ECE 111 - Homework #4

Week #4: Math 129 Linear Algebra.

N equations & N unknowns

1) Solve for {x, y}

$$3x + 6y = 5$$

$$2x + 5y = 0$$

Step 1: Express in matrix form

$$BA = Y$$

$$\begin{bmatrix} 3 & 6 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

In Matlab

```
>> B = [3,6 ; 2,5]

     3     6
     2     5

>> Y = [5;0]

     5
     0

>> A = inv(B)*Y

x    8.3333
y   -3.3333

>>
```

2) Solve for {x, y, z}

$$4x + y - 5z = 6$$

$$8x - 7y + 7z = -5$$

$$9x - 7y - 5z = 9$$

Step 1: Express in matrix form

$$BA = Y$$

$$\begin{bmatrix} 4 & 1 & -5 \\ 8 & -7 & 7 \\ 9 & -7 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -5 \\ 9 \end{bmatrix}$$

Step 2: Solve using Matlab

```
>> B = [4 1 -5 ; 8 -7 7 ; 9 -7 -5]

     4     1    -5
     8    -7     7
     9    -7    -5

>> Y = [6 ; -5 ; 9]

     6
    -5
     9

>> A = inv(B)*Y

x    0.1287
y   -0.2946
z   -1.1559
```

3) Solve for {a, b, c, d}

$$-3a - b + c + 5d = -9$$
$$-6a - 3b + 8c + 2d = 1$$
$$-5a + 7b - 4c + d = 6$$
$$2a + 2b + 5c - 8d = 9$$

Step 1: express in matrix form

$$\begin{bmatrix} -3 & -1 & 1 & 5 \\ -6 & -3 & 8 & 2 \\ -5 & 7 & -4 & 1 \\ 2 & 2 & 5 & -8 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -9 \\ 1 \\ 6 \\ 9 \end{bmatrix}$$

Step 2: solve using Matlab

```
>> B = [-3 -1 1 5 ; -6 -3 8 2 ; -5 7 -4 1 ; 2 2 5 -8]

    -3    -1     1     5
    -6    -3     8     2
    -5     7    -4     1
     2     2     5    -8

>> Y = [-9 ; 1 ; 6 ; 9]

    -9
     1
     6
     9

>> A = inv(B)*Y

a   -3.0652
b   -1.9565
c   -2.0000
d   -3.6304

>>
```

Global CO2 Levels

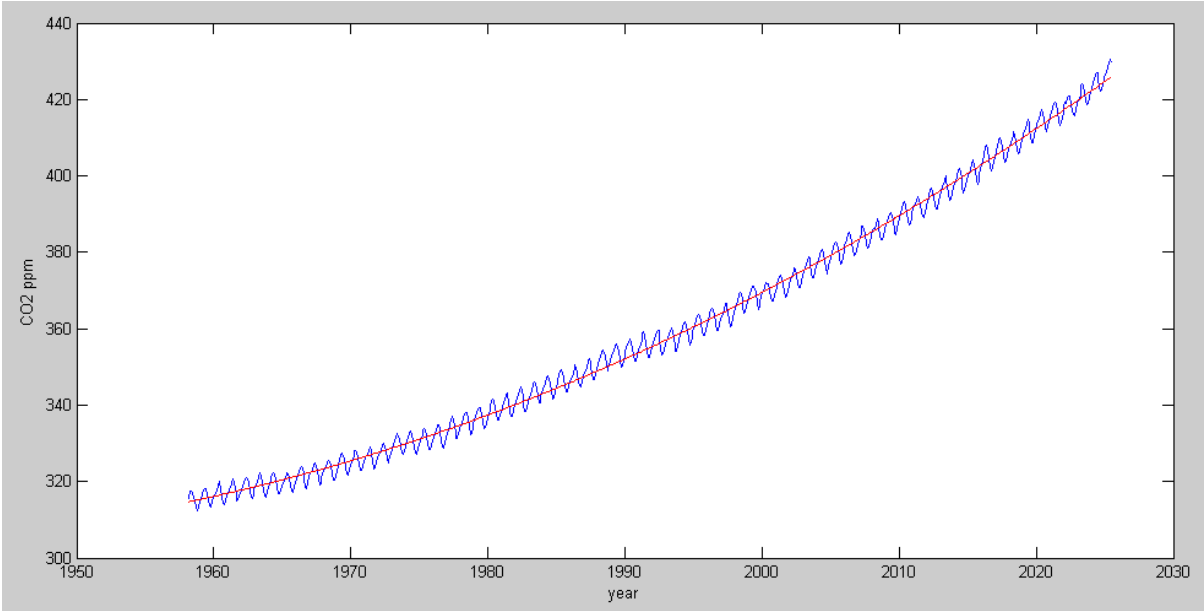
The CO2 levels measured at Mauna Loa observatory for the past 56 years are:

Problem 4) Determine a parabolic curve fit for this data in the form of

$$CO_2 \approx ay^2 + by + c$$

where 'y' is the year. Matlab Code

```
>> year = Data(:,3);
>> CO2 = Data(:,4);
>> B = [year.^2, year, year.^0];
>> format short e
>> A = inv(B'*B)*B'*CO2

a  1.3482e-002
b -5.2053e+001
c  5.0549e+004

>> plot(year,CO2,'b',year,B*A,'r')
>> xlabel('year');
>> ylabel('CO2 ppm')
>>
```



From this data, when do you predict that we will hit

- 350ppm?
- 500ppm?
- 1000 ppm?

Note: the *roots()* command in Matlab finds the zero crossings

$$ay^2 + by + c = 0$$

To get an answer different from zero,

$$ay^2 + by + c = 350$$

rewrite as

$$ay^2 + by + (c - 350) = 0$$

We hit 350ppm in the year 1988.7

```
>> roots(A - [0;0;350])

  1.9887e+003
  1.8724e+003
```

500ppm in the year 2051.0

```
>> roots(A - [0;0;500])

  2.0510e+003
  1.8101e+003
```

1000ppm in the year 2157.7

```
>> roots(A - [0;0;1000])

  2.1577e+003
  1.7034e+003
```

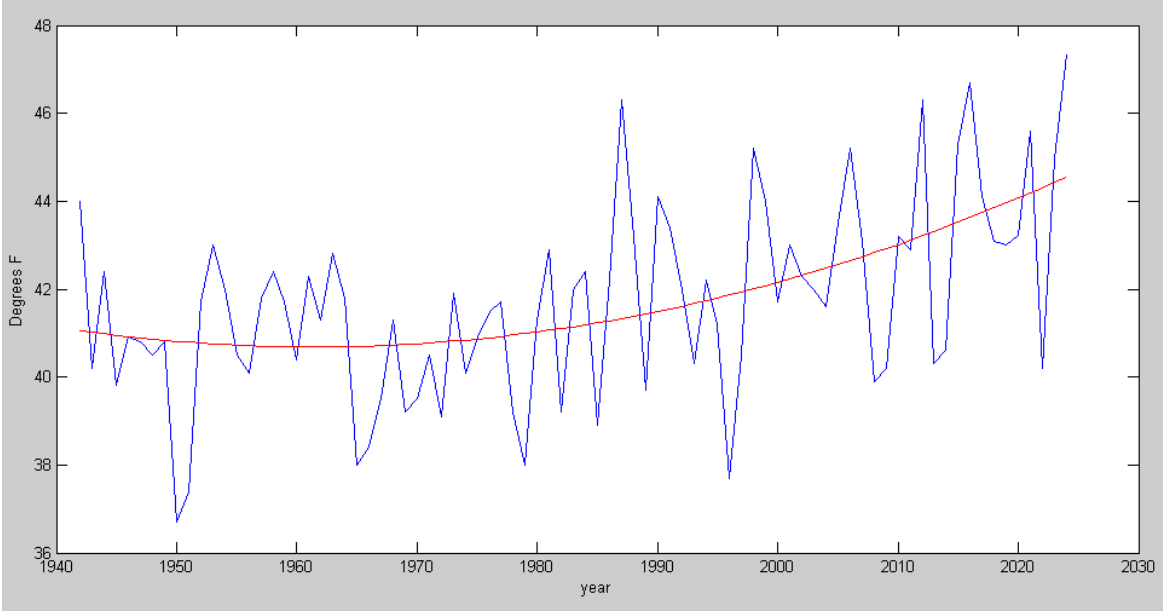
Fargo Temperatures

5) Using the average temperature in Fargo from 1942 to 2024:

5a) Determine a curve fit of the form of $T = ay^2 + by + c$

In Matlab

```
>> year = Data(:,1);
>> F = Data(:,14);
>> T = Data(:,14);
>> B = [year.^2, year, year.^0];
>> A = inv(B'*B)*B'*T

a   9.8667e-004
b  -3.8705e+000
c   3.8365e+003


>> plot(year,T,'b',year,B*A,'r')
>> xlabel('year');
>> ylabel('Degrees F')
```



5b) How much has Fargo warmed up over the past 80 years?

```
>> y = 1945;
>> T0 = [y^2 y 1] * A

T0 =   4.0950e+001

>> y = 2025;
>> T1 = [y^2 y 1] * A

T1 =   4.4678e+001

>> dT = T1 - T0

dT =   3.7276e+000
```

From this curve fit, Fargo's gotten 3.72F degrees warmer in the last 80 years

5c) What will the average temperature in Fargo be

- In the year 2050?
- In the year 2100?

```
>> y = 2050;
>> T2050 = [y^2 y 1] * A

T2050 =  48.433

>> y = 2100;
>> T2100 = [y^2 y 1] * A

T2100 =  59.642
```

Note: These results are somewhat suspect. You can use a t-test or an F-test to determine whether the data warrents a parabolic curve fit (it doesn't). This is covered in ECE 341 Random Processes (linked on Bison Academy).

Problem 6-7) Sea Ice: The area covered by sea ice is recored by the National Snow and Ice Data Center:

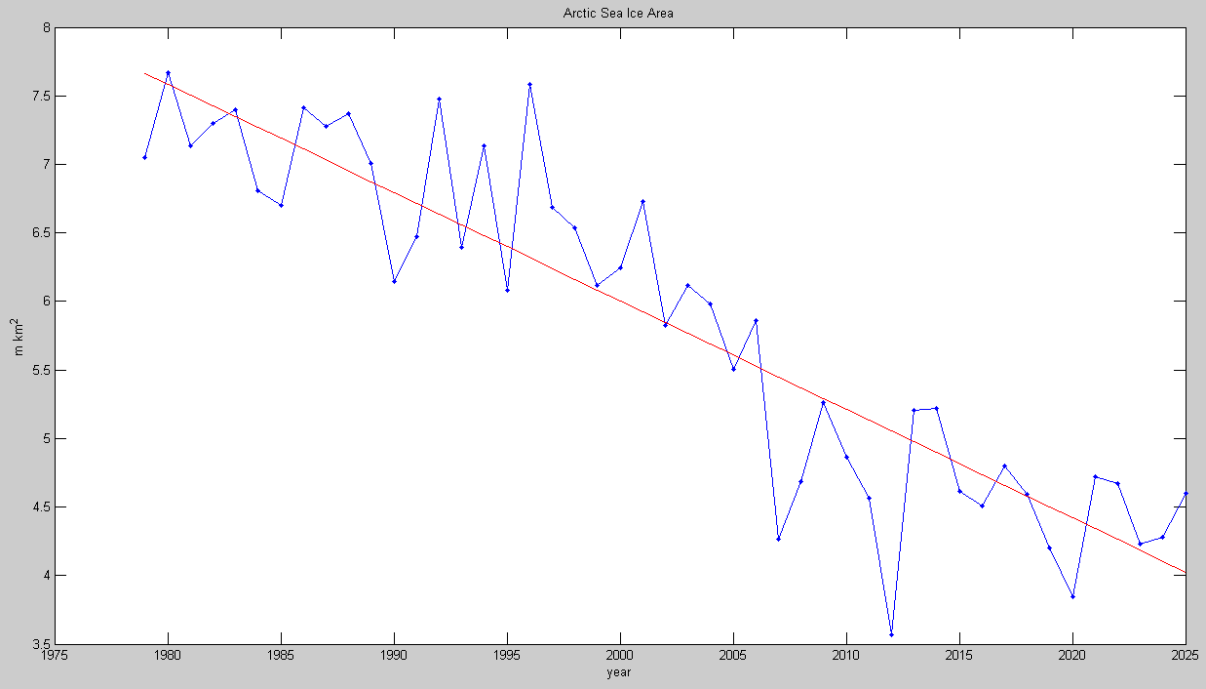
6) Approximate this data from the years 1979 - 2024 with a line

$$Area \approx ay + b$$

From this curve fit, when do you expect the Arctic to be ice free? (First time in 5 million years)

```
>> year = Data(:,1);
>> Ice = Data(:,2);
>> B = [year, year.^0];
>> A = inv(B'*B)*B'*Ice

a   -0.0791
b  164.2304

>> plot(year,Ice,'b.-',year,B*A,'r')
>> xlabel('year')
>> ylabel('m km^2');
>> title('Arctic Sea Ice Area')
>>
>> roots(A)

  2075.9
```

