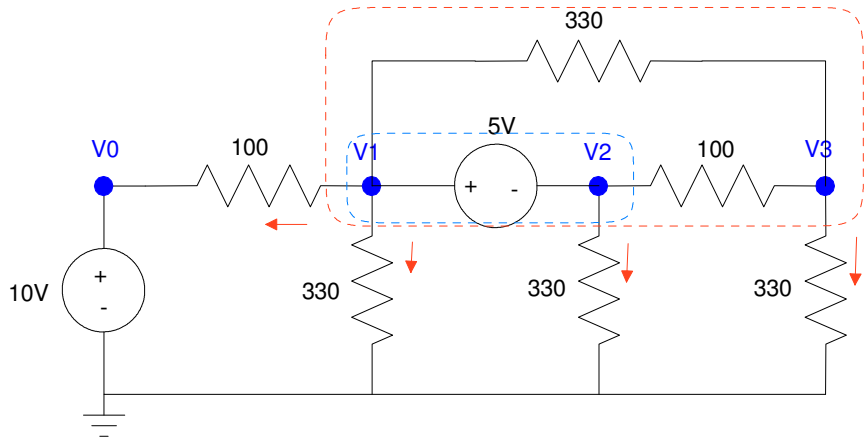
EE 206: Solution #4

Super Nodes, Current Loops. Due Mon, Feb 4th
Note: In this assignment just write N equations for N unknowns.

Super Nodes

1) Write the voltage node equations for the following circuit (N equations for N unknowns)



There are 4 voltage nodes. We need to write 4 equations to solve for 4 unknowns. Start with the easy ones

(1) $V_0 = 10$

(2) $V_1 - V_2 = 5$

Sum the currents from node V3 to zero

(3) $\left(\frac{V_3 - V_1}{330}\right) + \left(\frac{V_3 - V_2}{100}\right) + \left(\frac{V_3}{330}\right) = 0$

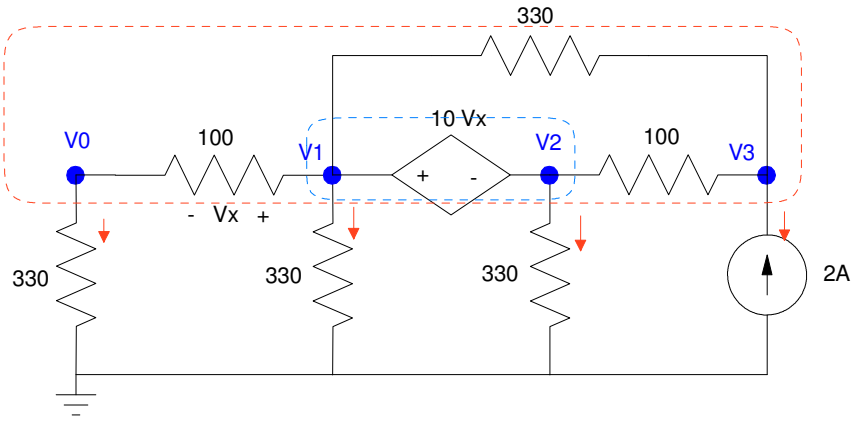
We need one more equation. Define a super node (red dotted line). The sum of the currents from the super node must be zero

(4) $\left(\frac{V_1 - V_0}{100}\right) + \left(\frac{V_1}{330}\right) + \left(\frac{V_2}{330}\right) + \left(\frac{V_3}{330}\right) = 0$

Note that other super nodes are also valid. If you use the blue dotted line, the 4th equation becomes

(4) $\left(\frac{V_1 - V_0}{100}\right) + \left(\frac{V_1 - V_3}{330}\right) + \left(\frac{V_1}{330}\right) + \left(\frac{V_2}{330}\right) + \left(\frac{V_2 - V_3}{100}\right) = 0$

2) Write the voltage node equations for the following circuit (N equations for N unknowns)



Three are 4 voltage nodes plus Vx. That makes 5 unknowns. Write 5 equations for 5 unknowns.

