ECE 331: Energy Conversion

ECE 111 - Week #16

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Objective of Today's Lecture:

- Transformer Operation
- Efficiency of a power grid
- Why we use AC instead of DC
- Why there are transformers & substations.

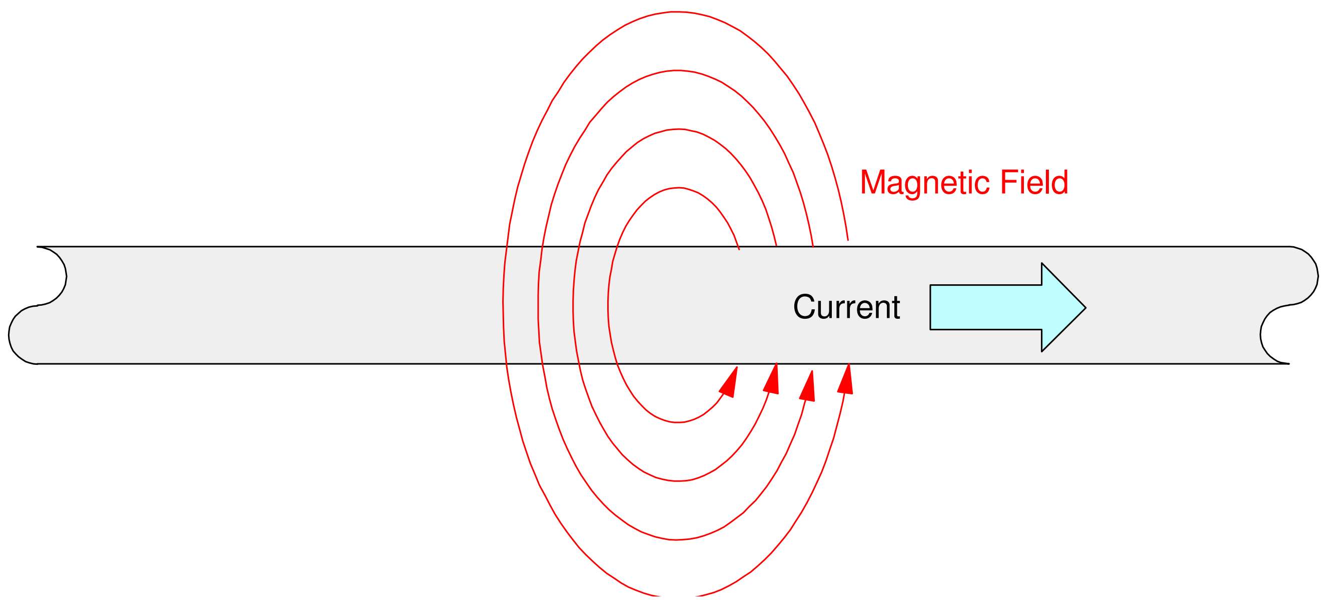
Magnetism and Current

Current and magnetism are inherently related

$$B = \frac{\mu I}{2\pi r} \qquad \textit{magnetic field strength}$$

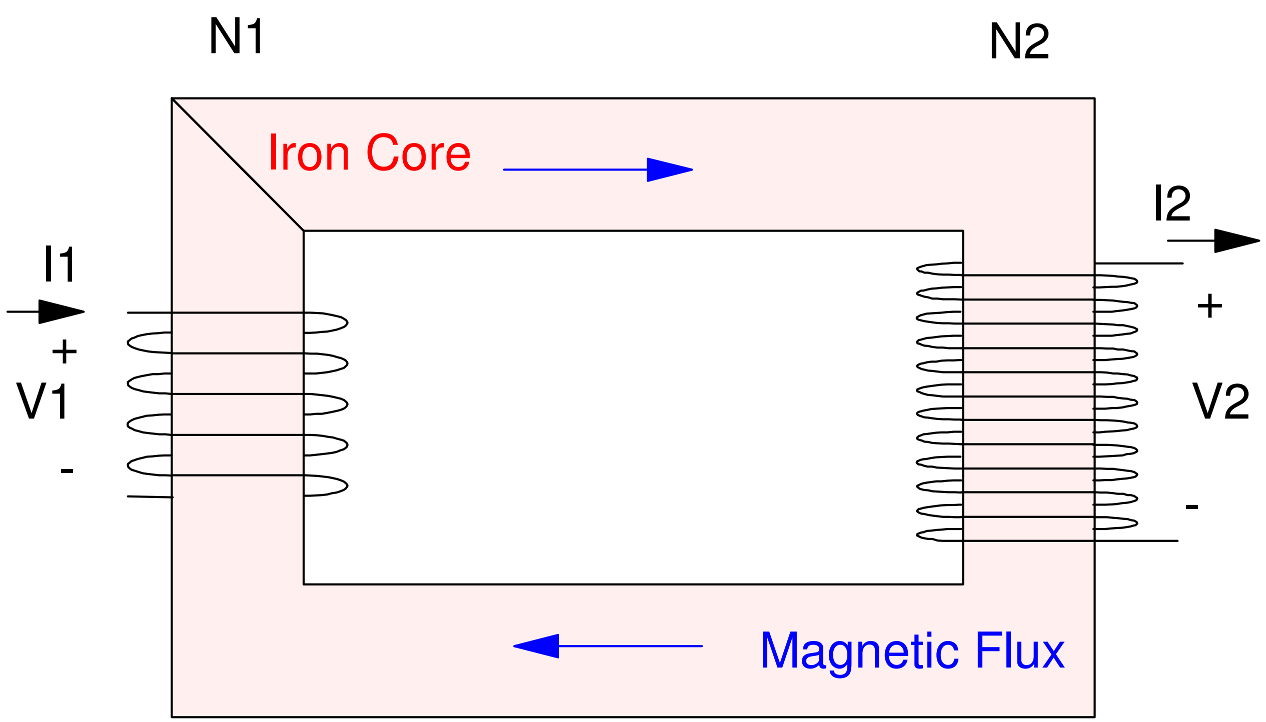
Changing magnetic fields produce voltage (Faraday's Law)

$$V = N\frac{d\phi}{dt}$$



Transformers

- If you apply current to a coil, you produce magnetic flux (think of it as magnetic current)
- If you have a changing magnetic flux flowing through a coil, you produce current



Transformer Equations

Flux Created

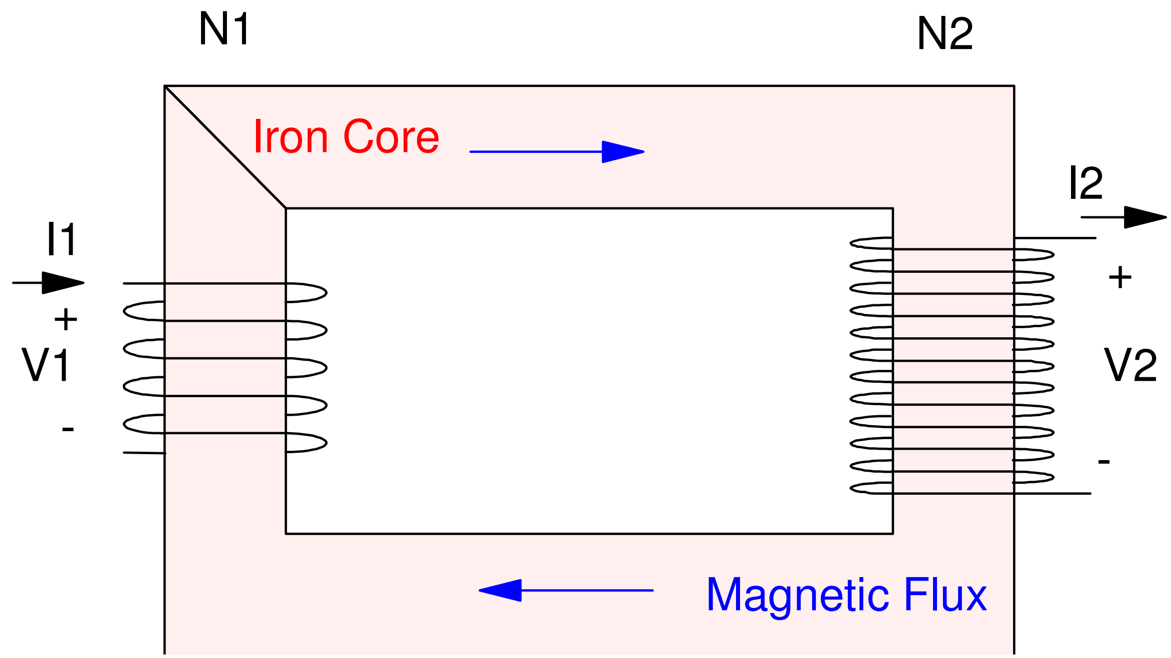
$$\phi = N_1 I_1 \quad \text{amp-turns}$$

