# EE 206: Homework #10 Solution

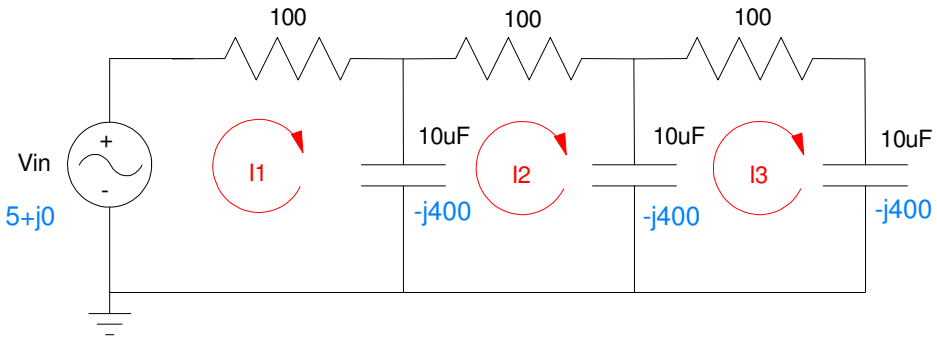
Current Loops, Op-Amps, Superposition with Phasors
Due Monday, April 20th

Problem 1) Assume

$$V_{in} = 10\cos(250t)$$

1a) Write the current-loop equations for the following circuit.

1b) Solve for I1, I2, and I3




Convert to phasor notation

$$s = j\omega = j250$$

$$V_{in} = 10 + j0$$

$$C \rightarrow \frac{1}{j\omega C} = -j400$$

Write the current loop equations

$$-5 + 100I_1 - j400(I_1 - I_2) = 0$$

$$-j400(I_2 - I_1) + 100I_2 - j400(I_2 - I_3) = 0$$

$$-j400(I_3 - I_2) + 100I_3 - j400I_3 = 0$$

Group terms

$$(100 - j400)I_1 + (j400)I_2 = 5$$

$$(j400)I_1 + (100 - j800)I_2 + (j400)I_3 = 0$$

$$(j400)I_2 + (100 - j800)I_3 = 0$$

Place in matrix form

$$\begin{bmatrix} (100-j400) & j400 & 0 \\ j400 & (100-j800) & j400 \\ 0 & j400 & (100-j800) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

Solve using Matlab

```
A =[100-j*400, j*400, 0 ; j*400, 100-j*800, j*400 ; 0, j*400, 100-j*800]

    100. - 400.i    400.i           0
    400.i           100. - 800.i    400.i
    0               400.i           100. - 800.i

B = [5;0;0]

    5.
    0.
    0.

I = inv(A)*B

I1    0.0171563 + 0.0163671i
I2    0.0130645 + 0.0081562i
I3    0.0069337 + 0.0032114i
```

meaning

$$i_1(t) = 0.0171\cos(250t) - 0.0163\sin(250t)$$

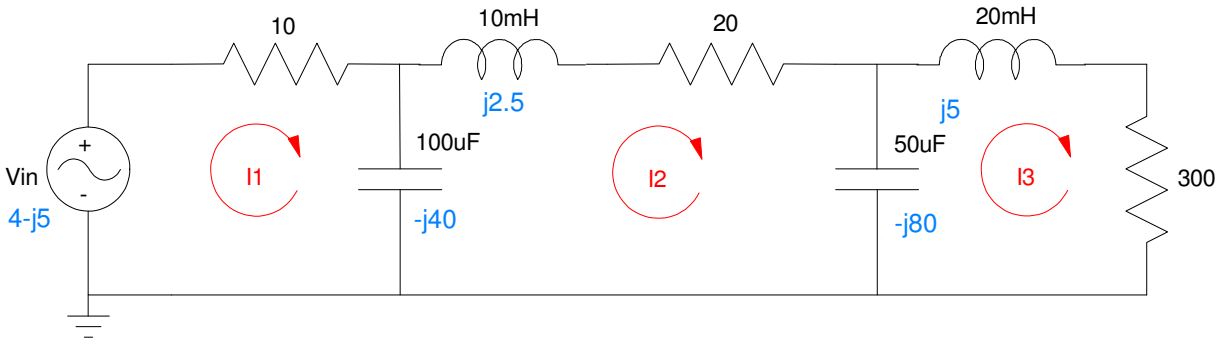
$$i_2(t) = 0.0130\cos(250t) - 0.0081\sin(250t)$$