Start with the easy ones

(1) $V_x = V_0 - V_1$

(2) $V_1 - V_2 = 10V_x$

Sum the current from node 0 to zero

(3) $\left(\frac{V_0}{330}\right) + \left(\frac{V_0 - V_1}{100}\right) = 0$

Sum the current from node 3 to zero

(4) $\left(\frac{V_3 - V_1}{330}\right) + \left(\frac{V_3 - V_2}{100}\right) - 2 = 0$

We need one more equation. Define a super-node (red dotted line)

(5) $\left(\frac{V_0}{330}\right) + \left(\frac{V_1}{330}\right) + \left(\frac{V_2}{330}\right) - 2 = 0$

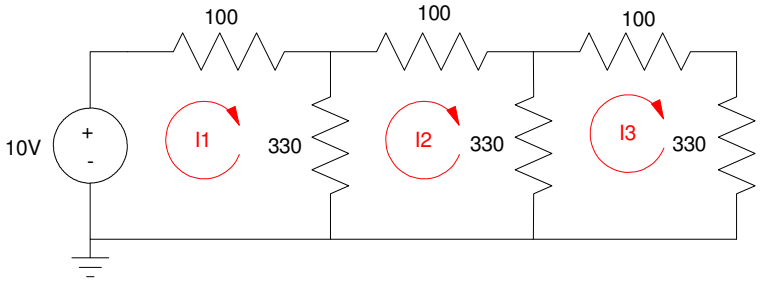
This gives 5 equations for 5 unknowns.

Note that you could use a different super node (blue dotted line). This makes the 5th equation:

(5) $\left(\frac{V_1 - V_0}{100}\right) + \left(\frac{V_1 - V_3}{330}\right) + \left(\frac{V_1}{330}\right) + \left(\frac{V_2}{330}\right) + \left(\frac{V_2 - V_3}{100}\right) = 0$

Current Loops

3) Write the current loop equations for the following circuit



With 3 unknown currents, we need to write 3 equations to solve for 3 unknowns

(1) $-10 + 100I_1 + 330(I_1 - I_2) = 0$

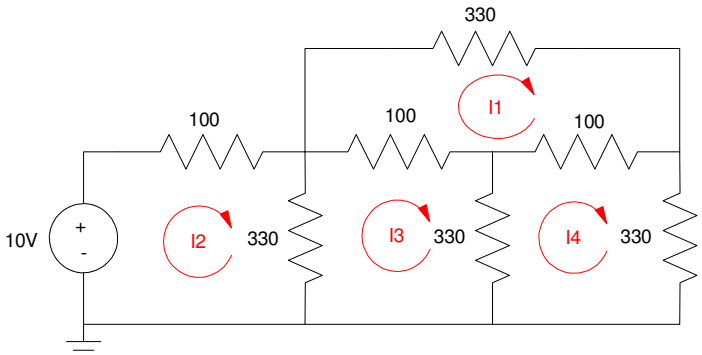
(2) $330(I_2 - I_1) + 100I_2 + 330(I_2 - I_3) = 0$

(3) $330(I_3 - I_2) + 100I_3 + 330I_3 = 0$

Note that around any loop Ix

- The sign is positive for Ix
- The sign is negative for all other currents

4) Write the current loop equations for the following circuit



With 4 unknown currents, we need to write 4 equations to solve for 4 unknowns

(1) $330I_1 + 100(I_1 - I_4) + 100(I_1 - I_3) = 0$

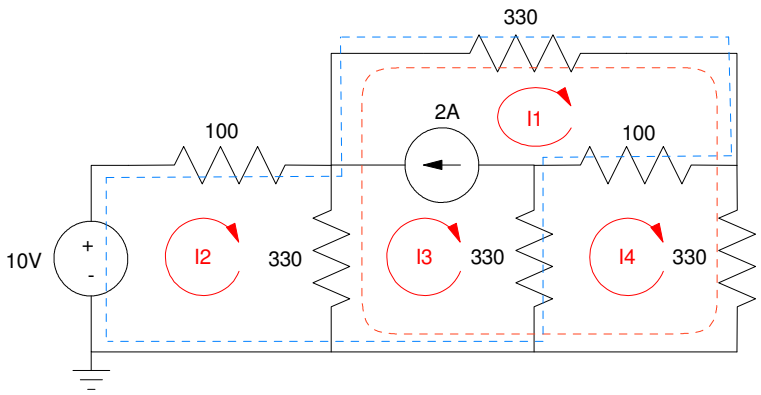
(2) $-10 + 100I_2 + 330(I_2 - I_3) = 0$

(3) $330(I_3 - I_2) + 100(I_3 - I_1) + 330(I_3 - I_4) = 0$

(4) $330(I_4 - I_3) + 100(I_4 - I_1) + 330I_4 = 0$

Super Loops

5) Write the current loop equations for the following circuit



With 4 unknown currents, we need to write 4 equations to solve for 4 unknowns. Start with the easy one (the current soruce)

