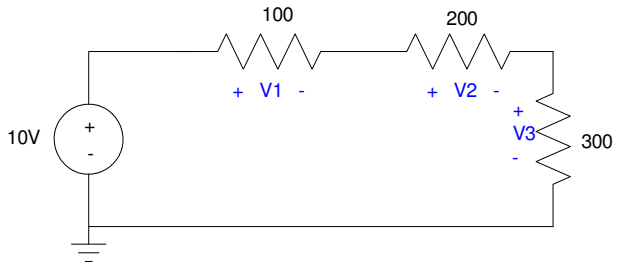
EE 206: Solution #3

Voltage and Current Division. Voltage Nodes. Due Mon, Jan 28th

Voltage Division:

1) Use votlage division to determine the voltages V1, V2, and V3



$$V = \left(\frac{\text{what you measuring}}{\text{total resistance}} \right) V_{in}$$

$$V_1 = \left(\frac{100}{100+200+300} \right) 10V = 1.6667V$$

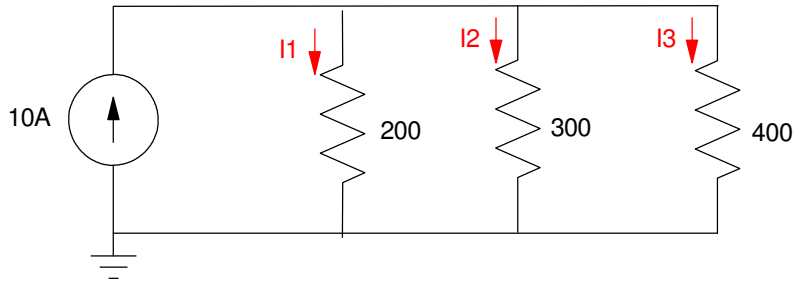
$$V_2 = \left(\frac{200}{100+200+300} \right) 10V = 3.3333V$$

$$V_3 = \left(\frac{300}{100+200+300} \right) 10V = 5.0000V$$

Note that

V1 + V2 + V3 = 10V

2) Use current division to determine the current I1, I2, and I3



$$I_x = \left(\frac{\text{Resistance of everything else}}{\text{Rx + Resistance of Everything Else}} \right) I_{in}$$

$$I_1 = \left(\frac{300||400}{200+300||400} \right) 10A$$

$$I_1 = \left(\frac{171.42}{200+171.42} \right) 10A = 4.6154A$$

$$I_2 = \left(\frac{200||400}{300+200||400} \right) 10A$$

$$I_2 = \left(\frac{133.33}{300+133.33} \right) 10A = 3.0769A$$

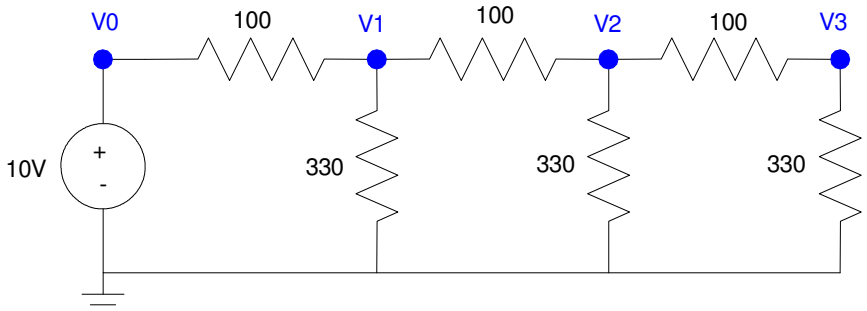
$$I_3 = \left(\frac{200||300}{400+200||300} \right) 10A$$

$$I_3 = \left(\frac{120}{400+120} \right) 10A = 2.3077A$$

Note that I1 + I2 + I3 = 10A

Voltage Nodes:

3) Write the votlage node equations for the following circuit.



