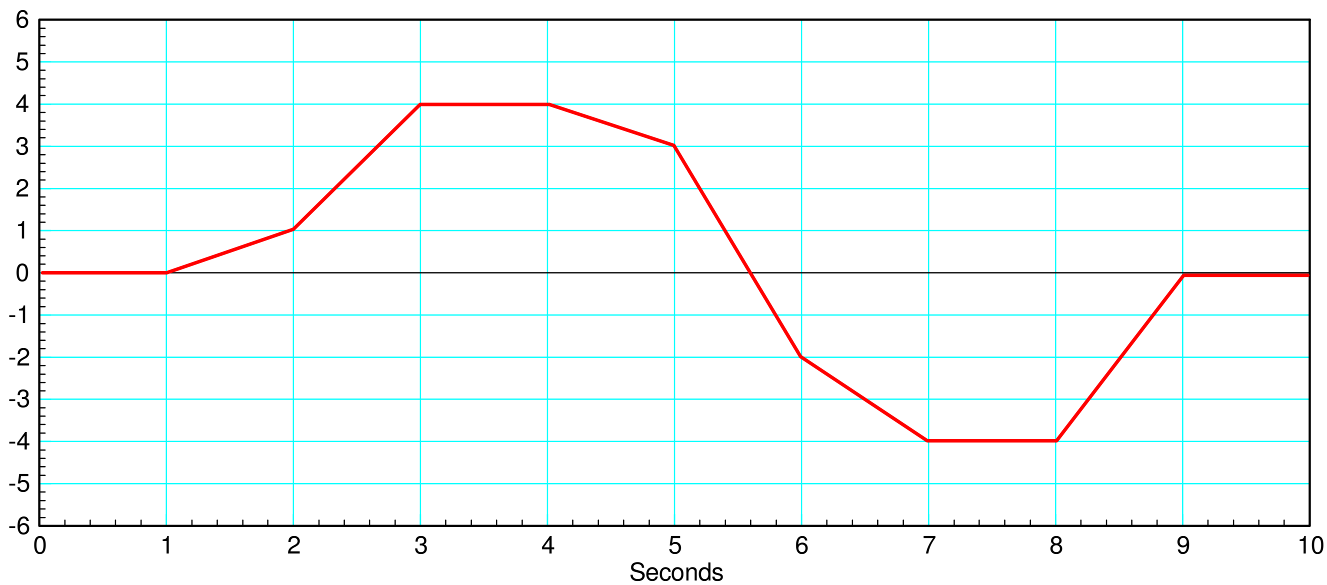
ECE 111 - Homework #6:

Math 165: Differentiation

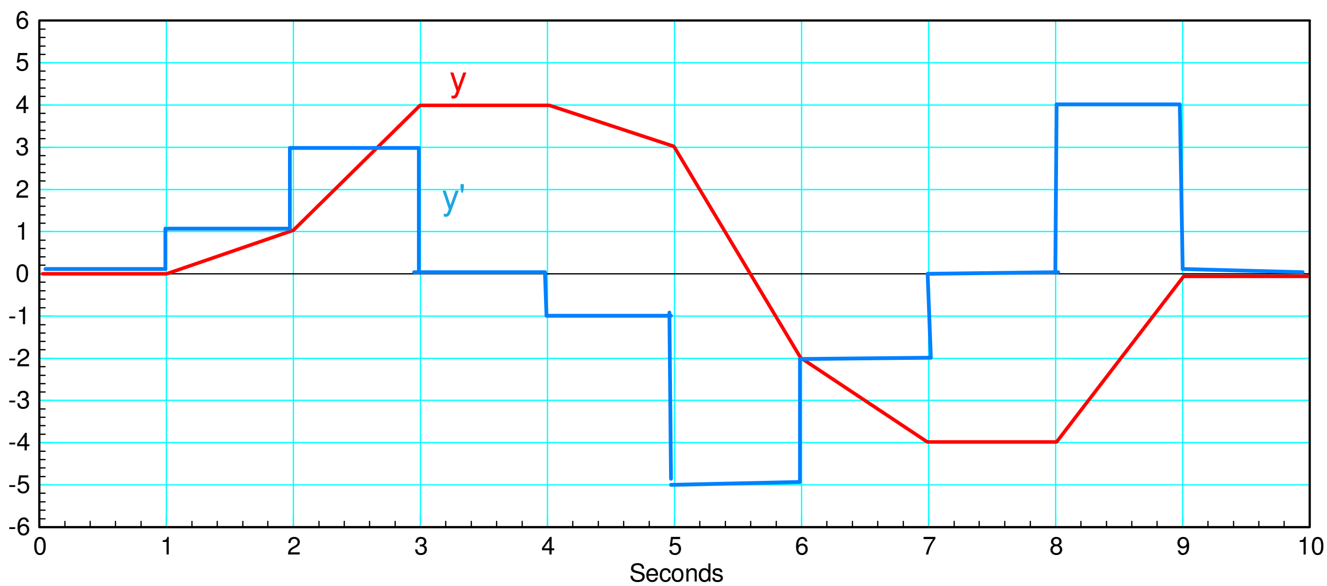
1) Sketch the derivative of the following funciton