(1) $I_1 - I_3 = 0$

Sum the voltages to zero around loop I2

(2) $-10 + 100I_2 + 330(I_2 - I_3) = 0$

Sum the voltages around loop I4

(3) $330(I_4 - I_3) + 100(I_4 - I_1) + 330I_4 = 0$

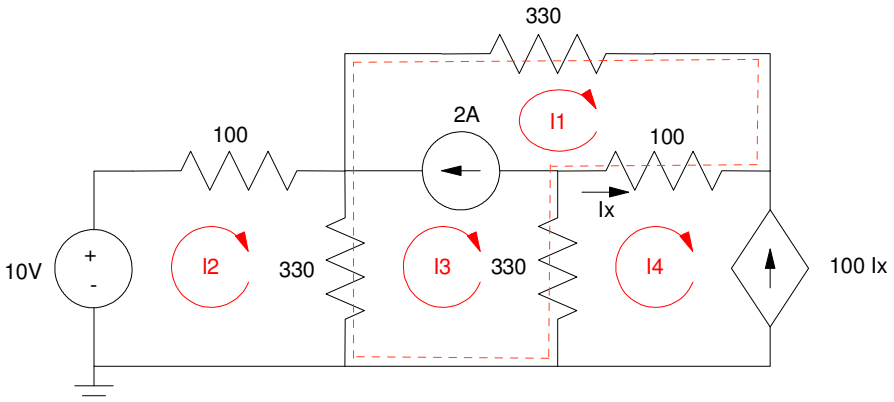
We need one more equation. Define a super-loop (shown in the red dotted line)

(4) $330(I_3 - I_2) + 330I_1 + 330I_4 = 0$

Other super loops are also valid. If you prefer the blue dotted line

(4) $-10 + 100I_2 + 330I_1 + 100(I_1 - I_4) + 330(I_3 - I_4) = 0$

6) Write the current loop equations for the following circuit



With 4 unknown current loops and Ix, we need to write 5 equations to solve for 5 unknowns. Start with the easy one (the current soruce)

(1) $I_x = I_4 - I_1$

(2) $I_1 - I_3 = 2$

(3) $I_4 = -100I_x$

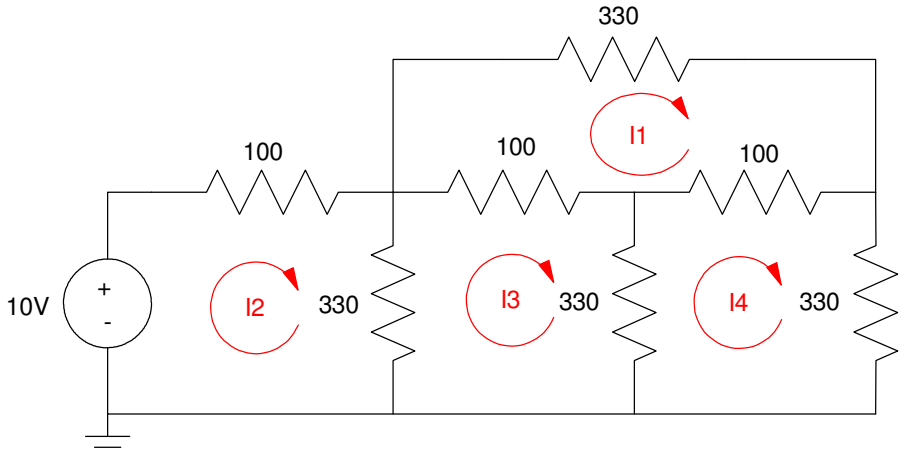
Sum the voltages around loop I2 to zero

(4) $-10 + 100I_2 + 330(I_2 - I_3) = 0$

We need one more equation. Pick a closed path which has known voltages (i.e. avoids current sources)

(5) $330(I_3 - I_2) + 330I_1 + 100(I_1 - I_4) + 330(I_3 - I_4) = 0$

For the circuit of problem #4:



7) Solve your sets of equations to find the currents. Recall from problem #4:

$$330I_1 + 100(I_1 - I_4) + 100(I_1 - I_3) = 0$$

$$-10 + 100I_2 + 330(I_2 - I_3) = 0$$

$$330(I_3 - I_2) + 100(I_3 - I_1) + 330(I_3 - I_4) = 0$$

$$330(I_4 - I_3) + 100(I_4 - I_1) + 330I_4 = 0$$

Grouping terms

$$530I_1 - 100I_3 - 100I_4 = 0$$

$$430I_2 - 330I_3 = 10$$

$$-100I_1 - 330I_2 + 760I_3 - 330I_4 = 0$$

$$-100I_1 - 330I_3 + 760I_4 = 0$$

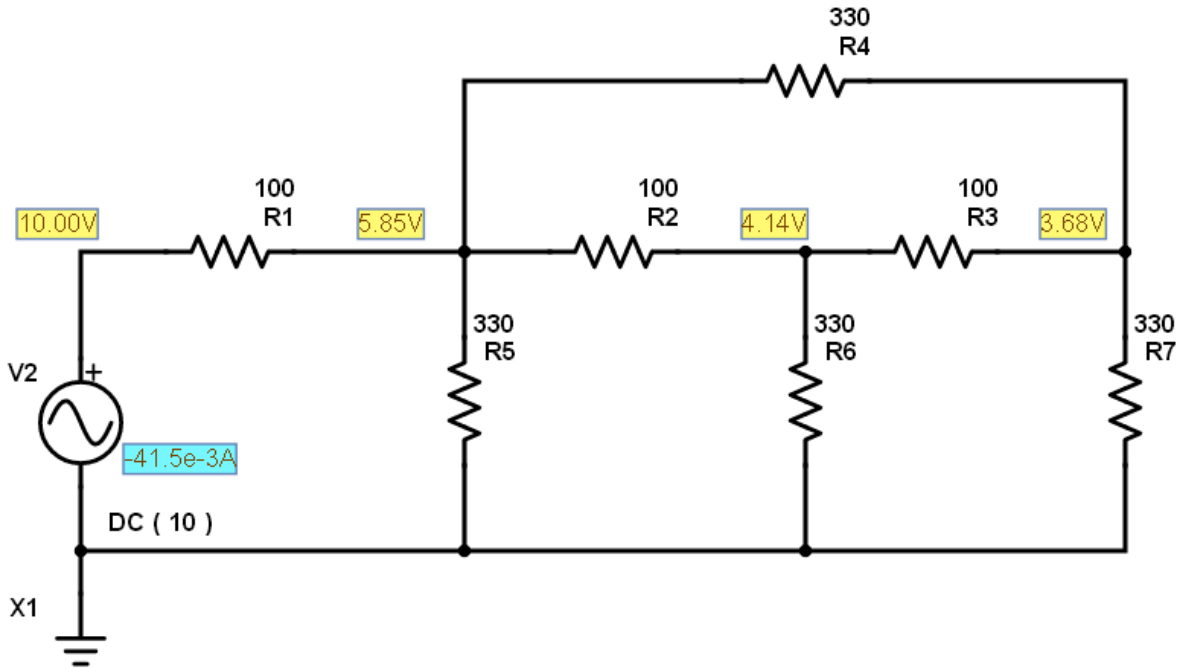
Place in matrix form

$$\begin{bmatrix} 530 & 0 & -100 & -100 \\ 0 & 430 & -330 & 0 \\ -100 & -330 & 760 & -330 \\ -100 & 0 & -330 & 760 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

```
A = [530,0,-100,-100 ; 0,430,-330,0 ; -100,-330,760,-330 ; -100,0,-330,760]

   530     0  -100  -100
     0   430  -330     0
  -100  -330   760  -330
  -100     0  -330   760

B = [0;10;0;0]

     0
    10
     0
     0

I = inv(A)*B

I1    0.0066
I2    0.0415
I3    0.0237
I4    0.0112
```

8) Simulate the circuit in PartSim. Check that the simulation results match your calculations.



This gives

- I2 = 41.5mA (the voltage source)

To find I1

$$I_1 = \left(\frac{5.85V - 3.68V}{330\Omega}\right) = 6.6mA$$

To find I4:

$$I_4 = \left(\frac{3.68V}{330\Omega}\right) = 11.2mA$$

To find I3,

$$I_2 - I_3 = \left(\frac{5.85V}{330\Omega}\right) = 17.7mA$$

$$I_3 = 23.8mA$$

	Calculated problem 7	Simulated problem 8
I1	0.0066	6.6 mA
I2	0.0415	41.5 mA
I3	0.0237	23.8 mA
I4	0.0112	11.2 mA