$$i_3(t) = 0.0069\cos(250t) - 0.0032\sin(250t)$$

Problem 2) Assume

$$V_{in} = 4\cos(250t) + 5\sin(250t)$$

2a) Write the votlage node equations for the following circuit




Convert to phasor notation (shown in blue)

$$V_{in} = 4 - j5$$

$$s = j\omega = j250$$

$$L \rightarrow j\omega L$$

$$C \rightarrow \frac{1}{j\omega C}$$

Write the current loop equations

$$-(4 - j5) + 10I_1 - j40(I_1 - I_2) = 0$$

$$-j40(I_2 - I_1) + (20 + j2.5)I_2 - j80(I_2 - I_3) = 0$$

$$-j80(I_3 - I_2) + j5I_3 + 300I_3 = 0$$

Group terms

$$(10 - j40)I_1 + (j40)I_2 = 4 - j5$$

$$(j40)I_1 + (20 - j117.5)I_2 + (j80)I_3 = 0$$

$$(j80)I_2 + (300 - j75)I_3 = 0$$

Place in matrix form

$$\begin{bmatrix} (10-j40) & j40 & 0 \\ j40 & (10-j117.5) & j80 \\ 0 & j80 & (300-j75) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4-j5 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

```
A = [10-j*40, j*40, 0 ; j*40, 10-j*117.5, j*80 ; 0, j*80, 300 - j*75]

    10. - 40.i    40.i           0
    40.i          10. - 117.5i   80.i
    0             80.i           300. - 75.i

B = [4-j*5 ; 0 ; 0]

    4. - 5.i
    0
    0

I = inv(A)*B

I1    0.2092021 + 0.0436046i
I2    0.0733009 - 0.0040949i
I3    0.0035715 - 0.0186540i
```

or in terms of mA

```
I = inv(A)*B * 1000

I1    209.20207 + 43.604619i    mA
I2    73.300916 - 4.094863i     mA
I3    3.5715428 - 18.654025i    mA
```

meaning

$$i_1(t) = 209.2\cos(250t) - 40.6\sin(250t)\ mA$$

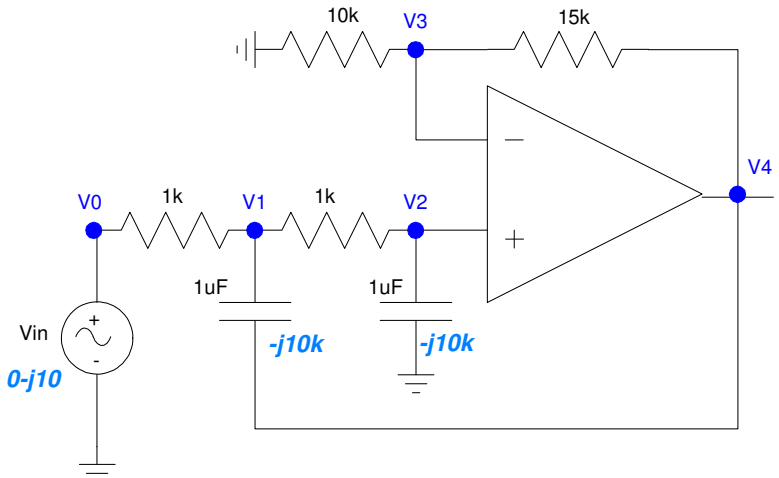
$$i_2(t) = 73.3\cos(250t) + 4.1\sin(250t)\ mA$$

$$i_3(t) = 3.6\cos(250t) + 18.6\sin(250t)\ mA$$

Problem 3) Assume

$$V_{in} = 10\sin(100t)$$

3a) Write the voltage node equations




$$V_2 = V_3$$

$$V_0 = 0 - j10$$

$$\left(\frac{V_1 - V_0}{1k}\right) + \left(\frac{V_1 - V_4}{-j10k}\right) + \left(\frac{V_1 - V_2}{1k}\right) = 0$$