You need to write 4 equations for 4 unknowns. The sum of the currents from the nodes must be zero.

$$V_0 = 10$$

$$\left(\frac{V_1 - V_0}{100}\right) + \left(\frac{V_1}{330}\right) + \left(\frac{V_1 - V_2}{100}\right) = 0$$

$$\left(\frac{V_2 - V_1}{100}\right) + \left(\frac{V_2}{330}\right) + \left(\frac{V_2 - V_3}{100}\right) = 0$$

$$\left(\frac{V_3 - V_2}{100}\right) + \left(\frac{V_3}{330}\right) = 0$$

Group terms.

$$V_0 = 10$$

$$-\left(\frac{1}{100}\right)V_0 + \left(\frac{1}{100} + \frac{1}{330} + \frac{1}{100}\right)V_1 - \left(\frac{1}{100}\right)V_2 = 0$$

$$-\left(\frac{1}{100}\right)V_1 + \left(\frac{1}{100} + \frac{1}{330} + \frac{1}{100}\right)V_2 - \left(\frac{1}{100}\right)V_3 = 0$$

$$-\left(\frac{1}{100}\right)V_2 + \left(\frac{1}{100} + \frac{1}{330}\right)V_3 = 0$$

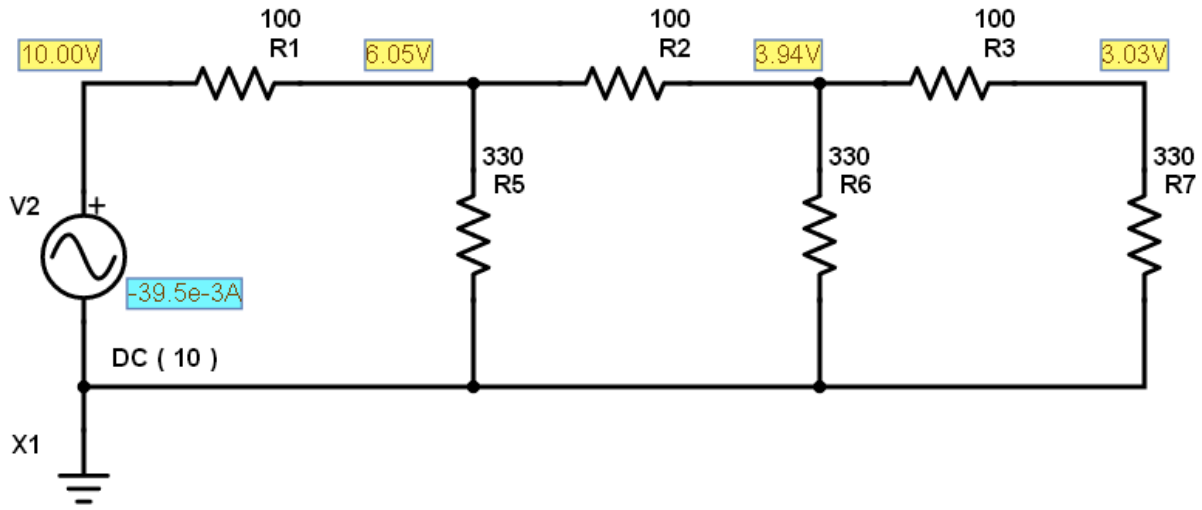
Place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -\left(\frac{1}{100}\right) & \left(\frac{1}{100} + \frac{1}{330} + \frac{1}{100}\right) & -\left(\frac{1}{100}\right) & 0 \\ 0 & -\left(\frac{1}{100}\right) & \left(\frac{1}{100} + \frac{1}{330} + \frac{1}{100}\right) & -\left(\frac{1}{100}\right) \\ 0 & 0 & -\left(\frac{1}{100}\right) & \left(\frac{1}{100} + \frac{1}{330}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