Based upon this data an a linear curve fit, the Arctic should be ice free in the year 2075



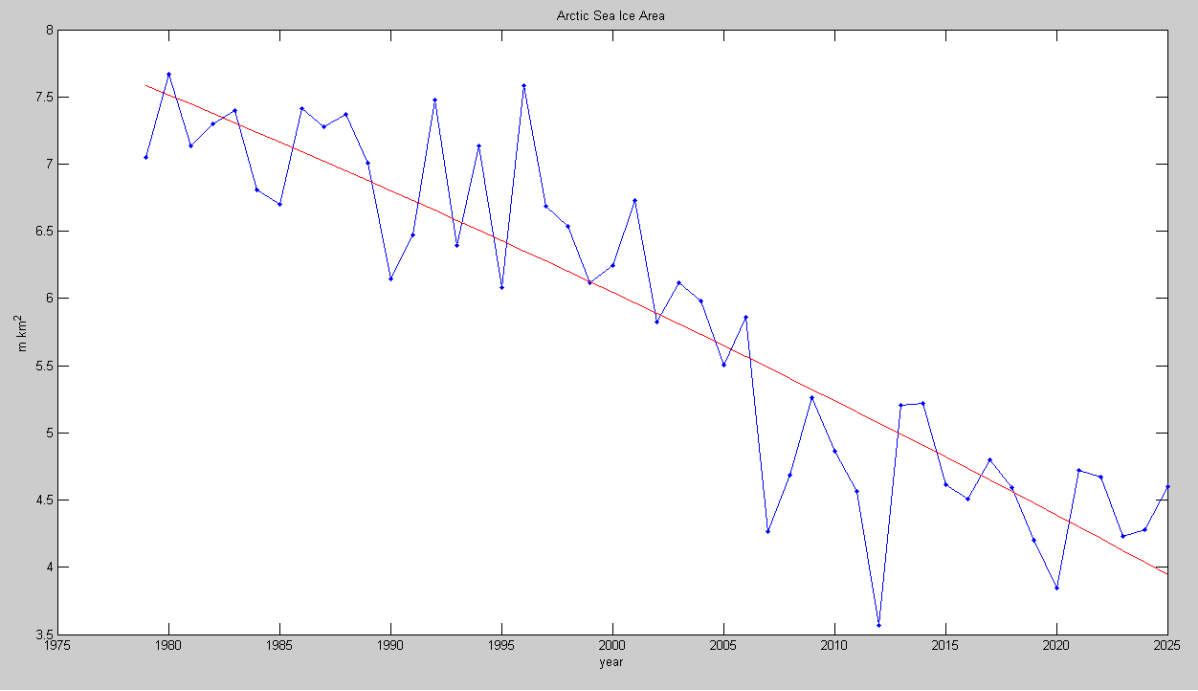
7) Approximate this data with a parabolic curve fit:

$$Area \approx ay^2 + by + c$$

From this curve fit, when do you expect the Arctic to be ice free?

```
>> B = [year.^2, year, year.^0];

>> year = Data(:,1);
>> Ice = Data(:,2);
>> B = [year.^2, year, year.^0];
>> A = inv(B'*B)*B'*Ice

a -2.3309e-004
b  8.5418e-001
c -7.6995e+002

>> plot(year,Ice,'b',year,B*A,'r')
>> xlabel('year');
>> ylabel('Area (M km^2)')
>> roots(A)

  2064.8
  1599.8
```

Assuming a parabolic curve fit, the Arctic should be ice free in the year 2064 (39 years from now)



Note: Again, the result is somewhat suspect as the data only warrents a linear curve fit (vs parabolic). You can check this with an F-test or a t-test (covered in ECE 341 random processes).

