Current Loops (Current Mesh)

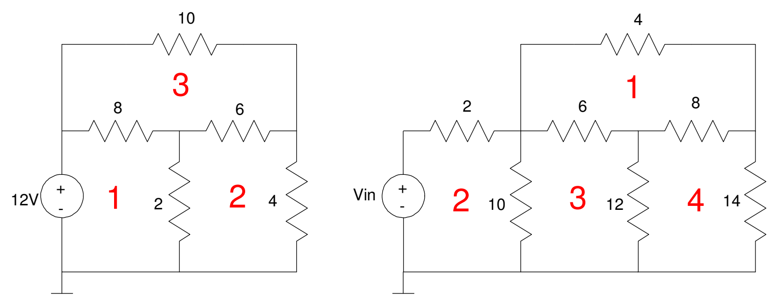
Kirchoff's Current Loops (KCL)

A second way to write N equations to solve for N unknowns is to write current mesh equations. This uses the concept that the sum of the voltages around a closed loop must equal zero.

Counting the Number of Current Loops

The first step in writing current loop equations is determining how many loop equations you need. One way to think of this is to draw the circuit on paper without any criss-crossing lines and count the number of 'windows' in the circuit.

For example, the following circuits have three and four 'windows'. Hence, you need to write three equations to solve for three unknowns.

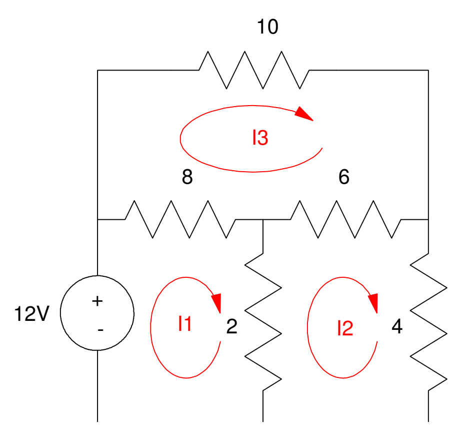


A circuit with 3 windows (left) and 4 windows (right).

Note that for some topologies, you can't draw the circuit on a flat piece of paper without crossing wires (for example, a resistor cube with one set of diagonals connected with a 13th resistor.) In these cases, you probably need to use voltage node techniques.

The number of 'windows' tell you how many current loop equations you need to write.

Current Loop Equations for Resistive Circuits:



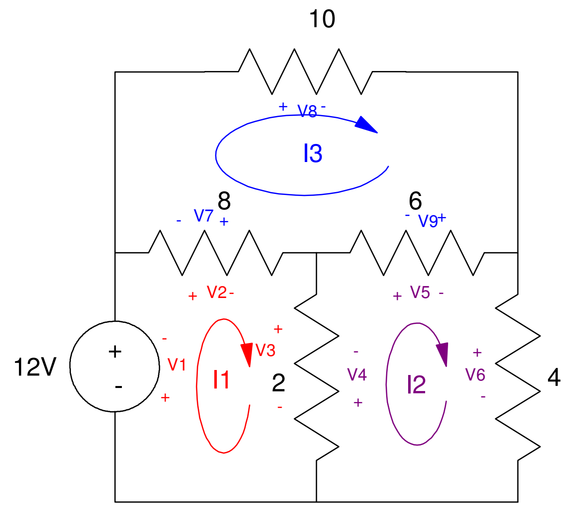
Like voltage nodes, if you have a circuit composed of resistors, the method to write N equations to solve for N unknowns is fairly straight forward.

Step 1: Define N current loops. Typically, count the number of 'windows' in the circuit and define that many current loops. Direction doesn't matter, but it is usually less confusing if you make all your current loops go the same direction (clockwise in this example).

Step 2: Go around each loop and sum the voltages as V = IR. The total must be zero. Note that

- If you encounter the - sign first as you go around the loop, subtract.
- If you encounter the + sign first, add.

Likewise, for loop I1, the 12V source is subtracted since you hit its minus sign first



Going around loop I1, you get

I1: V1 + V2 + V3 = 0

$$-12+(I_1-I_3)8+(I_1-I_2)2=0$$

I2: V4 + V5 + V6 = 0

$$(I_2-I_1)2+(I_2-I_3)6+(I_2)4=0$$

I3: V7 + V8 + V9 = 0

$$(I_3-I_1)8+(I_3)10+(I_3-I_2)6=0$$

Note:

- The sign of Ix is positive when you go around loop x. This is one way to check your answer for sign errors.
- There is a sign flip in equation I1 and I2 for V3 and V4. This is due to the direction that we travel across the 2 Ohm resistor. It's nothing to worry about.