$$\left(\frac{V_2 - V_1}{1k}\right) + \left(\frac{V_2}{-j10k}\right) = 0$$

$$\left(\frac{V_3}{10k}\right) + \left(\frac{V_3 - V_4}{15k}\right) = 0$$

3b) Solve for V1 .. V4. Group terms

$$V_2 - V_3 = 0$$

$$V_0 = 0 - j10$$

$$\left(\frac{-1}{1k}\right)V_0 + \left(\frac{1}{1k} + \frac{1}{j10k} + \frac{1}{1k}\right)V_1 - \left(\frac{1}{1k}\right)V_2 - \left(\frac{1}{j10k}\right)V_4 = 0$$

$$-\left(\frac{1}{1k}\right)V_1 + \left(\frac{1}{1k} - \frac{1}{j10k}\right)V_2 = 0$$

$$\left(\frac{1}{10k} + \frac{1}{15k}\right)V_3 - \left(\frac{1}{15k}\right)V_4 = 0$$

Place in matrix form. Multiply the last three equations by 1000 to clear the 'k' terms

$$\begin{bmatrix} 0 & 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ -1 & 2-j0.1 & -1 & 0 & j0.1 \\ 0 & -1 & 1+j0.1 & 0 & 0 \\ 0 & 0 & 0 & 0.1667 & -0.0667 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -j10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve

```
A = [0,0,1,-1,0 ; 1,0,0,0,0 ; -1,2-j*0.1,-1,0,j*0.1 ; 0,-1,1+j*0.1,0,0 ;
0,0,0,0.1667,-0.0667]

    0     0            1.          - 1.        0
    1.    0            0             0         0
  - 1.    2. - 0.1i  - 1.            0         0.1i
    0   - 1.           1. + 0.1i     0         0
    0     0            0             0.1667  - 0.0667

B = [0;-j*10;0;0;0]

    0
  - 10.i
    0
    0
    0

V = inv(A)*B

V0  - 10.i
V1  - 2.1786843 - 9.1461622i
V2  - 3.0626738 - 8.8398948i
V3  - 3.0626738 - 8.8398948i
V4  - 7.6543886 - 22.09311i
```

meaning

$$v_0(t) = 10\sin(100t)$$

$$v_1 = -2.178\cos(100t) + 9.146\sin(100t)$$

$$v_2 = -3.062\cos(100t) + 8.839\sin(100t)$$

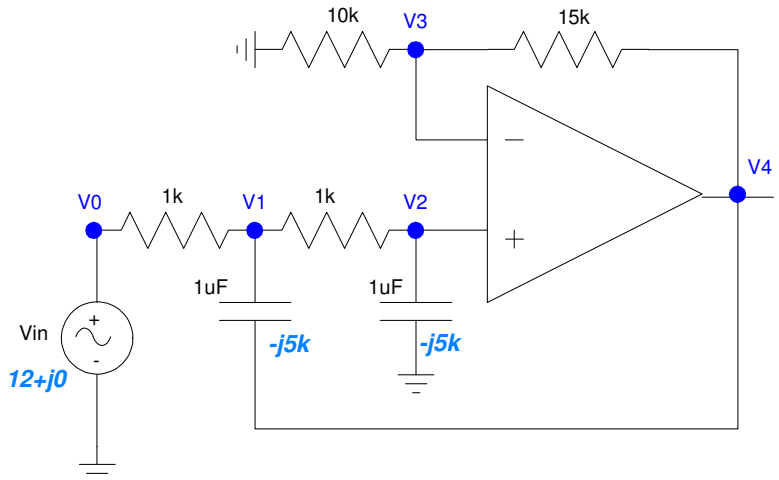
$$v_3 = -3.062\cos(100t) + 8.839\sin(100t)$$

$$v_4 = -7.654\cos(100t) + 22.093\sin(100t)$$

Problem 4) Assume

$$V_{in} = 12\cos(200t)$$

4a) Write the voltage node equations




Everything stays the same except for V0 and the capacitors (complex terms)

$$V_2 - V_3 = 0$$

$$V_0 = 12 + j0$$

$$\left(\frac{-1}{1k}\right)V_0 + \left(\frac{1}{1k} + \frac{1}{j5k} + \frac{1}{1k}\right)V_1 - \left(\frac{1}{1k}\right)V_2 - \left(\frac{1}{j5k}\right)V_4 = 0$$

$$-\left(\frac{1}{1k}\right)V_1 + \left(\frac{1}{1k} - \frac{1}{j5k}\right)V_2 = 0$$

$$\left(\frac{1}{10k} + \frac{1}{15k}\right)V_3 - \left(\frac{1}{15k}\right)V_4 = 0$$