Conservation of flux:

$$\phi = N_1 I_1 = N_2 I_2$$

meaning

$$I_2 = \left(\frac{N_1}{N_2}\right) I_1$$



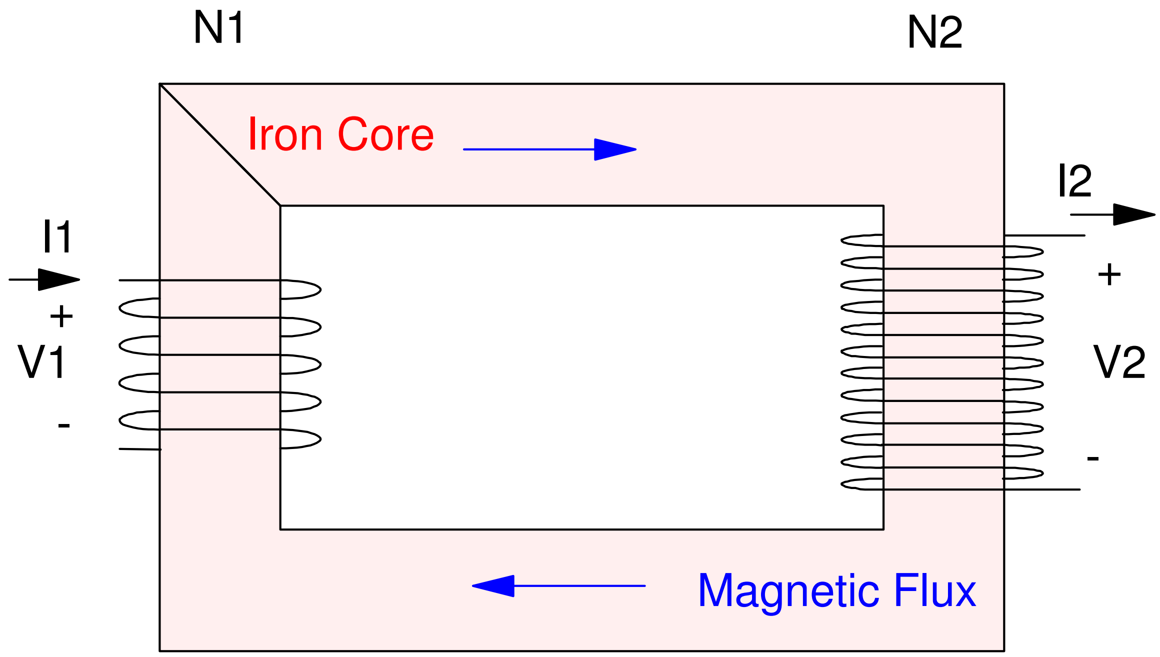
Conservation of energy

$$V_1 I_1 = V_2 I_1$$

Substituting:

$$V_2 = \left(\frac{N_2}{N_1}\right) V_1$$

Transformers convert one voltage to another by the turns-ratio



Impedances & Transformers

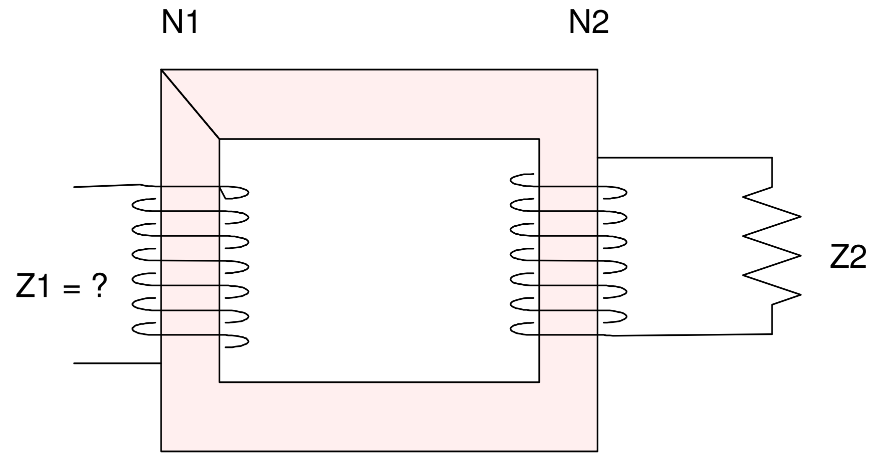
If you connect an impedance to side 2 of the transformer, the impedance on side 2 by definition is

$$Z_1 = \frac{V_1}{I_1}$$

Transferring this to side one gives

$$Z_1 = \frac{V_1}{I_1} = \frac{\left(\frac{N_1}{N_2}\right)V_2}{\left(\frac{N_2}{N_1}\right)I_2} = \left(\frac{N_1}{N_2}\right)^2\left(\frac{V_2}{I_2}\right)$$

$$\boxed{Z_1 = \left(\frac{N_1}{N_2}\right)^2 Z_2}$$



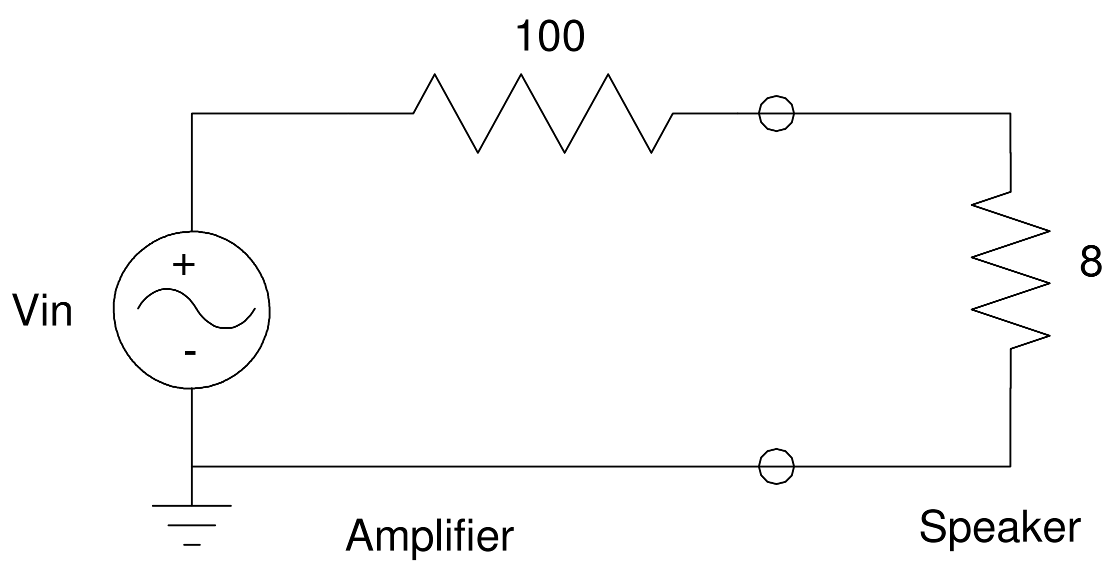
Impedances seen through a transformer scale by the square of the turns ratio

Applications (take 1)

Drive an 8 Ohm speaker

$$V_L = \left(\frac{8}{100+8}\right) V_{in}$$

Only 7.4% of the voltage gets to the speaker



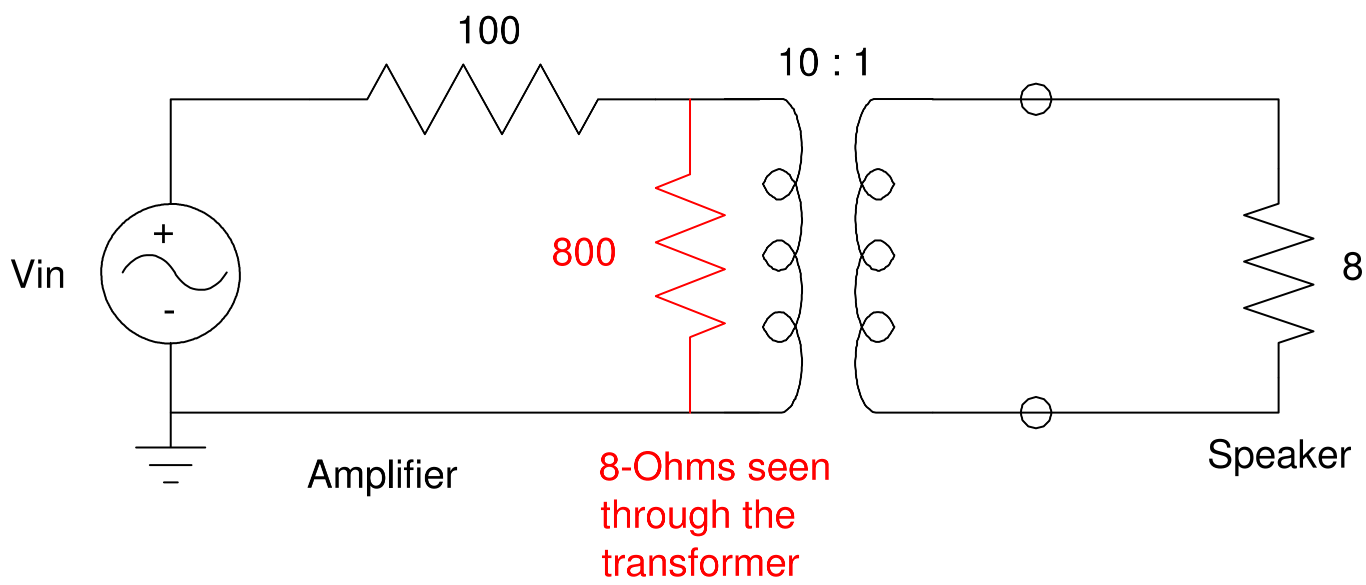
Add a Transformer:

The amplifier sees an 800 Ohm resistor

- Impedance increases as the square of the turns ratio

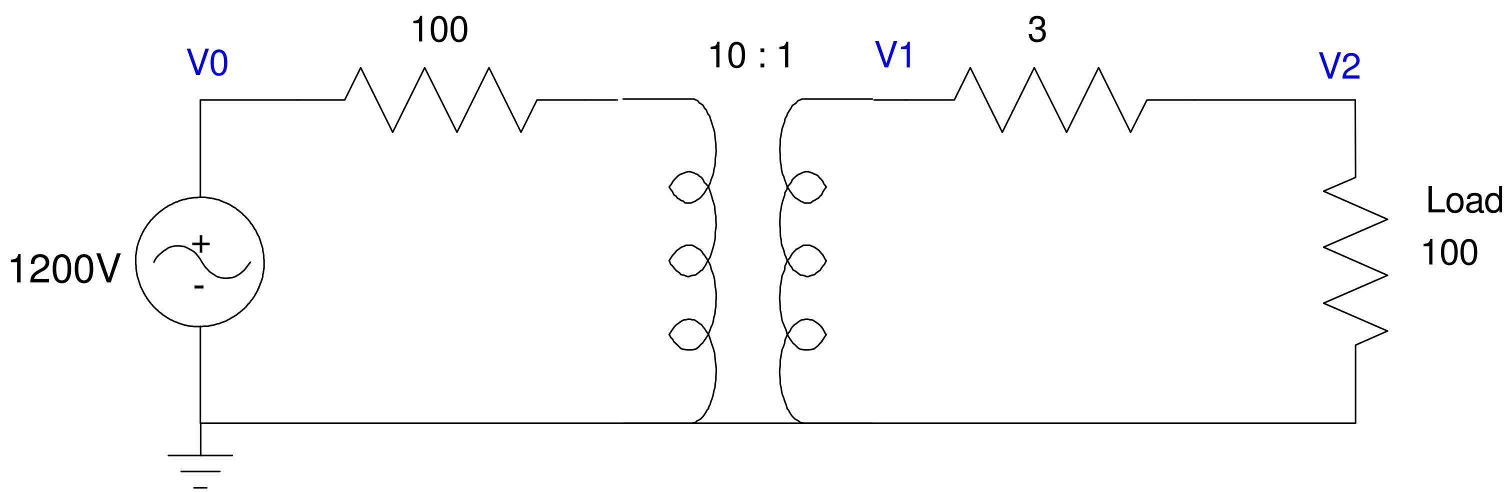
Now, 89% of the power goes to the speaker