If this is the balance of your checking account, how much money are you adding (positive) or withdrawing (negative) for the balance to be as shown?



The derivative is the slope

$$\frac{dy}{dx} = \frac{\text{change in y}}{\text{change in x}}$$



Numerical Differentiation:

2) Use numerical methods to determine the derivative of y:

$$y = \left(\frac{\cos(3x)}{1+x^2}\right)$$

$$z = \frac{d}{dx}(y)$$

for -10 < x < 10. (a plot is sufficient).

Start with a Matlab function

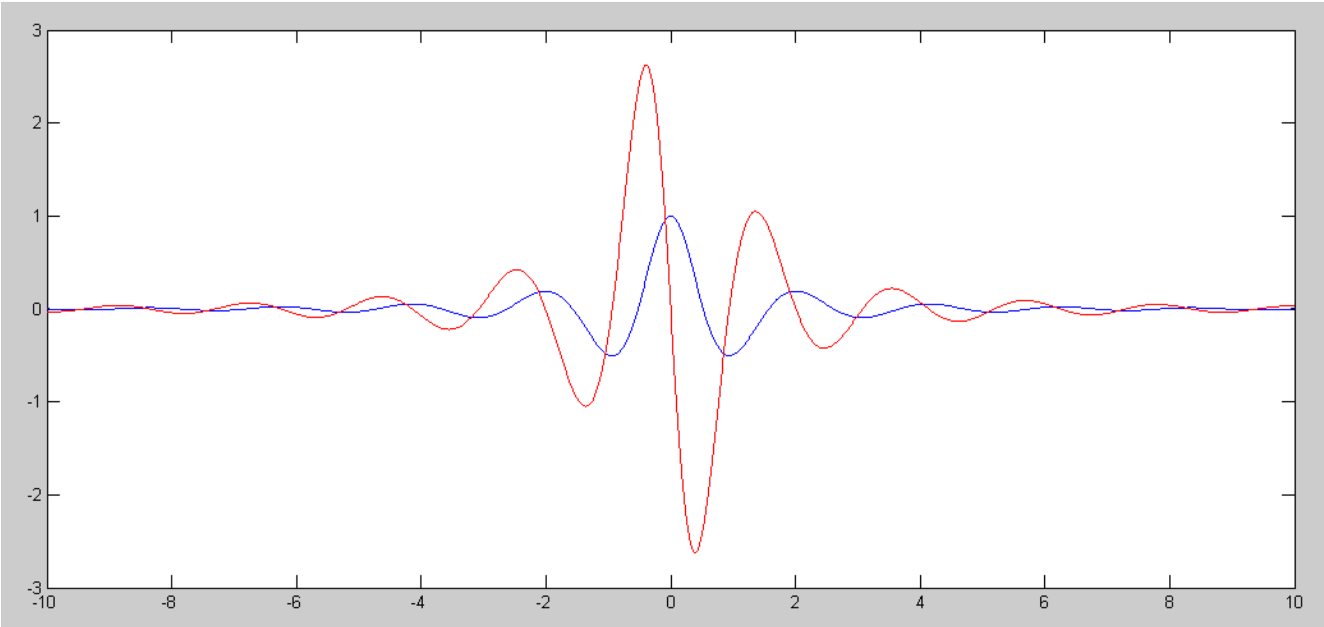
```
function [ dy ] = derivative( x, y )

dy = 0*y;
n = length(y);
for i=2:n-1
   dy(i) = ( y(i+1) - y(i-1) ) / ( x(i+1) - x(i-1) );
   end

dy(1) = 2*dy(2) - dy(3);
dy(n) = 2*dy(n-1) - dy(n-2);

end
```