In matrix form

$$\begin{bmatrix} 0 & 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ -1 & 2-j0.2 & -1 & 0 & j0.2 \\ 0 & -1 & 1+j0.2 & 0 & 0 \\ 0 & 0 & 0 & 0.1667 & -0.0667 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 12+j0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving in Matlab

```
A = [0,0,1,-1,0 ; 1,0,0,0,0 ; -1,2-j*0.2,-1,0,j*0.2 ; 0,-1,1+j*0.2,0,0 ;
0,0,0,0.1667,-0.0667]

    0     0            1.          - 1.         0
    1.    0            0             0          0
  - 1.    2. - 0.2i    - 1.          0          0.2i
    0     - 1.         1. + 0.2i     0          0
    0     0            0             0.1667     - 0.0667

B = [0;12+j*0;0;0;0]

    0
    12.
    0
    0
    0

V = inv(A)*B

V0    12.
V1    9.0109006 - 3.7560379i
V2    7.9420125 - 5.3444404i
V3    7.9420125 - 5.3444404i
V4    19.849078 - 13.357095i
```

meaning

$$v_0 = 12\cos(200t)$$

$$v_1 = 9.01\cos(200t) + 3.75\sin(200t)$$

$$v_2 = 7.94\cos(200t) + 5.34\sin(200t)$$

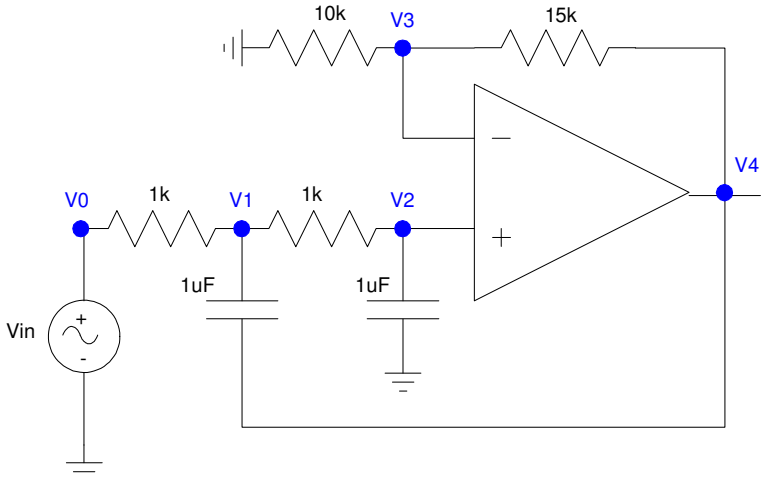
$$v_3 = 7.94\cos(200t) + 5.34\sin(200t)$$

$$v_4(t) = 19.85\cos(200t) + 13.36\sin(200t)$$

Problem 5) Assume

$$V_{in} = 10\sin(100t) + 12\cos(200t)$$

Determine V1 .. V4




Use superposition; add solution #4 to solution #5

problem 5 = problem 3 + problem 4

$$v_1 = -2.178\cos(100t) + 9.146\sin(100t) + 9.01\cos(200t) + 3.75\sin(200t)$$

$$v_2 = -3.062\cos(100t) + 8.839\sin(100t) + 7.94\cos(200t) + 5.34\sin(200t)$$

$$v_3 = -3.062\cos(100t) + 8.839\sin(100t) + 7.94\cos(200t) + 5.34\sin(200t)$$

$$v_4 = -7.654\cos(100t) + 22.093\sin(100t) + 19.85\cos(200t) + 13.36\sin(200t)$$