- But you have 10x the voltage on the amplifier side



Handout:

- Redraw the circuit as seen by the load (transfer everything to the right side of the transformer)
- Determine the voltages and current as seen by the load



Transformer Construction

An iron core is used to carry the magnetic flux

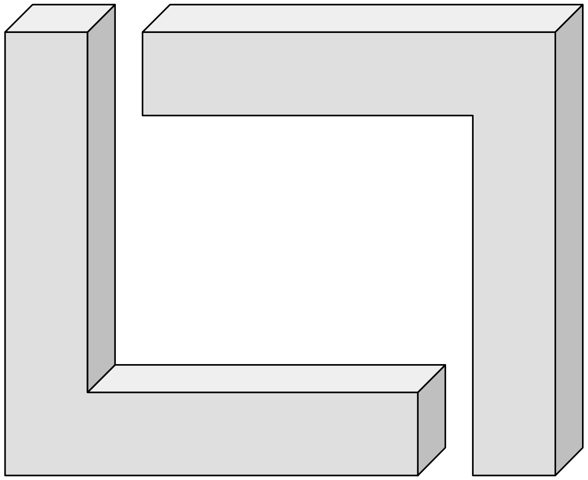
- Iron is 8,000 to 100,000 times more magnetic than air
- This makes transformers heavy

The core is

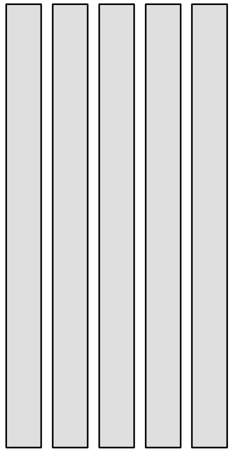
- Split to reduce Eddy currents
- Laminated to further reduce Eddy currents

You still have some losses in the core

- Transformers get warm when energized
- This is modeled as a resistance (lossy element)



Front View

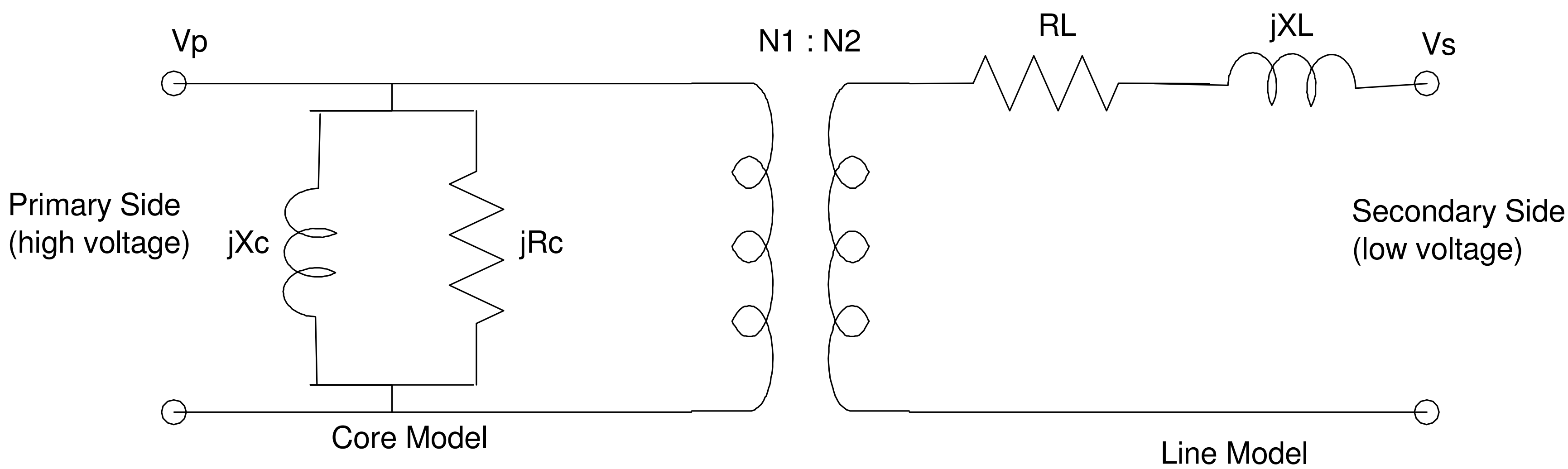


Side View

Transformer Modeling

Assume 60Hz:

- Core Model: Inductance of the transformer (jXc) & transformer losses (Rc)
- Line Model: Inductance of the wires (jX_L) & copper losses (R_L)



Power Factor

Often times in power systems, a parameter called *power factor* is used:

$$\text{Power Factor} = \frac{\text{real(power)}}{\text{abs(V)}\cdot\text{abs(I)}} = \cos\theta$$

where

$$R + jX = Z\angle\theta$$

Power is

$$P = \frac{V^2}{Z}\cdot pf = |V|\cdot|I|\cdot pf$$

The impedance as a function of power and power factor is:

$$Z = \left(\frac{V^2 \cdot pf}{P}\right)\angle\arccos(pf)$$

Transformer Testing

Open Circuit Test: (This measures Rc and jXc)

- Leave the secondary side open.
- Apply rated voltage to the primary side and measure the voltage, current, and power.

Short Circuit Test: (This measures R_l and jX_l)

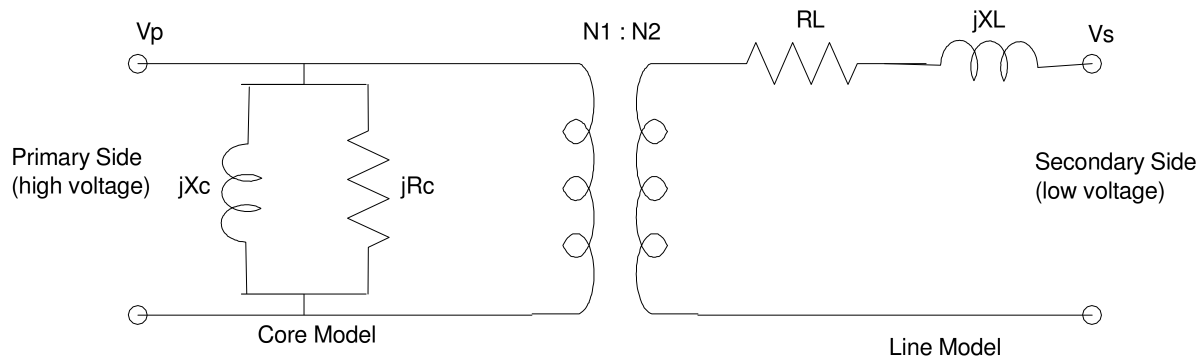
- Short the primary side.

Example: A 13kV : 240V transformer

	V	Power	pf
Open-Circuit Test	Vp = 13kV	100 W	0.02
Short-Curcuit Test	Vs = 10V	200 W	0.95

Open-Circuit Test:

- Secondary side = open
  current = 0
  RL & jXL don't matter
- Vp = 13.2kV
- P = 100W
- pf = 0.02
- Find the core model (Rc & jXc)



$$P = |V| \cdot |I| \cdot pf$$

$$100W = 13.2kV \cdot |I| \cdot 0.02$$

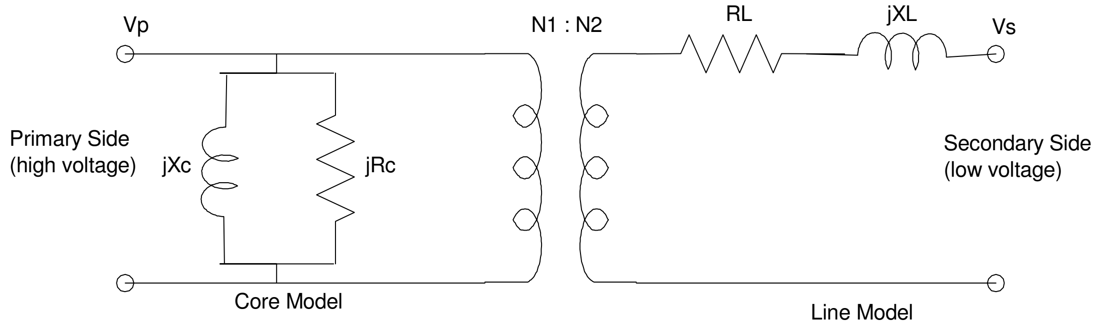
$$|I| = 0.3788A$$

Open-Circuit Test (cont'd)

$$Z = \left(\frac{|V|}{|I|}\right)\angle \arccos(pf)$$

$$Z = \left(\frac{13.2kV}{03788A}\right)\angle \arccos(0.02)$$

$$Z = 34,848\angle 88.854^0$$



$$Z = 696.96 + j34,841.03$$ *series model for R & L*

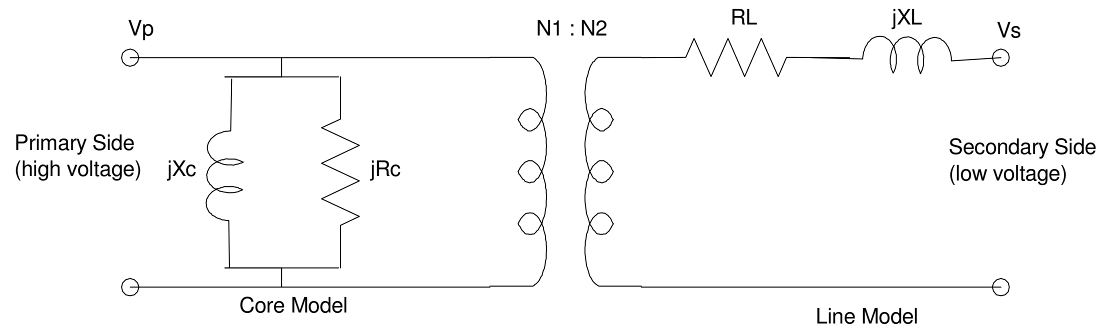
$$\frac{1}{Z} = \frac{1}{R_c} + \frac{1}{jX_c} = 5.739 \cdot 10^{-7} - j2.869 \cdot 10^{-5}$$

$$R_c = \frac{1}{5.739 \cdot 10^{-7}} = 1.742M\Omega$$ *parallel model for R & L*

$$jX_c = \frac{1}{-j2.869 \cdot 10^{-5}} = j34.854k\Omega$$

Short Circuit Test:

- Vp = 0V (short)
  Rc & jXc = 0 Ohms (shorted)
- Vs =10V
- P = 200W
- pf = 0.95
- Find RL & jXL



$$P = |V| \cdot |I| \cdot pf$$

$$200W = 10V \cdot |I| \cdot 0.95$$

$$|I| = 21.053A$$

Short Circuit Test (cont'd)

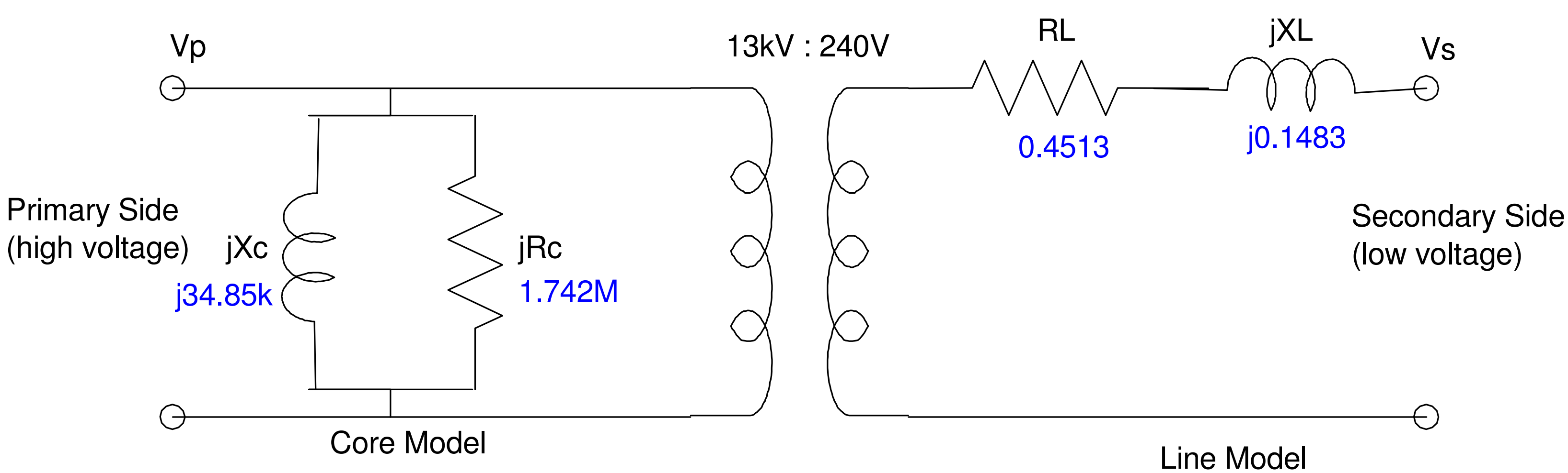
$$Z_L = \left(\frac{|V|}{|I|}\right) \angle \arccos(pf)$$

$$Z_L = \left(\frac{10V}{21.053A}\right) \angle \arccos(0.95)$$

$$Z_L = 0.4750 \angle 18.19^0$$

$$Z_L = 0.4513 + j0.1483$$

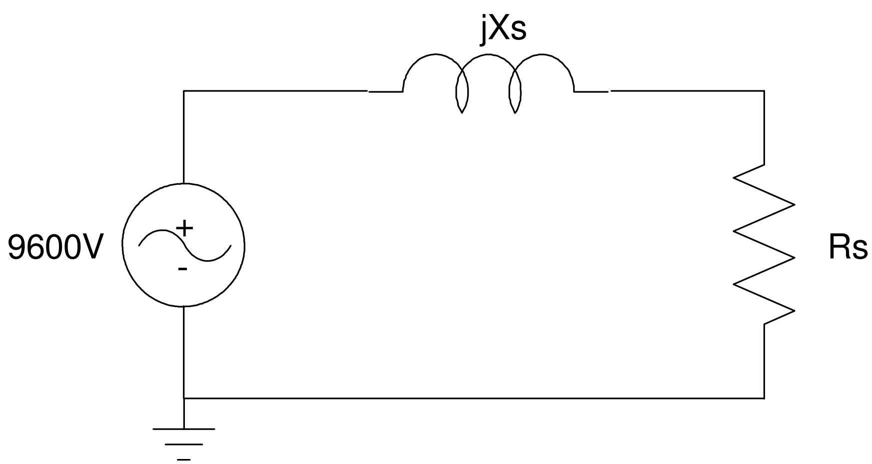
$$R_L = 0.4513\Omega \qquad jX_L = j0.1483\Omega$$

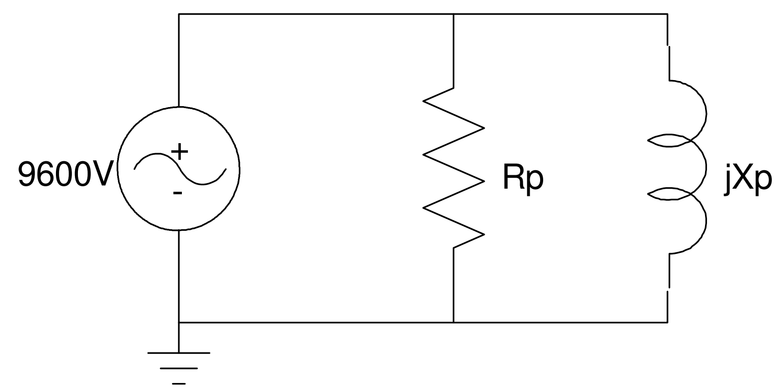


Handout:

Determine the series (Rs, jXs) and parallel (Rp, jXp) model for the load (Z)

- Vin = 9600V
- Power = 200 Watts
- pf = 0.05





Transmission Line Analysis:

US Energy and Information Administration

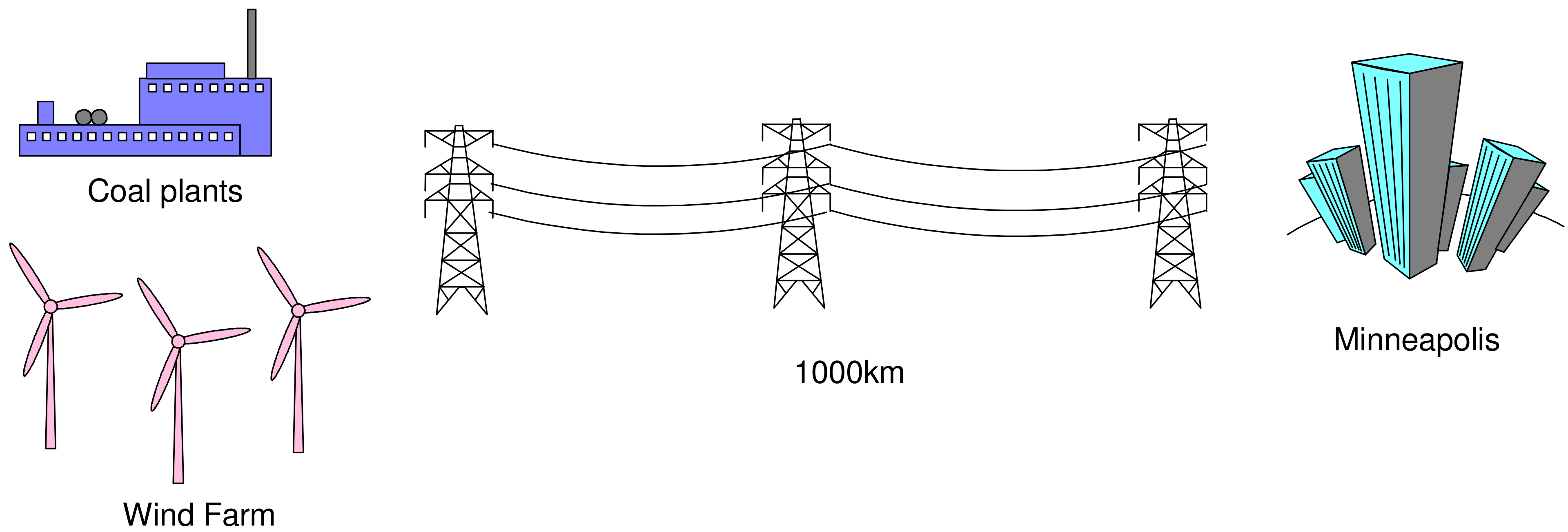
In 2024, North Dakota produced 3238 GWh of electricity:

- U.S. Energy and Information Administration
- At $0.11/kWh, this equates to $356 million.
- A large percentage of this energy is transmitted east to Minneapolis and Chicago.