From the command window

```
>> x = [-10:0.01:10]';
>> y = cos(3*x) ./ (1 + x.^2);
>> z = derivative(x, y);
>> plot(x,y,'b',x,derivative(x,y),'r')
```

Note

- If you can get the function into Matlab, you can find the derivative using numerical methods
- How to find the equation for z(x) is something you'll cover in Calculus I. Whatever it is, it looks like the red curve.



y (blue) & dy/dx (red)

3) Use numerical methods to determine the derivative of y:

$$y = \cos(x) + 0.1\sin(20x)$$

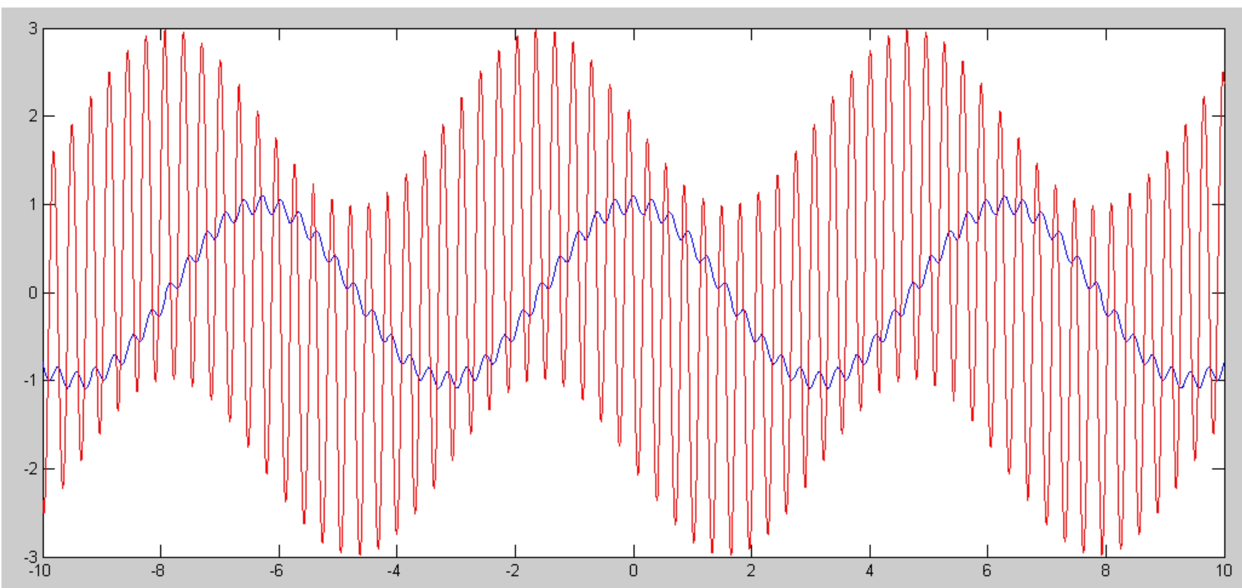
$$z = \frac{d}{dx}(y)$$

for -10 < x < 10. (a plot is sufficient).

```
>> x = [-10:0.01:10]';
>> y = cos(x) + 0.1*cos(20*x);
>> plot(x,y,'b',x,derivative(x,y),'r')
>>
```

Note

- Differentiation amplifies high-frequency terms
- If you can get the function into Matlab, you can find the derivative using numerical methods
- How to find the equation for z(x) is something you'll cover in Calculus I. Whatever it is, it looks like the red curve.