Once you have N equations for N unknowns, you can solve. Using MATLAB for the 3x3 circuit, first place is these equations in matrix form:

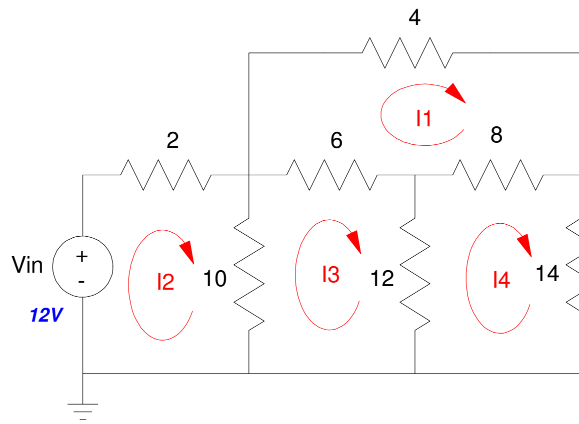
Solving in MATLAB:

```
-->A = [10,-2,-8;-2,12,-6;-8,-6,24]
 A  =

    10.  - 2.   - 8.
  - 2.     12.  - 6.
  - 8.   - 6.     24.


-->B = [12;0;0]
 B  =

    12.
    0.
    0.

-->inv(A)*B
 ans  =

    2.0655738
    0.7868852
    0.8852459
```

meaning

- I1 = 2.06A
- I2 = 0.786A
- I3 = 0.885A

Example 2: Find the currents for the 4-window circuit.
Assume Vin = +12V.



First, write 4 equations for 4 unknowns:

I1: $4I_1 + 8(I_1 - I_4) + 6(I_1 - I_3) = 0$

I2: $-12 + 2I_2 + 10(I_2 - I_3) = 0$

I3: $10(I_3 - I_2) + 6(I_3 - I_1) + 12(I_3 - I_4) = 0$

I4: $12(I_4 - I_3) + 8(I_4 - I_1) + 14(I_4) = 0$

Next, group terms:

I1: $18I_1 - 6I_3 - 8I_4 = 0$

I2: $12I_2 - 10I_3 = 12$

I3: $-6I_1 - 10I_2 + 28I_3 - 12I_4 = 0$

I4: $-8I_1 - 12I_3 + 34I_4 = 0$

Place in matrix form:

$$\begin{bmatrix} 18 & 0 & -6 & -8 \\ 0 & 12 & -10 & 0 \\ -6 & -10 & 28 & -12 \\ -8 & 0 & -12 & 34 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ 0 \\ 0 \end{bmatrix}$$

Solving in MATLAB:

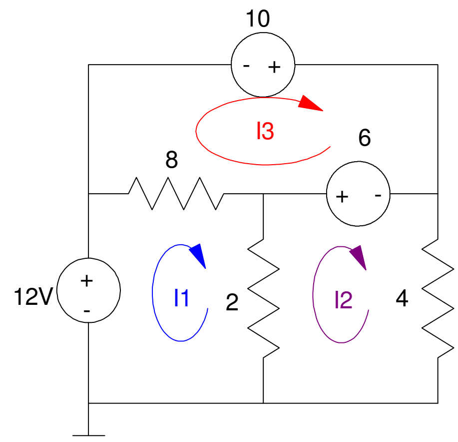
```
-->A = [18,0,-6,-8; 0,12,-10,0; -6,-10,28,-12; -8,0,-12,34]
    18.     0.    - 6.    - 8.
    0.      12.   - 10.     0.
  - 6.    - 10.     28.   - 12.
  - 8.      0.    - 12.     34.

-->B = [0;12;0;0]
    0.
    12.
    0.
    0.

-->inv(A)*B
I1:    0.5164104
I2:    1.7860913
I3:    0.9433096
I4:    0.4544411
```

These are the four values of I1, I2, I3, and I4.

Current Loops with Voltage Sources

Voltage sources don't cause any problems with writing the current loops equations. Instead of using V = IR, just use the voltage.

- If you encounter the + sign first, add the voltage.
- If you encounter the - sign first, subtract the voltage.



Example: Change some of the resistors to voltage sources. The loop equations become:

I1: $-12+8(I_1-I_3)+2(I_1-I_2)=0$

I2: $2(I_2-I_1)+6+4(I_2)=0$

I3: $-10+6+8(I_3-I_1)=0$

Note that as you go around loop I3

- You encounter the - sign on the 10V source first, so you subtract 10.
- You encounter the + sign on the 6V source first, so you add 6.