Source	GWh	% of Total
Gas	250	7.7%
Hydro	126	3.9%
Wind / Solar	1,187	36.7%
Coal	1,675	51.7%
Total	3,238	**100%**

Problem:

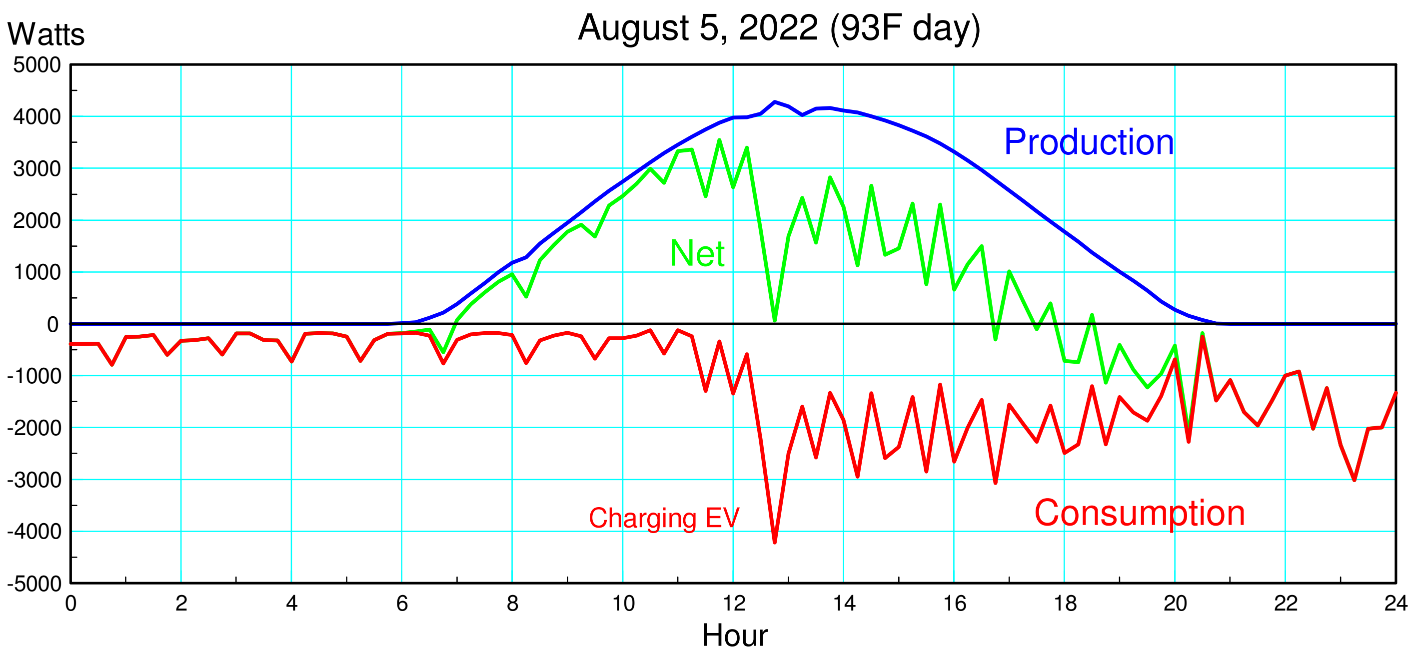
How do you transmit large amounts of energy over a distance of 1000km (Minneapolis) or 2000km (Chicago)?



How much power does a home use?

Case Study: My house

- 2000 square feet
- 5400W solar panels on roof
- August 5, 2022 (93F day)
- About 3kW (worst case)



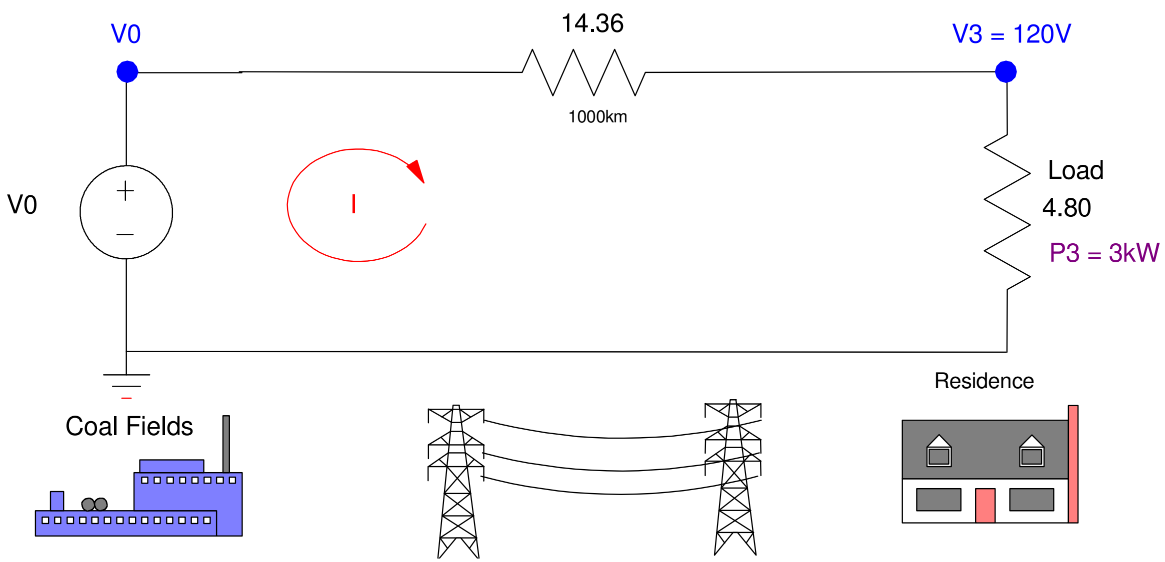
Option 1: DC Power Grid

Thomas Edison's vision

- Produce and transmit DC power
- Place dynamos (DC generator) every 2-3 miles

What happens if you try to transmit power 1000km?

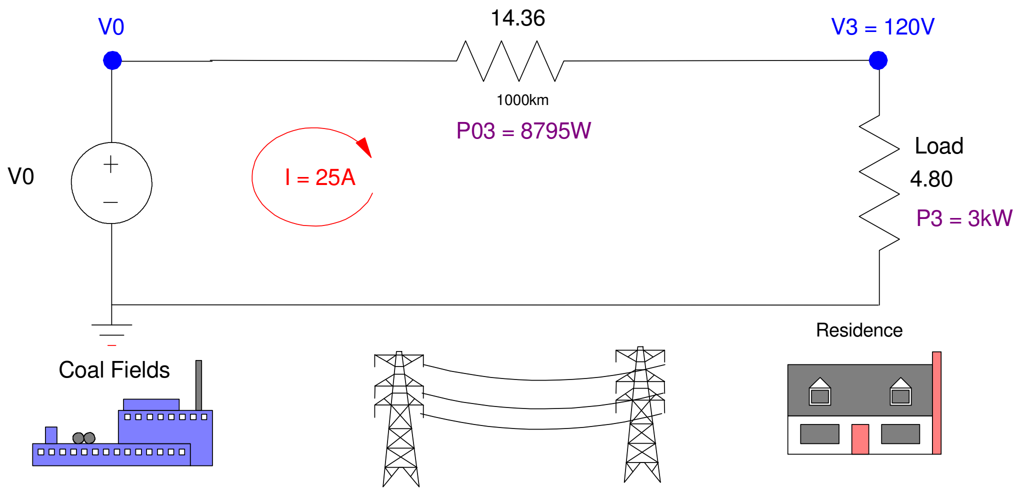
- 1000km of 1cm dia aluminum wire = 14.36 Ohms



DC Power Grid Analysis: 1 Customer

Each customer = 3kW

$$I = \frac{3kW}{120V} = 25A$$



Transmission Line Losses:

$$P_{03} = I^2R = (25A)^2 \cdot 14.36\Omega$$

$$P_{03} = 8795W$$

Efficiency

$$\eta = \frac{\text{power to load}}{\text{total power}} = \frac{3kW}{3kW+8795W} = 25.4\%$$

DC Power Grid Analysis: N Customers

- Power to load increases by N
- Line losses increase by N^2

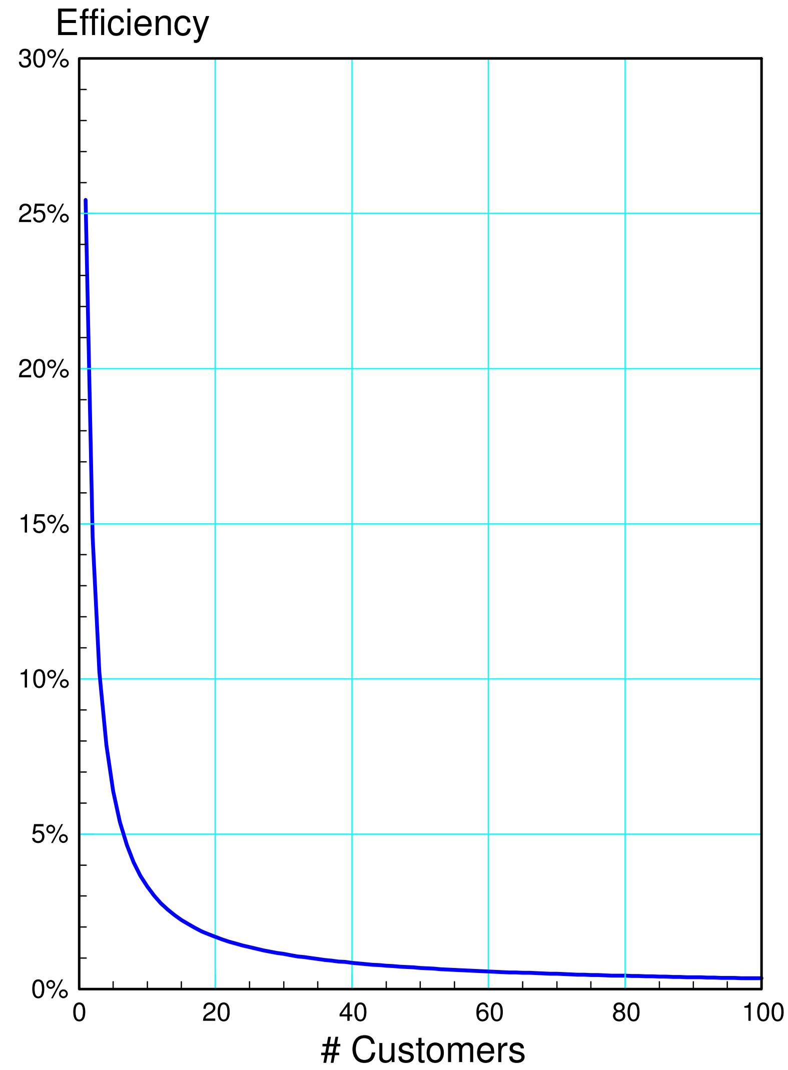
Efficiency

$$\eta = \left(\frac{3kW \cdot N}{3kW \cdot N + 8795 \cdot N^2}\right) \cdot \left(\frac{1/N}{1/N}\right)$$

