Thevenin Equivalents with Phasors

EE 206 Circuits I

Jake Glower

03/19/20

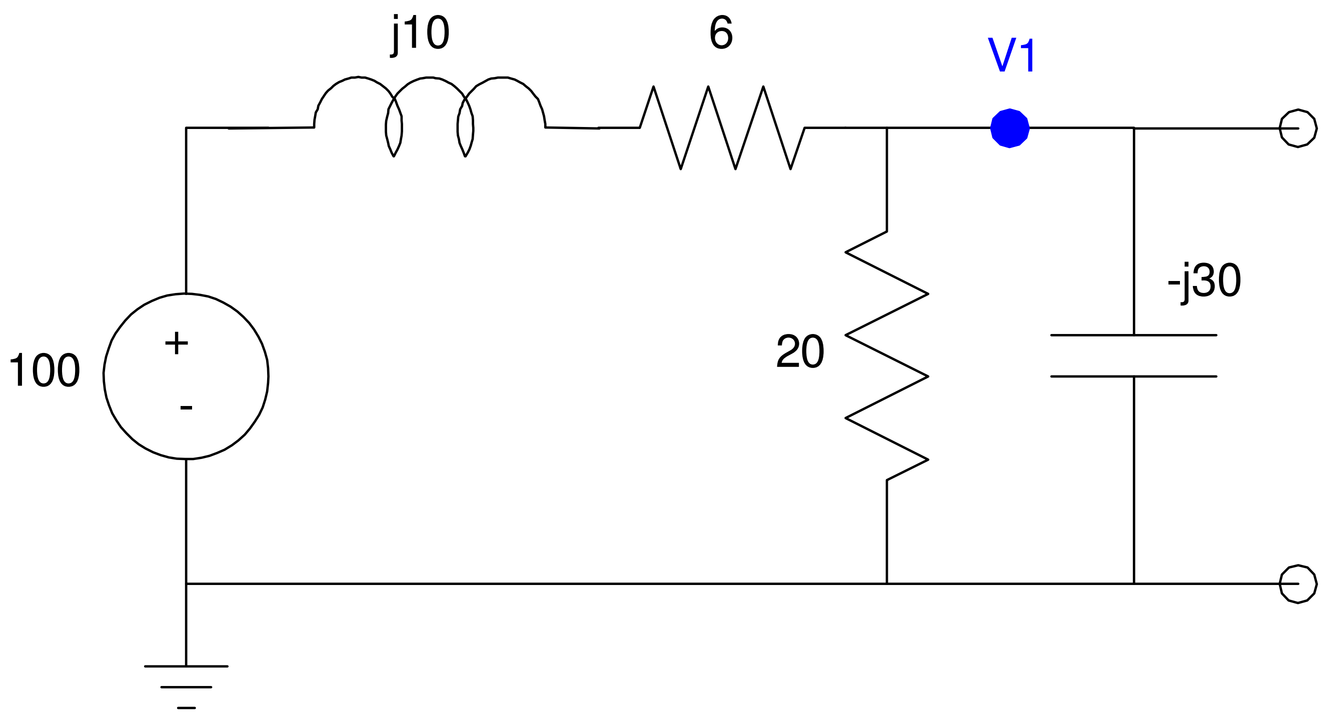
Please visit Bison Academy for corresponding lecture notes, homework sets, and solutions

Thevenin equivalents also work with phasors - only you get complex numbers for the Thevenin voltage and Thevenin resistance.

	VI relationship	Phasor Notation
Voltage	$v(t) = a\cos(\omega t) + b\sin(\omega t)$	$V = a - jb$
Resistor	$v = iR$	$Z_R = R$
Inductor	$v = L\frac{di}{dt}$	$Z_L = j\omega L$
Capacitor	$i = C\frac{dv}{dt}$	$Z_C = \frac{1}{j\omega C}$

Example 1: Determine

- The Thevenin equivalent for the following circuit,
- ZL for max power transfer, and
- The maximum power to a load



