y = (x-2)(x+1)$$

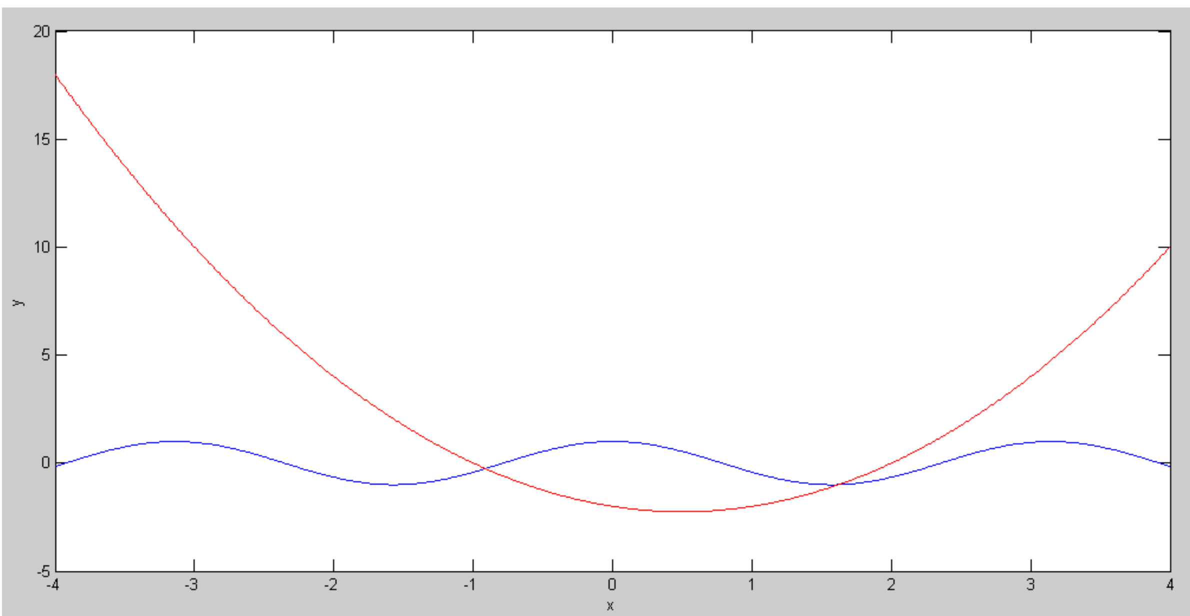
3) Find all solutions in the range of (-4 < x < 4) using graphical methods. (Plot both functions on the same graph. The solution is when the two functions intersect.)

In Matlab

```
>> x = [-4:0.01:4]';
>> y1 = cos(2*x);
>> y2 = (x-2).*(x+1);
>> plot(x,y1,'b',x,y2,'r')
>> xlabel('x');
>> ylabel('y');
>>
```

The graphs intersect at two locations (two solutions)

- x = 0.9, y = -0.3
- x = 1.6 y = -1.0



4) Find the solutions to problem #3 using Newton's method.

Start with a funcion where you pass x and it returns the error

```
function [e] = Prob3(x)
   y1 = cos(2*x);
   y2 = (x-2)*(x+1);
   e = y1 - y2;
end
```

Reuse the previous Newton's method routine

```
x1 = 0;
y1 = Prob3(x1);
for n=1:10
    disp([n, x1, y1])
    pause(0.1);
    x2 = x1 + 0.01;
    y2 = Prob3(x2);
    x1 = x1 - (x2-x1)/(y2-y1) * y1;
    y1 = Prob3(x1);
end
```

Starting at x = -1

n	x	error
1	-1.000000000000000	-0.416146836547142
2	-0.913605057651165	-0.001892486348330
3	-0.913207206067950	0.000001933805819
4	-0.913207612640883	-0.000000002062341
5	-0.913207612207287	0.000000000002200
6	-0.913207612207749	-0.000000000000002
7	-0.913207612207748	-0.000000000000000
8	-0.913207612207748	-0.000000000000000
9	-0.913207612207748	-0.000000000000000
10	**-0.913207612207748**	**-0.000000000000000**

Starting at x = +1:

n	x	error
1	1.000000000000000	1.583853163452858
2	1.561619993942688	0.123131393903411
3	1.618891878891919	0.002703761505062
4	1.620219983899382	-0.000011393117663
5	1.620214380300255	0.000000055442565
6	1.620214407569018	-0.000000000269648
7	1.620214407436394	0.000000000001311
8	1.620214407437039	-0.000000000000006
9	1.620214407437036	-0.000000000000000
10	**1.620214407437036**	**-0.000000000000000**