$$\eta = \left(\frac{3kW}{3kW + 8795 \cdot N}\right)$$

The efficiency gets worse as you add customers

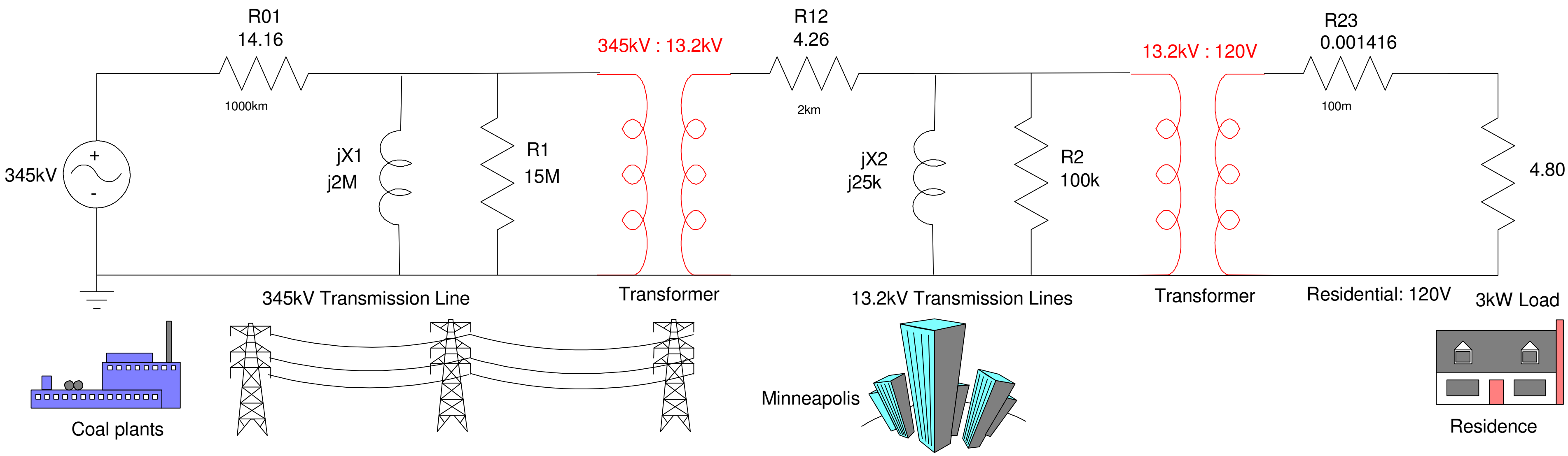
A 120V DC power grid would grid required generators every 2-3 miles



Option 2: AC Power Grid

Tesla's vision

- Bump up the voltage to 345kV at the coal plant
- Step down to 13.2kV at a substation near the city, and then
- Stepped down to 120V close to the residence:

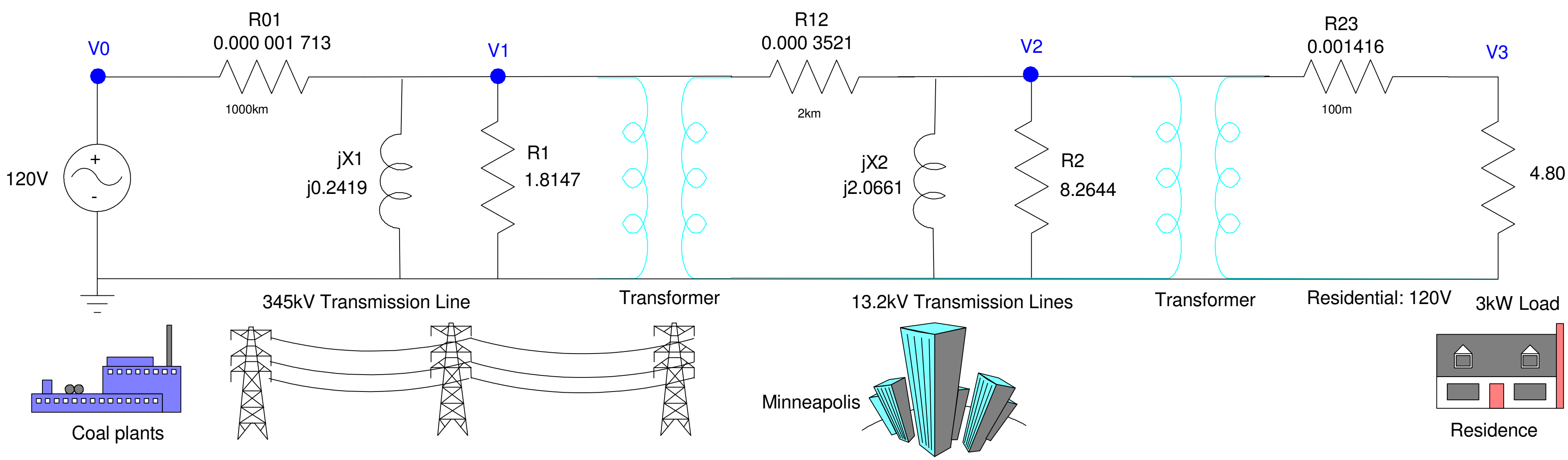


Analysis of AC Power Grid

Step 1: Remove the transformers

$$V_2 = \left(\frac{N_2}{N_1}\right) V_1$$

$$Z_2 = \left(\frac{N_2}{N_1}\right)^2 Z_1$$

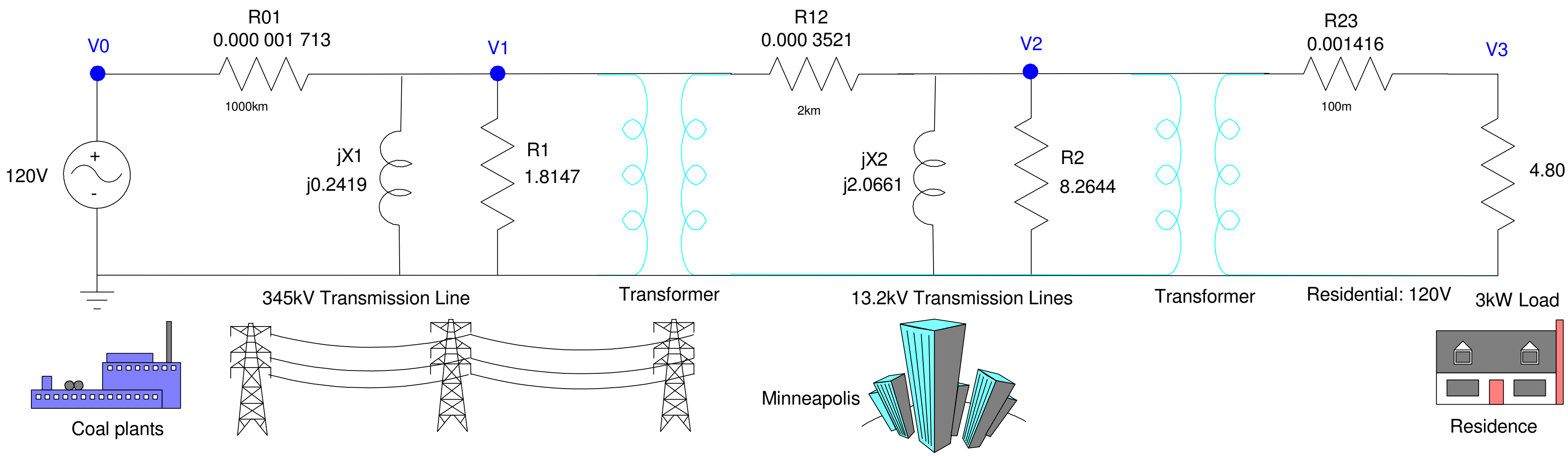


Step 2: Write the voltage node equations:

$$\left(\frac{V_1 - V_0}{R_{01}}\right) + \left(\frac{V_1}{jX_1}\right) + \left(\frac{V_1}{R_1}\right) + \left(\frac{V_1 - V_2}{R_{12}}\right) = 0$$

$$\left(\frac{V_2 - V_1}{R_{12}}\right) + \left(\frac{V_2}{jX_2}\right) + \left(\frac{V_2}{R_2}\right) + \left(\frac{V_2 - V_3}{R_{23}}\right) = 0$$

$$\left(\frac{V_3 - V_2}{R_{23}}\right) + \left(\frac{V_3}{R_3}\right) = 0$$



Step 3: Place in matrix form and solve in Matlab.