Solution: Combine the 20 Ohms and -j30 Ohms in parallel:

$$20 || -j30 = (13.846 - j9.231)\Omega$$

The Thevenin voltage by voltage division is

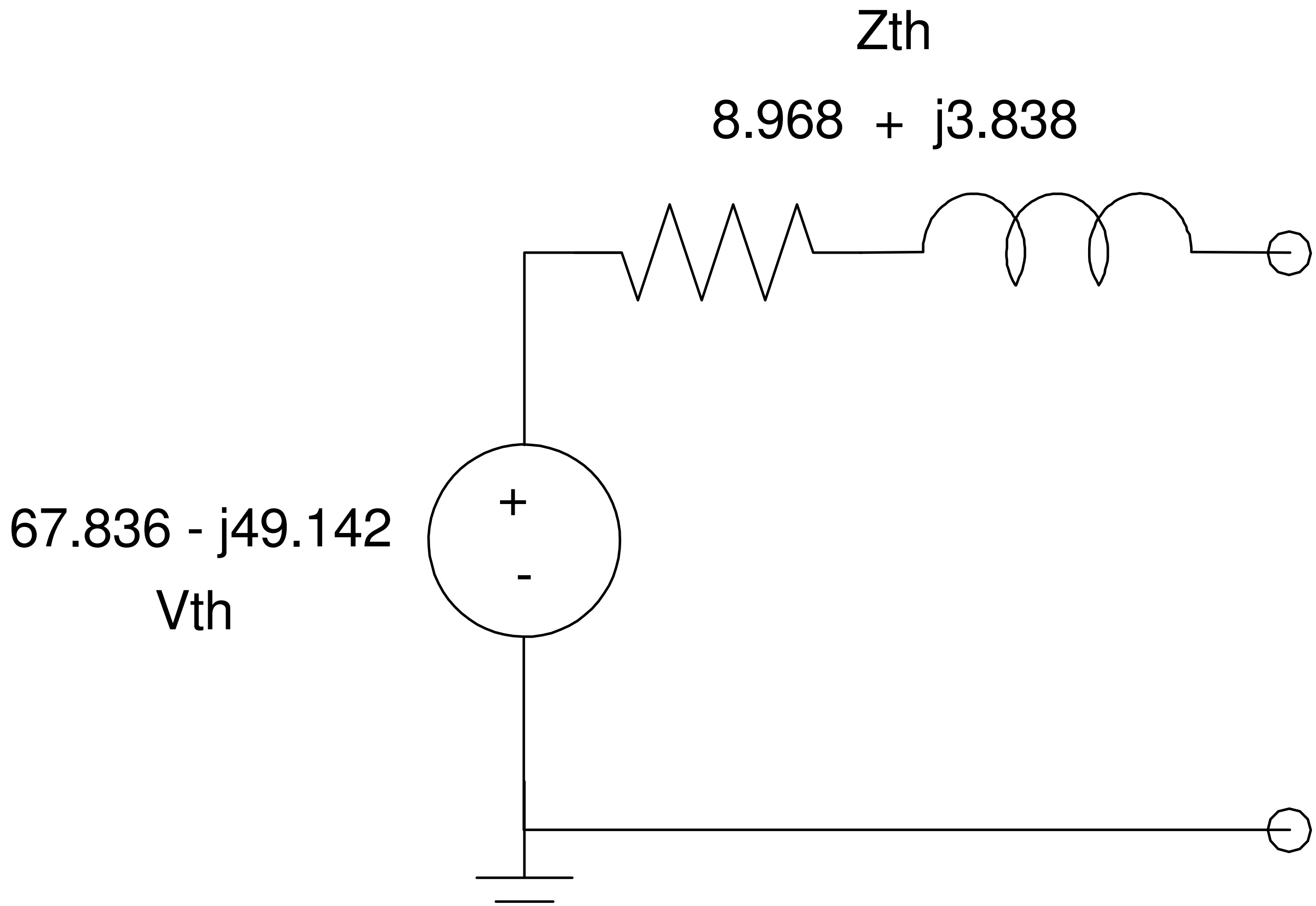
$$V_{th} = \left(\frac{(13.846 - j9.231)}{(13.846 - j9.231) + (6 + j10)}\right) 100 = 67.836 - j49.142$$

The Thevenin resistance is (turn off the voltage source and measure the resistance looking in:

$$Z_{th} = (-j30) || (20) || (6 + j10)$$

$$Z_{th} = 8.968 + j3.838$$

So the Thevenin equivalent is



AC Power

At DC, power is

$$P = VI = \frac{V^2}{R} = I^2 R$$

For AC

rms units

$$P = V_{rms} \cdot I^*_{rms}$$
$$= \left|I_{rms}\right|^2 \cdot Z$$
$$= \frac{\left|V_{rms}\right|^2}{Z^*}$$