Note:

- The answer matches up with problem #3, only with lots more decimal places
- Your initial guess determines which solution Newton's method finds

Newton's Method with a Temperature Sensor

Assume the light - resistance relationship of a temperature sensor is:

$$R = 5000 \cdot \exp\left(\frac{4100}{T+273} - \frac{4100}{298}\right)\Omega$$

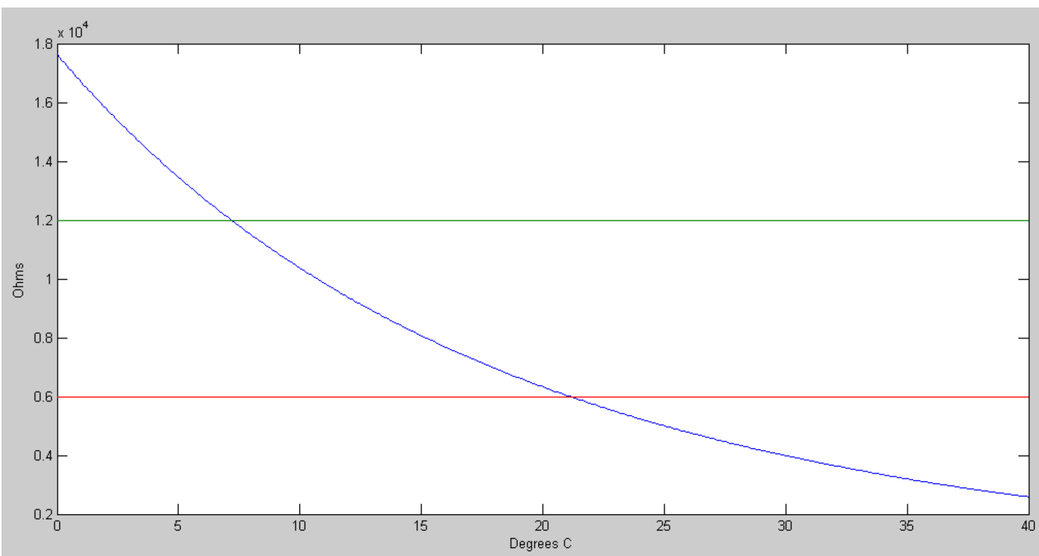
```
T = [0:0.01: 50]';
R = 5000 * exp(4100 ./ (T + 273) - 4100/298);
plot(T,R)
```

5) Use Newton's method to find the temperature when

- R = 12,000 Ohms
- R = 6,000 Ohms

(optional: graph the function): It helps to know what the result should look like. Plotting resistance vs. temperature

```
>> T = [0:0.1:40]';
>> R = 5000 * exp(4100 ./ (T+273) - 4100/298);
>> plot(T,R,T,0*R+12000,T,0*R+6000)
>> xlabel('Degrees C');
>> ylabel('Ohms');
>>
```



From the graph, the answers should be about + 7C and +21C

Start with a Matlab function where you guess T and it returns the error in resistance

```
function [e] = Prob5(T)
   R = 5000 * exp(4100/(T+273) - 4100/298);
   e = R - 12000;
end
```

Drive the error using Newton's Method

```
x1 = +1;
y1 = Prob5(x1);
for n=1:10
    disp([n, x1, y1])
    pause(0.1);
    x2 = x1 + 0.01;
    y2 = Prob5(x2);
    x1 = x1 - (x2-x1)/(y2-y1) * y1;
    y1 = Prob5(x1);
end
```

Result for 12000 Ohms

- T = 7.172187009910332C

n	T	error
1	1.000000000000	4685.703923644607
2	6.1437755087552	664.7201308248878
3	7.141579138792292	19.201841104642881
4	7.172168293011883	0.011731382292055
5	7.172187015456469	-0.000003476190614
6	7.172187009908712	0.000000001005901
7	7.172187009910317	0.000000000025466
8	7.172187009910358	-0.000000000016371
9	7.172187009910332	0.000000000025466
10	**7.172187009910373**	-0.000000000016371

Result for 6000 Ohms

- T = 21.102650386209412C

n	x	error
1	0.100000000000	10685.70392364461
2	12.730331884371	3027.204831631265
3	19.4097954657375	504.3211928958454
4	21.027220593688799	21.496530429883933
5	21.102516765246616	0.038002607832823
6	21.102650421940044	-0.000010161960745
7	21.102650386199763	0.000000002762135
8	21.102650386209476	-0.000000000009095
9	21.102650386209444	-0.000000000009095
10	**21.102650386209412**	0.000000000011823