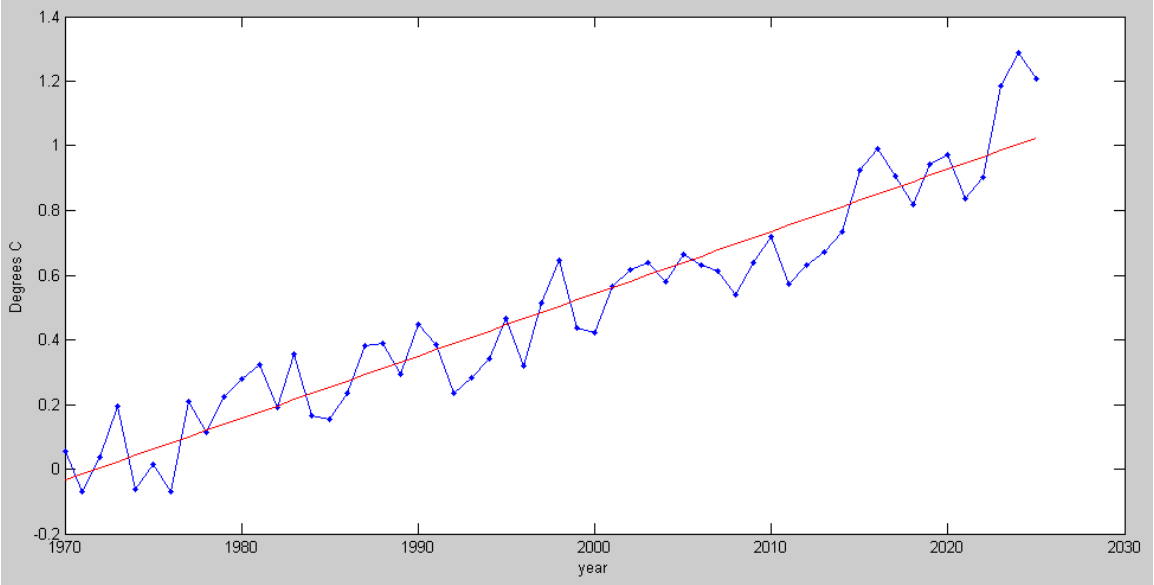
Problem 8-9: World Temperatures. NASA Goddard has been keep records since 1880 (144 years of data).

8) Determine a least-squares curve fit for this data from the year 1970 - 2024 in the form of

$$\delta T = ay + b$$

In Matlab

```
>> year = Data(:,1);
>> dT = Data(:,2);
>> B = [year, year.^0];
>> A = inv(B'*B)*B'*dT

a   1.9247e-002
b -3.7951e+001

>> plot(year,dT,'b.-',year,B*A,'r')
>> xlabel('year');
>> ylabel('Degrees C')
```



Based upon this data, what will the temperature be in the year 2050?

```
>> y = 2050;
>> T2050 = [y, 1] * A

T2050 =  1.5051e+000
```