peak units

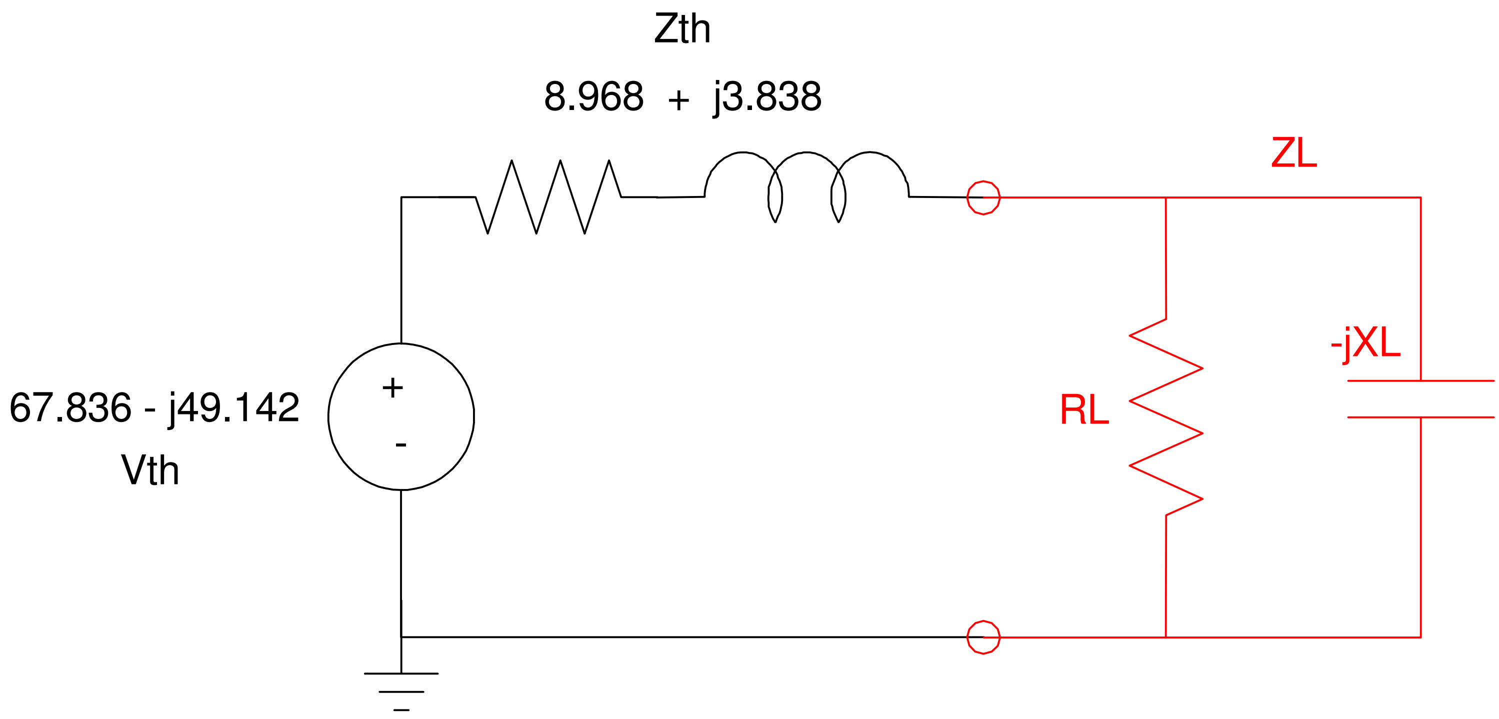
$$P = \frac{1}{2} V_p \, I^*_p$$
$$= \frac{1}{2} \left|I_p\right|^2 Z$$
$$= \frac{1}{2} \frac{\left|V_p\right|^2}{Z^*}$$

- The real part of P is the work done (or heat produced),
- The complex part of P is the energy that bounces back and forth.

Maximum Power to the Load

Maximum power to the load is when the load is the complex conjugate of Zth:

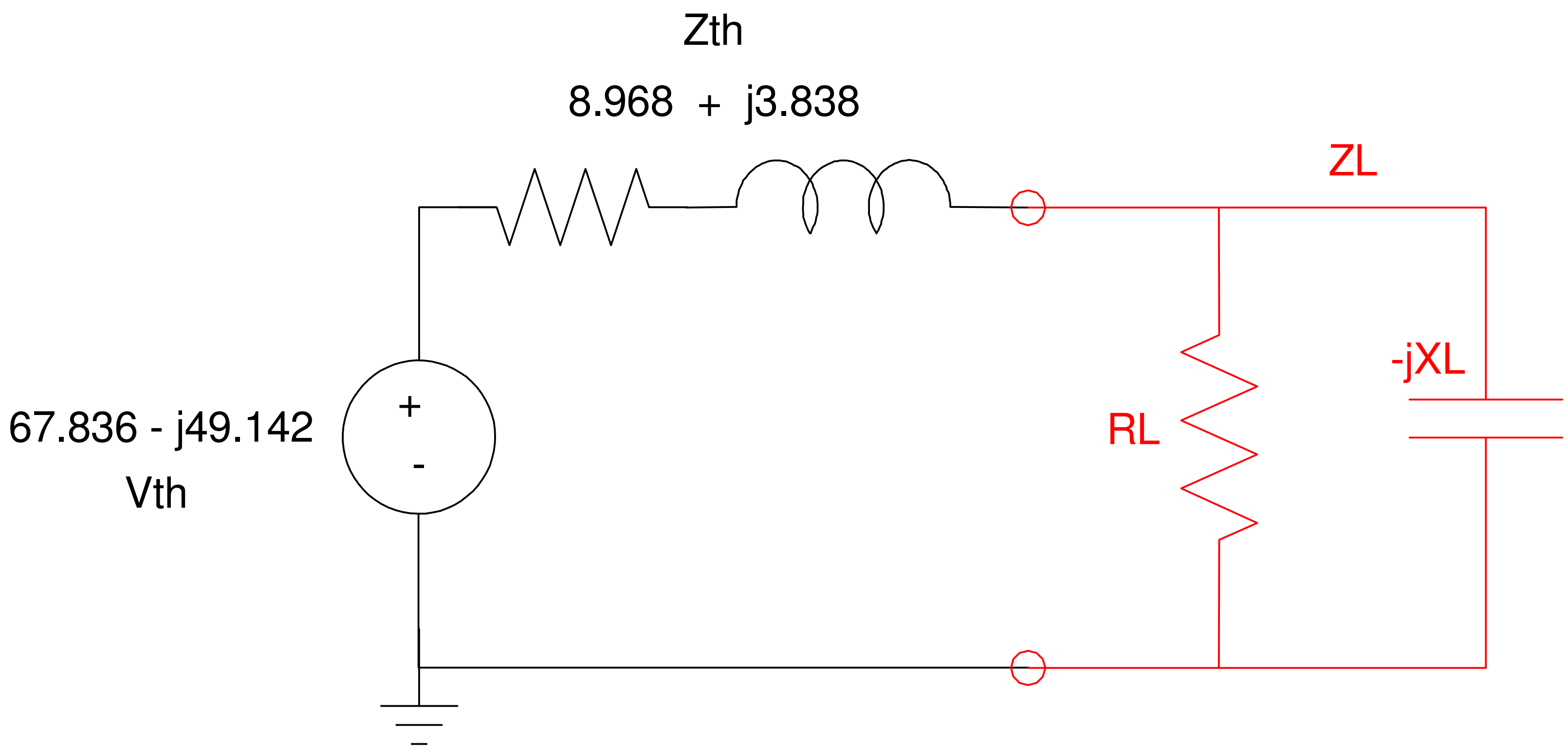
$$Z_L = Z_{th}^*$$



Example:

Determine

- The load, ZL, which maximimizes the power to the load, and
- The power to the load (real and complex power)