Path Planning

4) Assume a motor's angle is as follows:

$$\theta = \begin{cases} 0 & t < 0 \\ t/4 & 0 < t < 4 \\ 1 & t > 4 \end{cases}$$

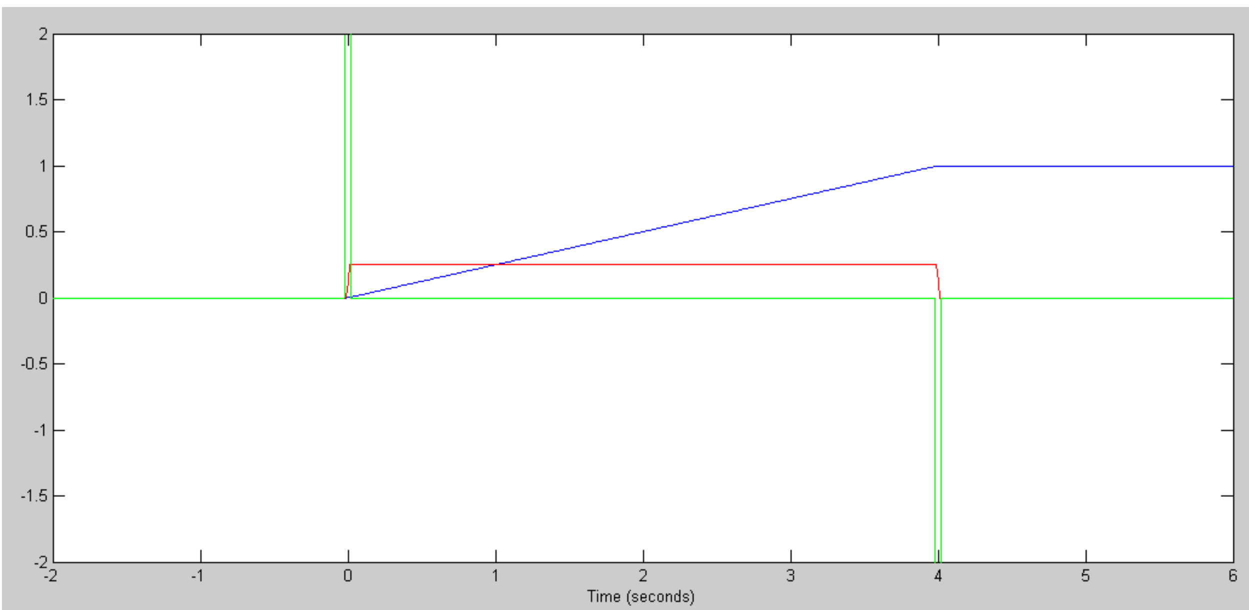
Calculate and plot using Matlab and numerical differentiation:

- The velocity vs. time (i.e. the voltage to the motor), and
- The acceleration vs. time (i.e. the current to the motor).

```
>> t = [-2:0.01:6]' + 1e-6;
>> q4 = t/4 .* (t>0) .* (t<4) + 1*(t>4);
>> q = t/4 .* (t>0) .* (t<4) + 1*(t>4);
>> dq = derivative(t, q);
>> ddq = derivative(t, dq);
>> plot(t,q,'b',t,dq,'r',t,ddq,'g')
>> ylim([-2,2])
>> xlim([-2,6])
>> xlabel('Time (seconds)')
>>
```

Note:

- The acceleration has spikes (delta functions) at the start and end of the motion
- This implies infinite accelearation and infinite current to the motors
- This isn't a good way to go from point A to point B



Angle (blue), Velocity (red), Acceleration (green)

5) Assume a motor's angle is as follows:

$$\theta = \begin{cases} 0 & t < 0 \\ \frac{1}{2} - \frac{1}{2}\cos\left(\frac{1}{4}\pi t\right) & 0 < t < 4 \\ 1 & t > 4 \end{cases}$$