6) Determine how many iterations are required to get the answer within

1 degree C

- 4 iterations

0.001 degree C

- 5 iterations

0.000 001 degree C

- 6 iterations

Newton's Method and a Voltage Divider

Assume

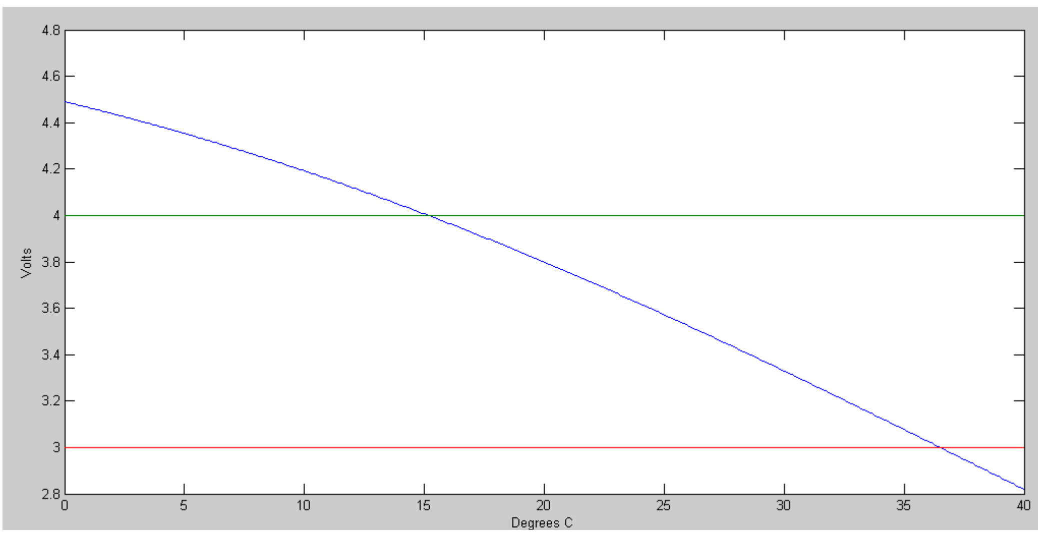
$$V = \left(\frac{R}{R+2000}\right) \cdot 5V$$

7) Use Netwon's method to determine the temperature when

- V = 4.00V
- V = 3.00V

(optional): Plot the fucntion to get an idea of what the answer should be:

```
>> T = [0:0.1:40]';
>> R = 5000 * exp(4100 ./ (T+273) - 4100/298);
>> V = R ./ (R + 2000) * 5;
>> plot(T,V,T,0*V+4,T,0*V+3)
>> xlabel('Degrees C');
>> ylabel('Volts');
>>
```



The answer should in the area of +15C and +36C

Check using Newton's method

First, write a routine which

- is passed the temperature, and
- returns the error in voltage

```
function [e] = Prob7(T)
   R = 5000 * exp(4100/(T+273) - 4100/298);
   V = R / (R+2000) * 5;
   e = V - 4;
end
```

Modify the Newton's search to find the zero crossing

```
x1 = +1;
y1 = Prob7(x1);
for n=1:10
    disp([n, x1, y1])
    pause(0.1);
    x2 = x1 + 0.01;
    y2 = Prob7(x2);
    x1 = x1 - (x2-x1)/(y2-y1) * y1;
    y1 = Prob7(x1);
end
```

Solution for 4.00V

- T = 15.156226174783603C

n	T	Voltage Error
1	1.000000000000000	0.464831507506327
2	18.807783737414393	-0.150057431356021
3	15.288761166179729	-0.005243245690512
4	15.156439397265190	-0.000008422710469
5	15.156226199480233	-0.000000000975563
6	15.156226174786422	-0.000000000000110
7	15.156226174783635	-0.000000000000000
8	15.156226174783624	-0.000000000000000
9	15.156226174783614	-0.000000000000000
10	**15.156226174783603**	-0.000000000000000

Solution for 3.00V

- T = 36.490870324702826 C

n	T	Voltage error
1	1.000000000000000	1.464831507506327
2	57.117974526647863	-1.020162858650172
3	34.442085267326746	0.104943051713067
4	36.497890215690994	-0.000360581118905
5	36.490870449635338	-0.000000006417188
6	36.490870324704169	-0.000000000000068
7	36.490870324702847	0.000000000000003
8	36.490870324702897	-0.000000000000002
9	36.490870324702861	-0.000000000000002
10	**36.490870324702826**	0.000000000000003