Solution: The load should be the complex conjugate of Zth

$$Z_L = (8.968 + j3.838)^*$$

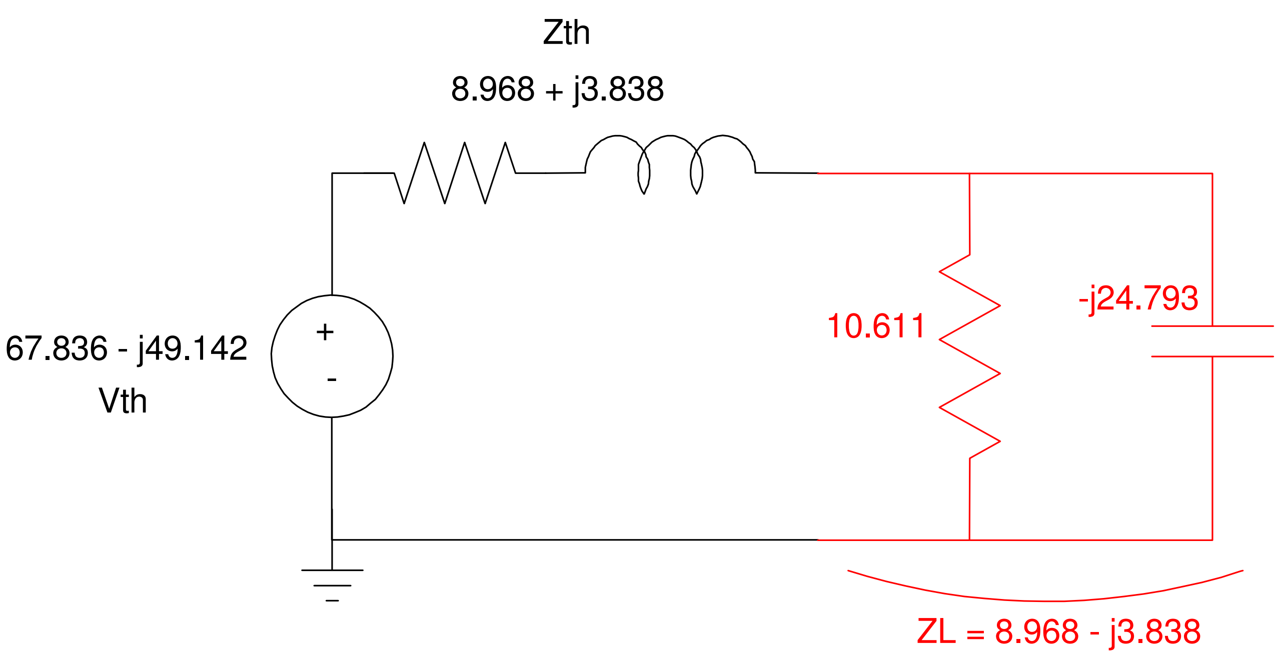
$$Z_L = 8.968 - j3.838$$

To find RL and jXL, add the inverses (since they are in parallel):

$$\frac{1}{Z_L} = \frac{1}{R_L} + \frac{1}{-jX_L}$$

$$R_L = 10.611\Omega$$

$$-jX_L = -j24.793\Omega$$



The power to the load is then

$$V_L = \left(\frac{(8.968 - j3.838)}{(8.969 - j3.838) + (8.968 + j3.838)} \right) \cdot (67.836 - j49.142)$$

$$V_L = 23.402 - j39.087$$

Assuming units are rms:

$$P = \frac{|V_{rms}|^2}{Z^*} = \frac{|23.402 - j39.087|}{(8.969 - j3.838)^*}$$

$$= \frac{(45.557)^2}{8.969 + j3.838}$$

$$= 4.293 - j1.837 \quad \text{Watts}$$

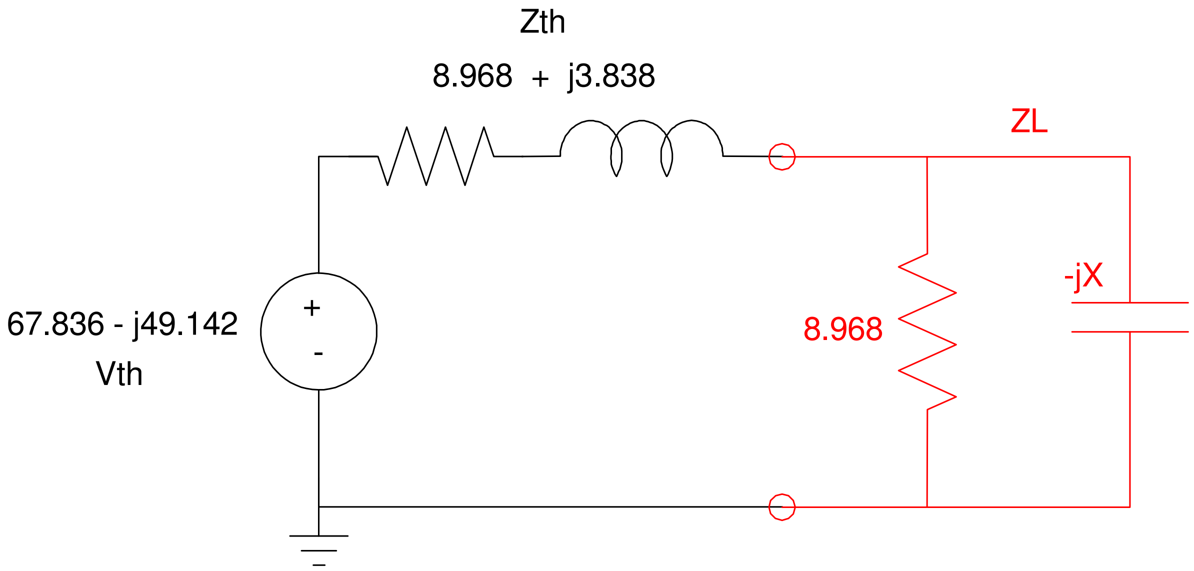
The real part is the power to the load (driving a motor, heating a resistor)