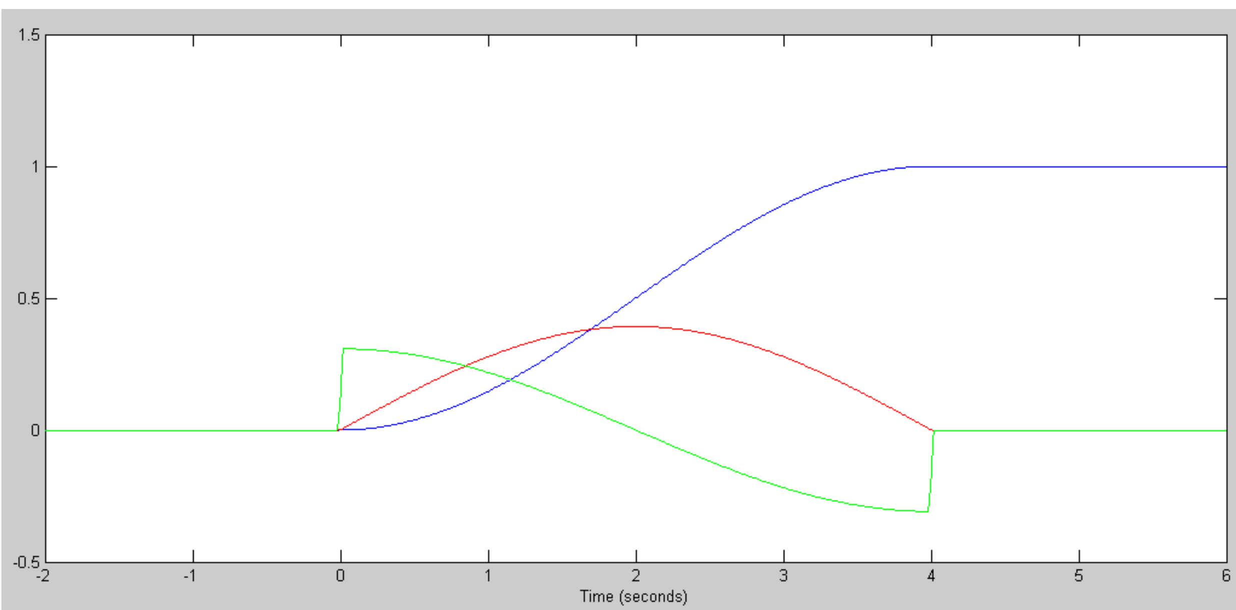
Calculate and plot using Matlab and numerical differentiation:

- The velocity vs. time (i.e. the voltage to the motor), and
- The acceleration vs. time (i.e. the current to the motor).

```
>> t = [-2:0.01:6]' + 1e-6;
>> q = (0.5 - 0.5*cos(pi*t/4)) .* (t>0) .* (t<4) + 1*(t>4);
>> dq = derivative(t, q);
>> ddq = derivative(t, dq);
>> plot(t,q,'b',t,dq,'r',t,ddq,'g')
>> xlabel('Time (seconds)')
>> xlim([-2,6])
>> ylim([-2,2])
>> ylim([-0.5,1.5])
>>
```

Note:

- This is a much more reasonable way to go from point A to point B
- The velocity (red) is roughly motor voltage - and is easy to apply
- The acceleration (green) is roughly motor current. It has jump discontinuities - but that's not a problem for the hardware.



6) Assume a motor's angle is as follows:

$$\theta = \begin{cases} 0 & t < 0 \\ \frac{1}{8}t^2 & 0 < t < 2 \\ -\frac{1}{8}t^2 + t - 1 & 2 < t < 4 \\ 1 & t > 4 \end{cases}$$

Calculate using Matlab and numerical differentiation:

- The velocity vs. time (i.e. the voltage to the motor), and
- The acceleration vs. time (i.e. the current to the motor).

```
>> t = [-2:0.01:6]' + 1e-6;
>> q = t.^2/8 .* (t>0) .* (t<2) + (-t.^2/8+t-1).*(t>2).*(t<4) + 1*(t>4);
>> dq = derivative(t, q);
>> ddq = derivative(t, dq);
>> plot(t,q,'b',t,dq,'r',t,ddq,'g')
>> xlim([-2,6])
>> ylim([-0.5,1.5])
>> xlabel('Time (seconds)')
>>
```

Note:

- This is another reasonable way to go from point A to point B
- The velocity (red) is roughly motor voltage - and is easy to apply
- The acceleration (green) is roughly motor current. It has jump discontinuities - but that's not a problem for the hardware.