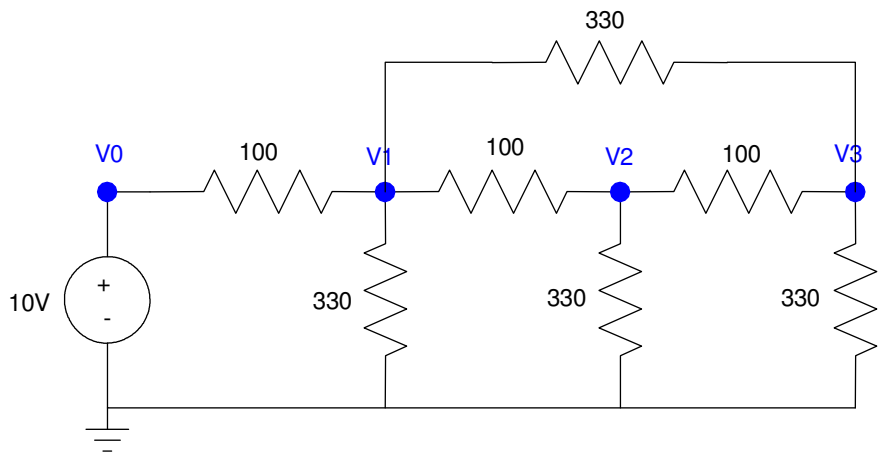
Solve in Matlab

```
a1 = [1,0,0,0];
a2 = [-1/100, 1/100+1/330+1/100, -1/100, 0];
a3 = [0, -1/100, 1/100+1/330+1/100, -1/100];
a4 = [0, 0, -1/100, 1/100+1/330];

A = [a1;a2;a3;a4]

    1.0000         0         0         0
   -0.0100    0.0230   -0.0100         0
         0   -0.0100    0.0230   -0.0100
         0         0   -0.0100    0.0130

B = [10;0;0;0]

    10
     0
     0
     0

V = inv(A)*B

V0   10.0000
V1    6.0539
V2    3.9424
V3    3.0256
```

4) Check your answers in PartSim (or similar program)



5) Write the votlage node equations for the following circuit. Solve for V1 .. V3 using Matlab (or similar program)



With 4 voltage nodes, you need to write 4 equations for 4 unkowns (3 also works if you set V0 = 10)

$$V_0 = 10$$

$$\left(\frac{V_1 - V_0}{100}\right) + \left(\frac{V_1}{330}\right) + \left(\frac{V_1 - V_2}{100}\right) + \left(\frac{V_1 - V_3}{330}\right) = 0$$

$$\left(\frac{V_2 - V_1}{100}\right) + \left(\frac{V_2}{330}\right) + \left(\frac{V_2 - V_3}{100}\right) = 0$$

$$\left(\frac{V_3 - V_2}{100}\right) + \left(\frac{V_3}{330}\right) + \left(\frac{V_3 - V_1}{330}\right) = 0$$

Group terms

$$V_0 = 10$$

$$-\left(\frac{1}{100}\right)V_0 + \left(\frac{1}{100} + \frac{1}{330} + \frac{1}{100} + \frac{1}{330}\right)V_1 - \left(\frac{1}{100}\right)V_2 - \left(\frac{1}{330}\right)V_3 = 0$$

$$-\left(\frac{1}{100}\right)V_1 + \left(\frac{1}{100} + \frac{1}{330} + \frac{1}{100}\right)V_2 - \left(\frac{1}{100}\right)V_3 = 0$$

$$-\left(\frac{1}{330}\right)V_1 - \left(\frac{1}{100}\right)V_2 + \left(\frac{1}{100} + \frac{1}{330} + \frac{1}{330}\right)V_3 = 0$$