The complex part is power that bounces back and forth

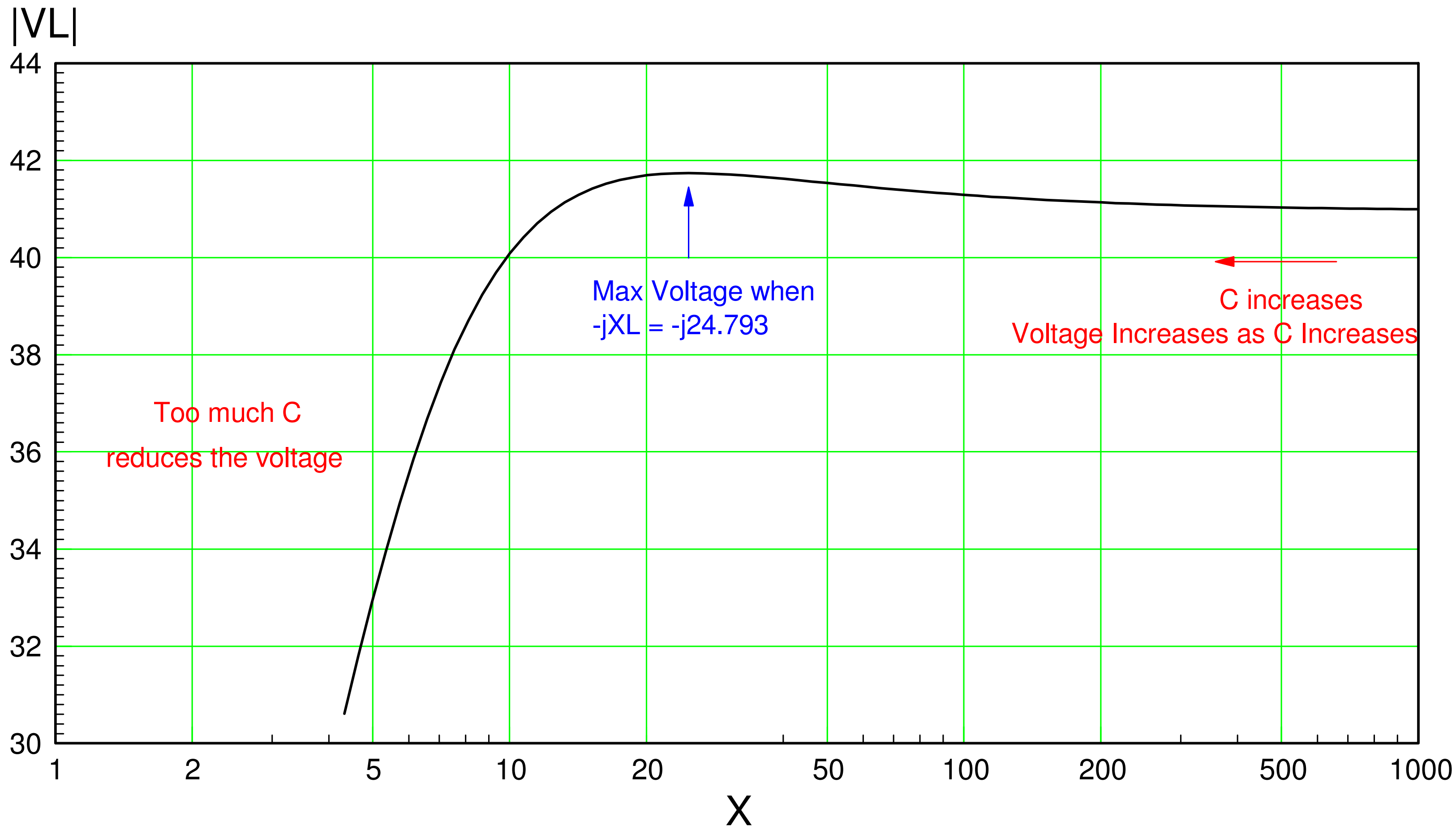
"Capacitors add Voltage"

If Zth is inductove, then adding capacitors to the load,

- Cancels the complex part of Zth, which
- Reduces the overall impedance, which
- Increases the current to the load, which
- Increases the voltage at the load.