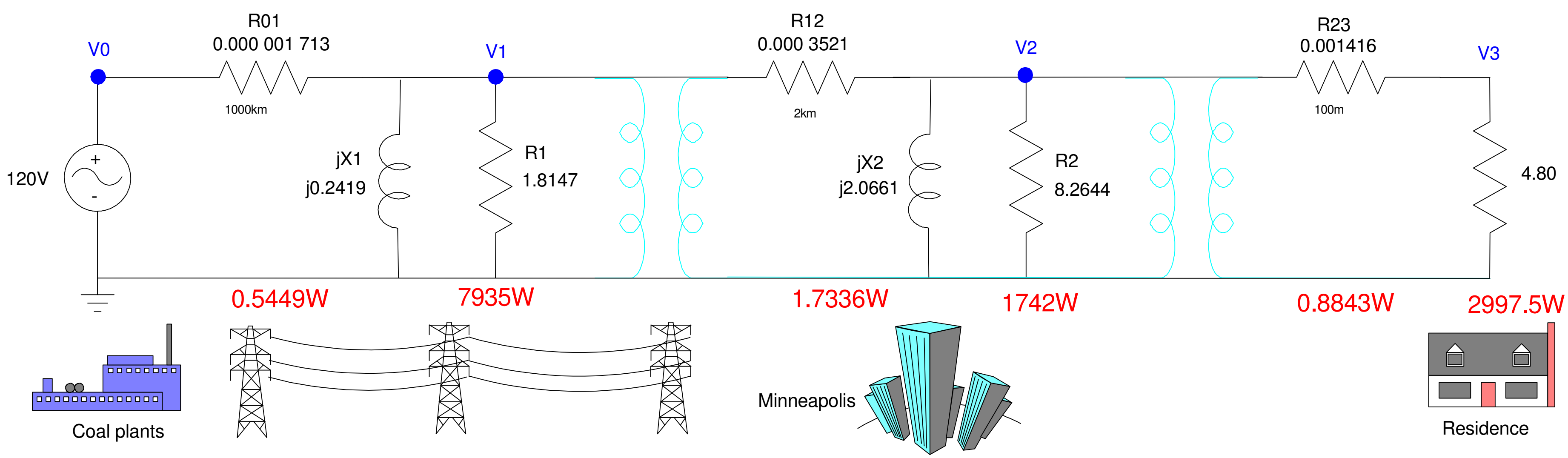
```
>> a1 = [1,0,0,0];
>> a2 = [-1/R01, 1/R01+1/R1+1/(j*X1)+1/R12, -1/R12, 0];
>> a3 = [0,-1/R12, 1/R12+1/R2+1/(j*X2)+1/R23,-1/R23];
>> a4 = [0,0,-1/R23,1/R23+1/R3];
>> A = [a1;a2;a3;a4]
>> B = [120;0;0;0]
>> V = inv(A)*B

V0  120.000
V1  119.999 + 0.0094i
V2  119.985 + 0.2139i
V3  119.950 + 0.2138i
```

Note: The voltage at the customer has drooped

- On hot days with high demand, the last customer's voltage can drop down to 110V
- The utility can adjust the voltage output from the residential transformer to compensate

Step 4: Calculate the power dissipated in each resistor

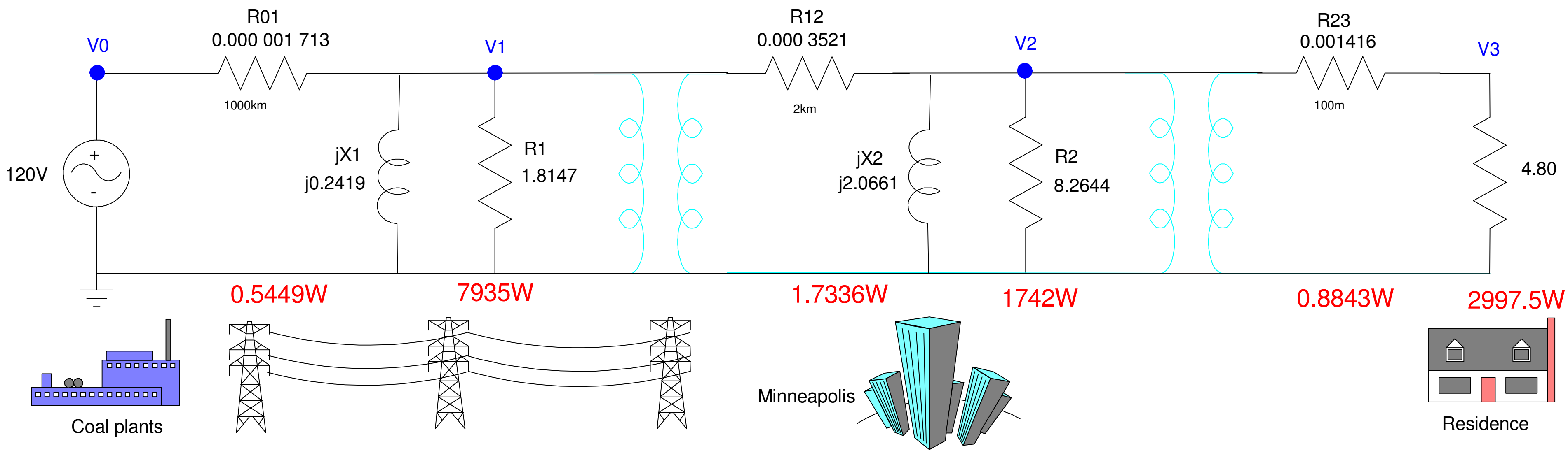
```
>> V0 = V(1);
>> V1 = V(2);
>> V2 = V(3);
>> V3 = V(4);
>> P01 = abs(V0 - V1)^2 / R01
>> P1 = abs(V1)^2 / R1
>> P12 = abs(V1 - V2)^2 / R12
>> P2 = abs(V2)^2 / R2
>> P23 = abs(V2 - V3)^2 / R23
>> P3 = abs(V3)^2 / R3
```



Step 5: Efficiency of AC system with one customer

- Include the core losses
- Efficiency = 23.64%

```
>> eff = P3 / (P01 + P1 + P12 + P2 + P23 + P3)
    eff = 0.2364
```



Efficiency of AC System with N customers

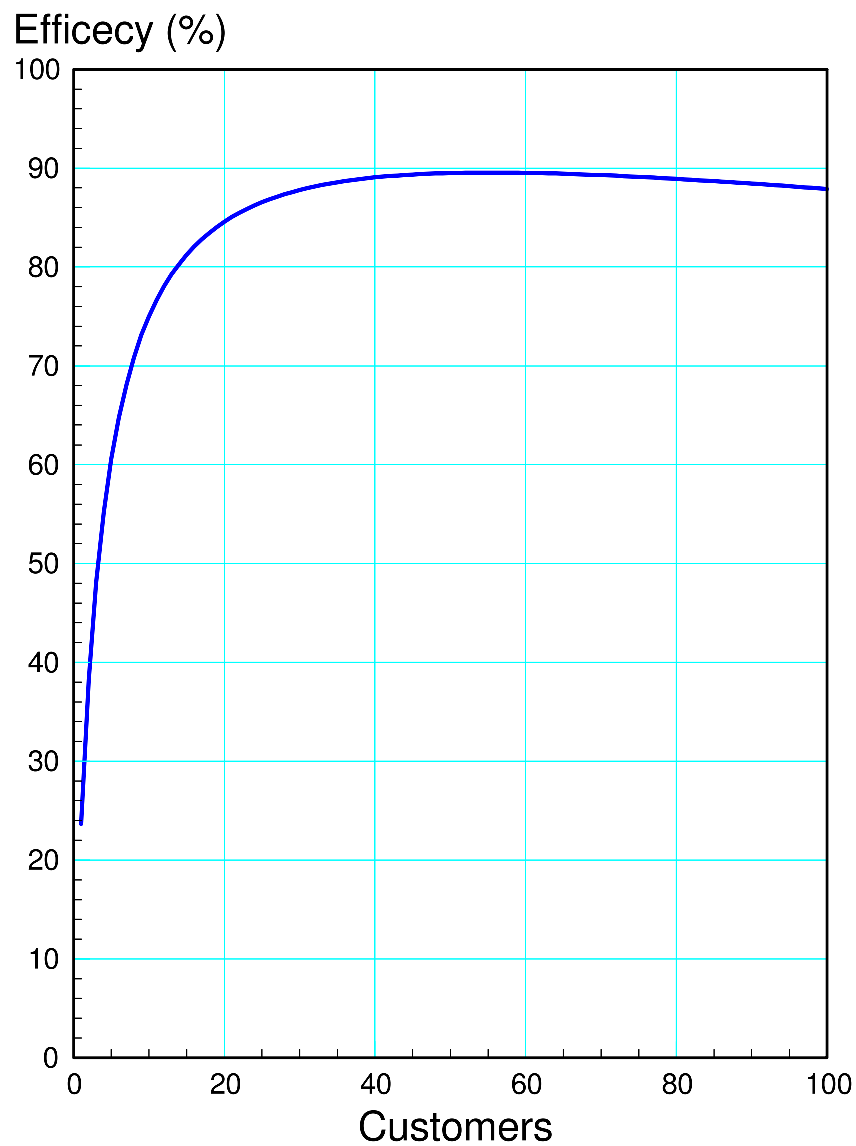
- Per-customer basis
- Line losses scale by N (I^2R)
- Core losses scale by 1/N

```
eff = zeros(100,1);
for N=1:100
  eff(N) = P3 /
     (P3+(P01+P12+P23)*N+(P1+P2)/N);
  end
plot(eff)
```

Efficiency approaches 90%

Efficiency drops off slightly with demand

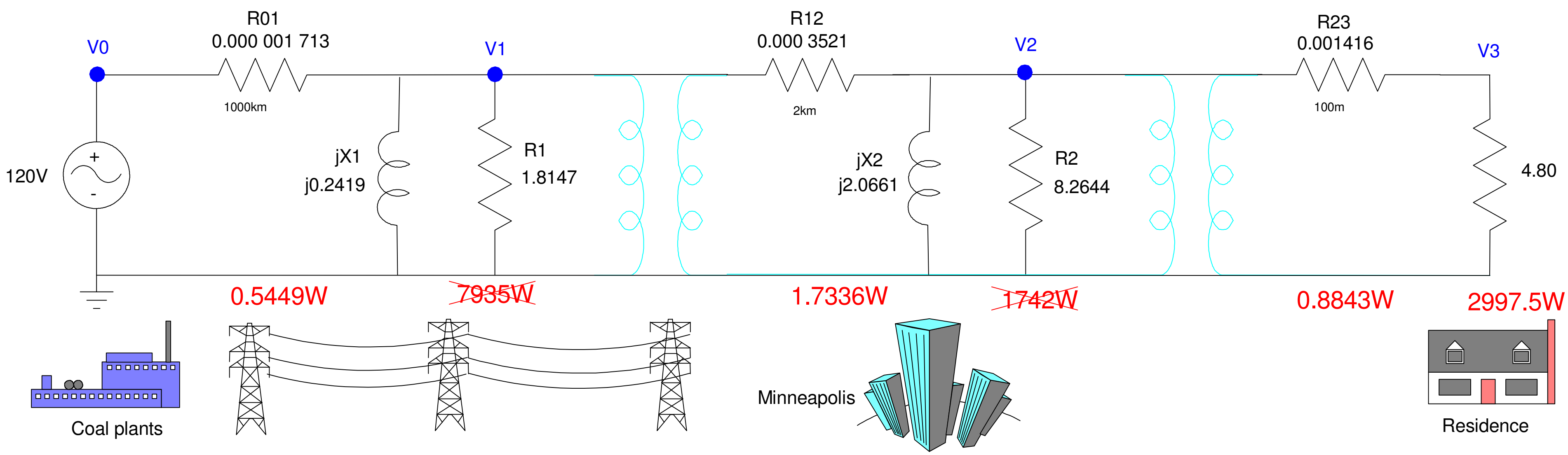
- Hot summer days
- Cold winter days
- 6pm (peak demand)