Place in matrix form

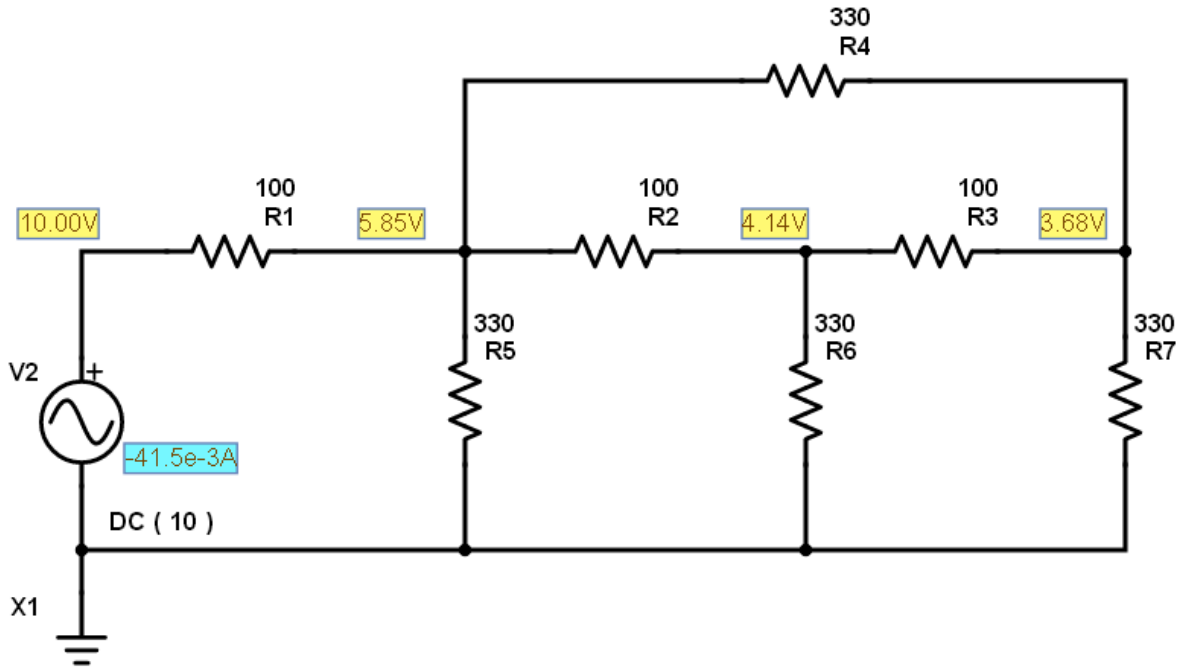
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -\left(\frac{1}{100}\right) & \left(\frac{1}{100}+\frac{1}{330}+\frac{1}{100}+\frac{1}{330}\right) & -\left(\frac{1}{100}\right) & -\left(\frac{1}{330}\right) \\ 0 & -\left(\frac{1}{100}\right) & \left(\frac{1}{100}+\frac{1}{330}+\frac{1}{100}\right) & -\left(\frac{1}{100}\right) \\ 0 & -\left(\frac{1}{330}\right) & -\left(\frac{1}{100}\right) & \left(\frac{1}{100}+\frac{1}{330}+\frac{1}{330}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

```
a1 = [1,0,0,0];
a2 = [-1/100, 1/100+1/330+1/100+1/330, -1/100, -1/330];
a3 = [0, -1/100, 1/100+1/330+1/100, -1/100];
a4 = [0, -1/330, -1/100, 1/100+1/330+1/330];

A = [a1;a2;a3;a4]

B = [10;0;0;0]

V = inv(A)*B
```

```
A =

    1.0000         0         0         0
   -0.0100    0.0261   -0.0100   -0.0030
         0   -0.0100    0.0230   -0.0100
         0   -0.0030   -0.0100    0.0161


B =

    10
     0
     0
     0


V =

V0   10.0000
V1    5.8547
V2    4.1415
V3    3.6833

>>
```

6) Check your answers in PartSim (or similar program)



	Calculated prob 5	Simulated prob 6
V1	5.8547V	5.85 V
V2	4.1415V	4.14 V
V3	3.6833V	3.68 V