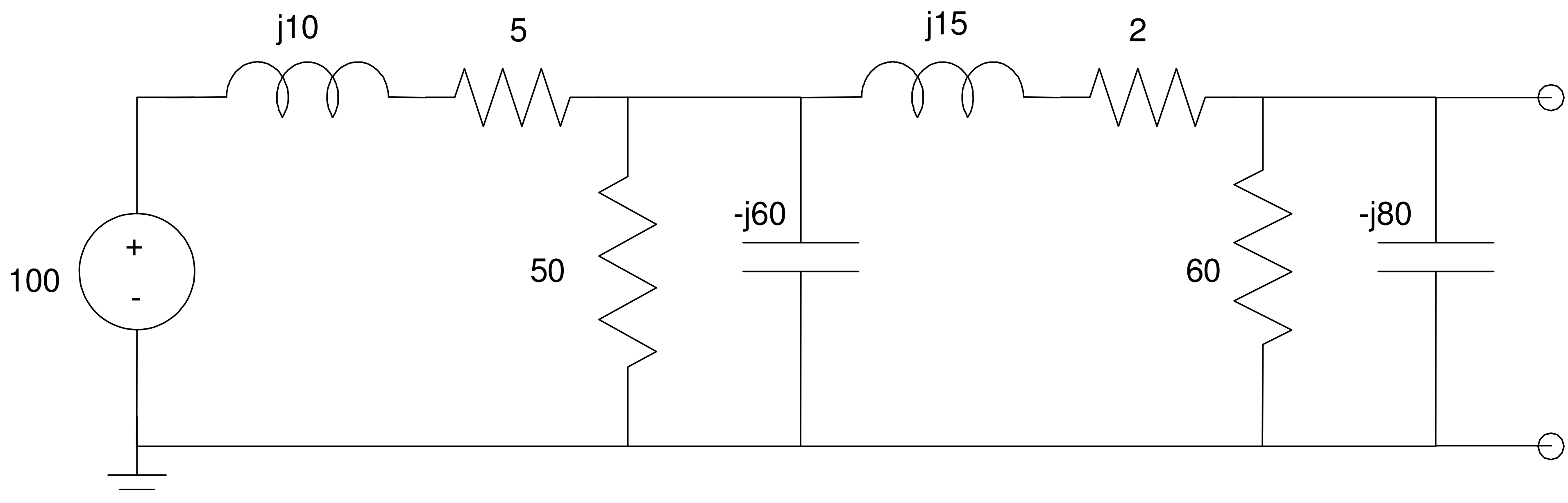


This only works up to a point: once you have cancelled all of the inductance (+jX), adding more capacitors will actually redice the voltage.



Example 3: Source Transformations

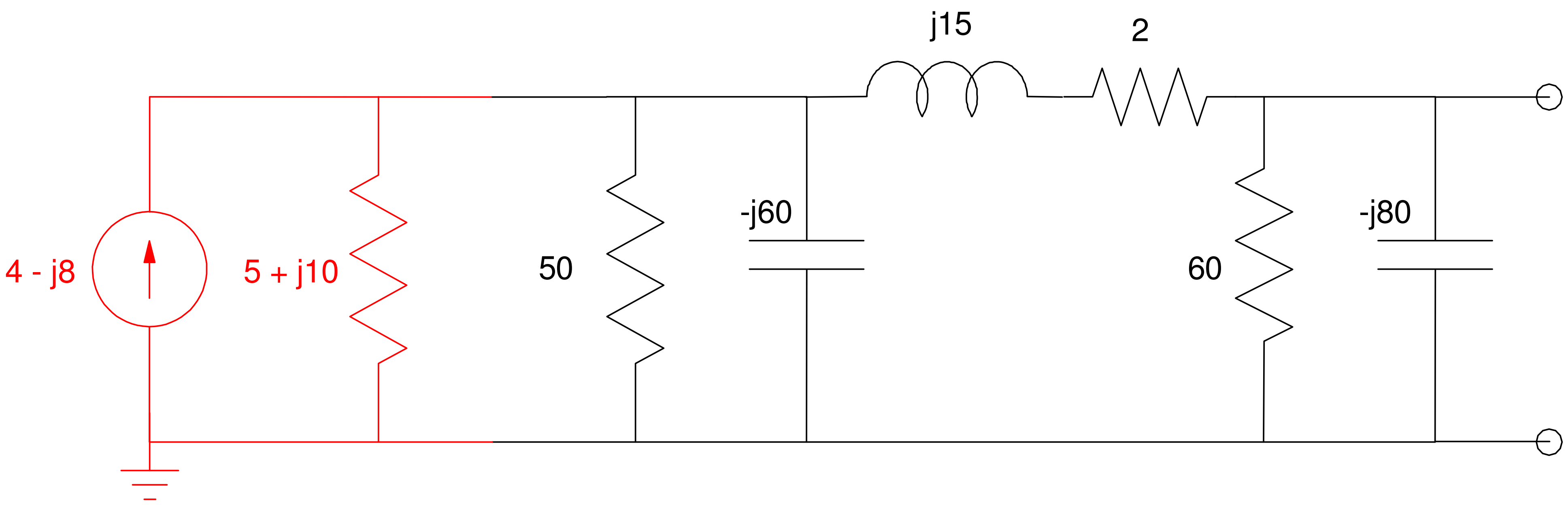
Source transformations also work with complex numbers. For example, determine the Thevenin equivalent for the following circuit:



Step 1: Convert to a Norton equivalent

$$Z_N = Z_{th} = 5 + j10$$

$$I_N = \frac{V_{th}}{Z_{th}} = 4 - j8$$



Combine impedances in parallel

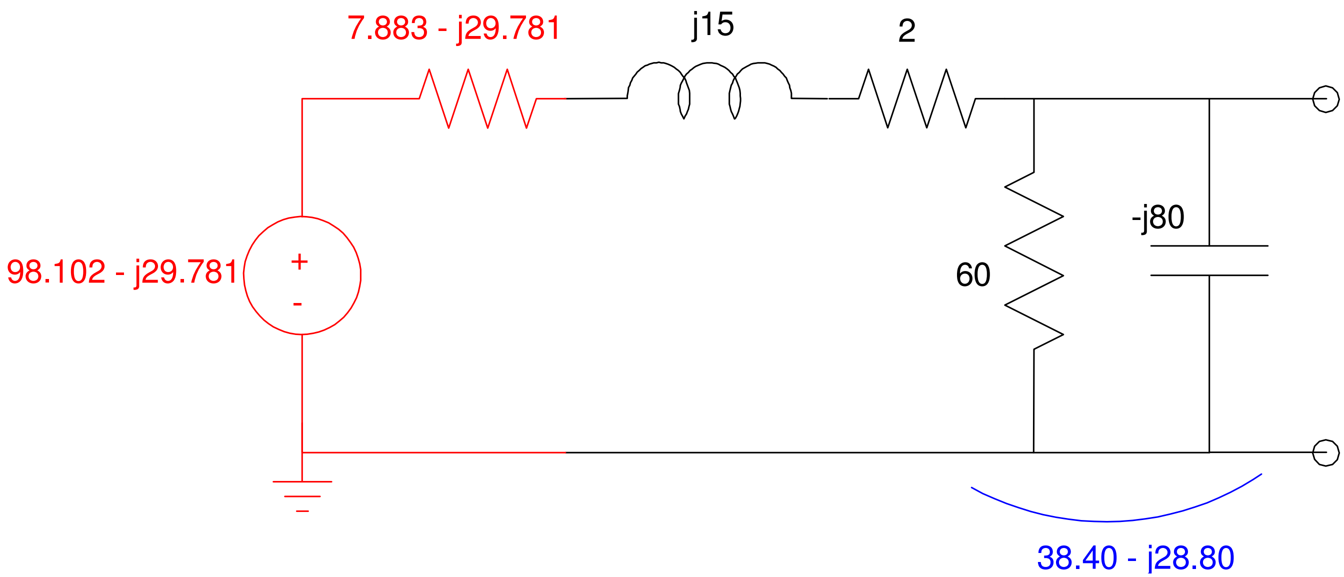
$$(5+j10)||(50)||(-j60) = 7.883 + j8.321$$

Convert to Thevenin

$$Z_{th} = Z_N = 7.883 + j8.321$$

$$V_{th} = I_N \cdot Z_N = (4-j8)\cdot(7.883+j8.321)$$

$$V_{th} = 98.102 - j29.781$$



Now find the Thevenin equivalent.

By voltage division:

$$V_{th} = \left(\frac{(38.40 - j28.80)}{(38.40 - j28.80) + (7.883 - j29.781) + (2 + j15)}\right)(98.102 - j29.781)$$

$$V_{th} = 68.701 - j74.405$$

Zth: Turn off the source and measure the impedance

$$Z_{th} = ((7.83 - j29.781) + (2 + j15)) || (60) || (-j80)$$

$$Z_{th} = 20.077 + j14.931$$