Efficiency of AC system with many customers

- Approximate
- Ignore core losses (they are shared by a large number of customers)

```
>> eff = P3 / (P01 + P12 + P23 + P3)
    eff = 0.9989
```



Conclusion

Our power grid runs at 60Hz (AC)

- DC power grid (Edison's plan) limits transmission lines to about 2 miles
- AC power grids (Tesla's plan) allows longer transmission lines

The power grid is actually really efficient.

- 80% to 90% efficient under normal operating conditions

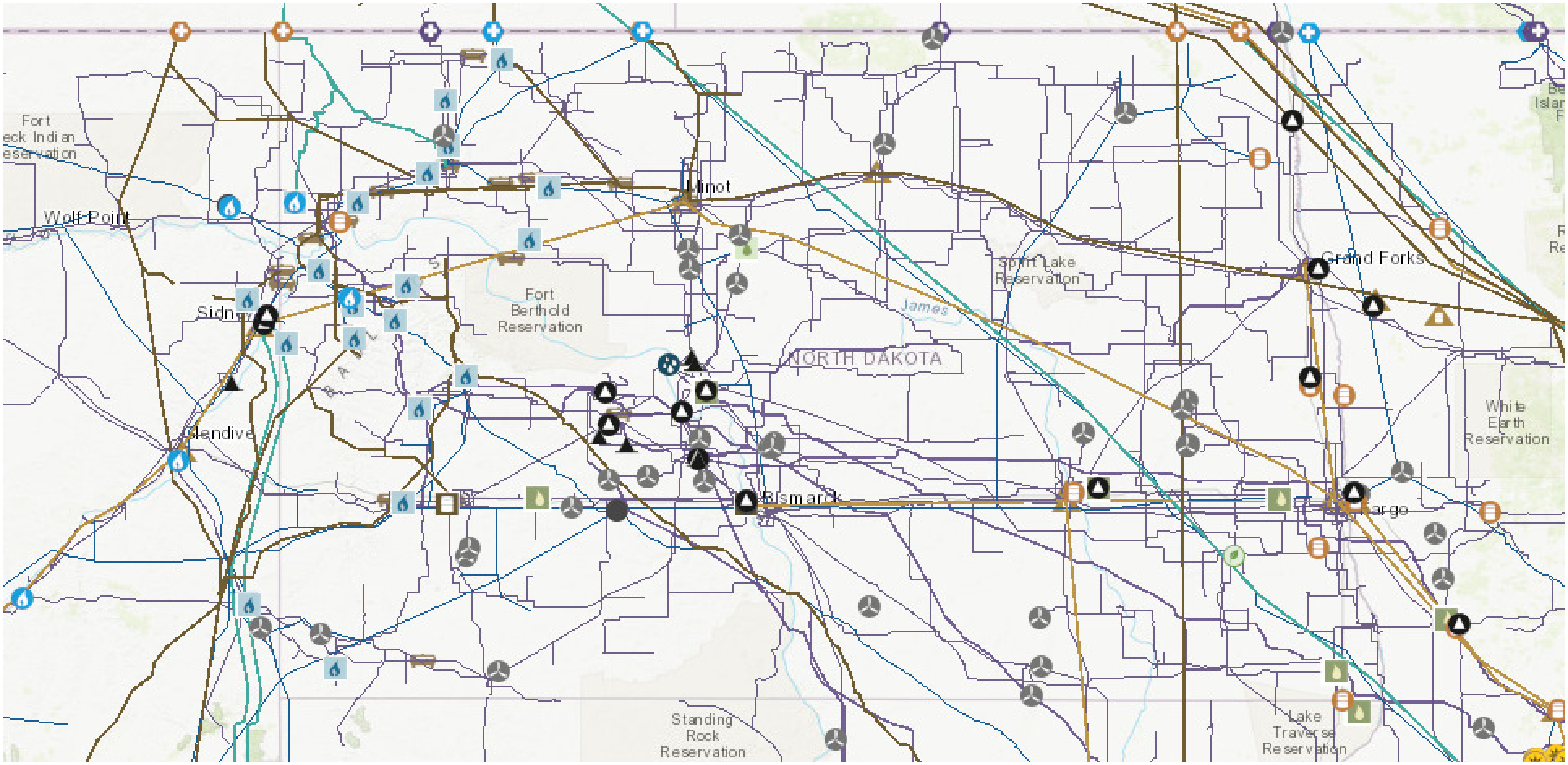
It is impractical for a utility to run a power line to a single customer.

- It's more efficient to have that customer run a diesel generator for really remote locations.

It costs the utility money even if the customers are not using any power.

- It takes energy (and money) to keep the grid powered up.

Note #1: This is why we have high-voltage transmission lines



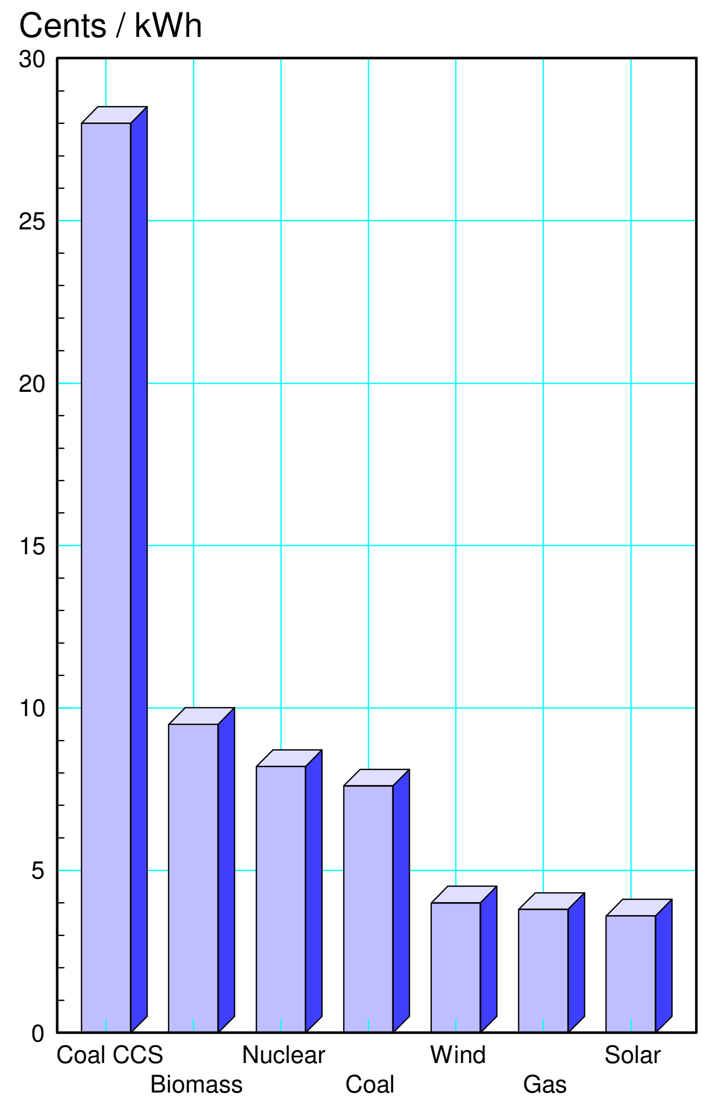
Power plants and transmission lines (345kV or more) in North Dakota. Source: http://www.eia.gov/state/?sid=ND

Note #2: Solar and Wind Energy are Inexpensive

- It's hard to compete with an energy source where the fuel cost is $0
- This is why so many solar and wind farms are being built in North Dakota, South Dakota, Minnesota, etc.

- Solar[1] 3.6 cents / kWh
- Gas[1] 3.8 cents / kWh
- Wind[1] 4.0 cents / kWh
- Coal/Steam[1] 7.6 cents / kWh
- Nuclear[1] 8.2 cents / kWh
- Biomass[1] 9.5 cents / kWh
- Coal w/ CCS[2] 25 - 32 cents / kWh

(1) Bank of America, 2023
(2) US Energy and Information Administration



Note #3: Wind Energy Benefits Farmers

NDSU study reported in www.Sciencing.com

Wind Rights:

- $2,000 to $10,000 / acre
- Paid to the farmer
- 20-year lease typical

Production Royalties

- $4,000 to $6,000 per MW per year

Example: 2MW wind turbine

- $100,000 up front (wind rights)
- $10,000/year (production royalties)
- Plus, you can keep using the land

photo: https://thumbs.dreamstime.com/b/cow-wind-turbine-12454919.jpg

Note #4: North Dakota is windy

- North Dakota has enough wind to produce 1.182TWh of electricity each year
  `National Renewable Energy Administration`
  `https://www.nrel.gov/docs/fy00osti/28054.pdf`
- Enough to supply 25% of the nation's electricity needs
- Enough to produce $118 billion in revenue each year
  at $0.10 / kWh
- Enough to double the state's GDP
  `http://www.deptofnumbers.com/gdp/north-dakota/`

To do this, we'll need more high-voltage transmission lines to get this energy out of the state