The year 2100?

```
>> y = 2100;
>> T2100 = [y, 1] * A

T2100 =  2.4675e+000
```

When will be hit +6C?

```
>> roots(A - [0;6])

ans =  2.2835e+003
```

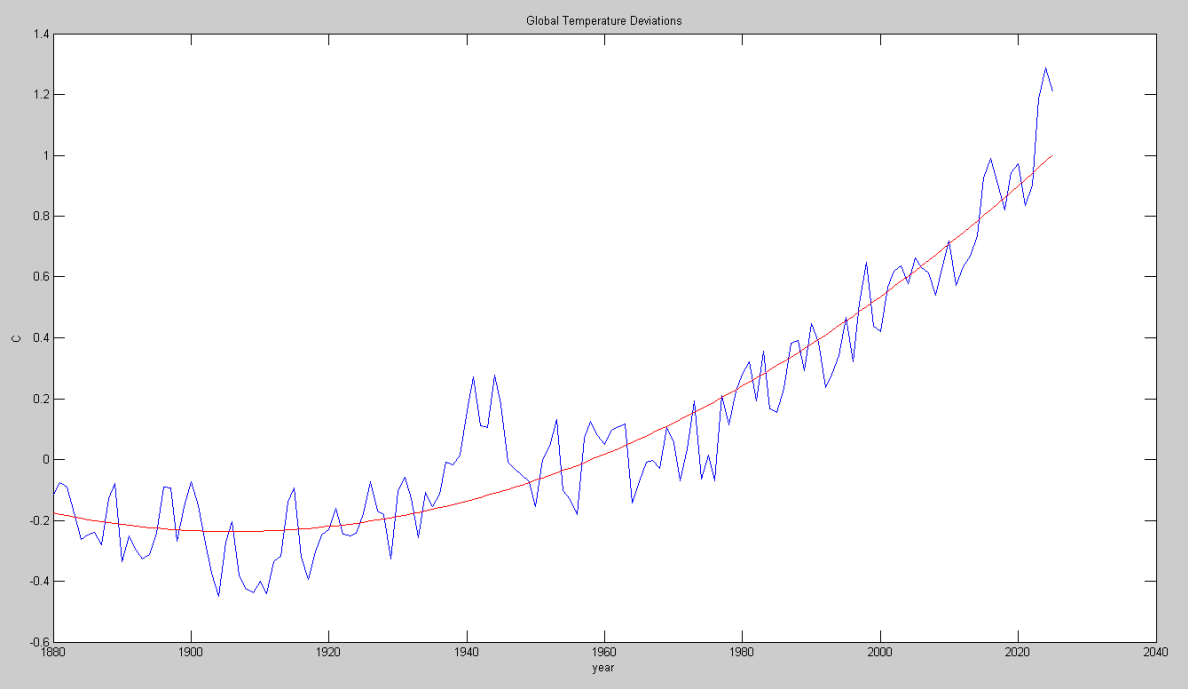
9) Determine a least-squares curve fit for this data from the year 1880 - 2024 in the form of

$$\delta T \approx ay^2 + by + c$$

```
>> y = Data(:,1);
>> dT = Data(:,2);
>> B = [y.^2, y, y.^0];
>> format short e
>> A = inv(B'*B)*B'*dT
```

```
a  8.7829e-005
b -3.3484e-001
c  3.1891e+002
```

```
>> plot(y,dT,'b',y,B*A,'r')
>> xlabel('year')
>> ylabel('C');
>> title('Global Temperature Deviations')
```



Based upon this data, what will the temperature be in the year 2050?

```
>> y = 2050;
>> [y^2 y 1]*A
```

```
1.5786
```

In the year 2100?

```
>> y = 2100;
>> [y^2 y 1]*A
```

```
3.0609
```

When will we hit +6C?

```
>> roots(A - [0;0;6])
```

```
2172.7
1639.7
```

10) What does a temperature rise of 10 degrees mean for the planet?

not graded - too political

The Permian Extinction Event suggests that it's not good: no animals larger than a mouse survived the Permian Extinction - which was triggered by CO2 levels at 2000ppm and a +10 degree C temerature rise.

One Degree: 2024 Summers like 2003 where a heat wave in France caused 10,000 deaths become the norm. Flows of the Po and Rhine river decrease. Crop production drops.

```
-->roots(A - [0;0;1])

  2024.9
  1787.6
```

Two Degrees: 2065. Oceans absorb less CO2 (too hot) and soils start to release CO2. Vacations to the Mediterranean in the summer are just too hot. Crop failures in Africa and Central America cause mass migration. Coastal cities flood. 1/3rd of species face extinction.

```
-->roots(A - [0;0;2])

  2065.8
  1746.6
```

Three Degrees: 2098. Crop failures in China cause the migration of more than 1 billion people. Collapse of equatorial governments.

```
-->roots(A - [0;0;3])

  2098.2
  1714.3
```

Four Degrees: 2125. Spain becomes a desert. Mass migration to Northern latitudes. Rain forests burn up.

```
-->roots(A - [0;0;4])

  2125.9
  1686.6
```

Six Degrees: 2172. Ice caps are gone. Methane hydrates become unstable raising temperatures in a positive-feedback loop. Ocean circulation stops. Hydrogen sulfide producing bacteria flourish poisoning the air. The Ozone layer dissipates leaving the land sterilized with UV radiation. End-Permian-like conditions make life nearly impossible.

```
-->roots(A - [0;0;0;6])

  2172.7
  1639.7
```

Scary? Yes. That's why the rest of the world sees the Paris Climate Accord as being important. That's why the United Nations sees Global Warming as the #1 threat - far greater than terrorism. Far greater than COVID.