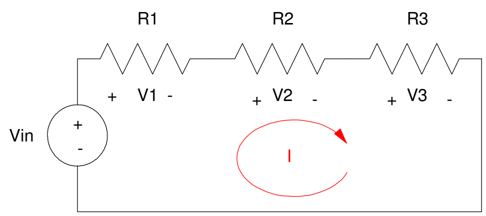
Resistors in Series and Parallel

Some circuits you can simplify by combining resistors

Resistors in Series:

Problem: Find the net resistance of this circuit:



Using Kirchoff's voltage law:

Vin = V1 + V2 + V3

$$V_{in} = I \cdot R_1 + I \cdot R_2 + I \cdot R_3$$

$$V_{in} = I \cdot (R_1 + R_2 + R_3)$$

$$V_{in} = I \cdot R$$

Resistors in series add

Problem 1: Let R1 = 100 Ohms, R2 = 200 Ohms, R3 = 300 Ohms. Find the total resistance.

Answer: Resistors in series add:

$$R_{net} = R_1 + R_2 + R_3$$

$$R_{net} = 100\Omega + 200\Omega + 300\Omega$$

$$\boxed{R_{net} = 600\Omega}$$

Problem 2: Let R1 = 100 Ohms, R2 = 200 Ohms, and the total resistance be 1000 Ohms. Find R3.

Answer: Resistors in series add:

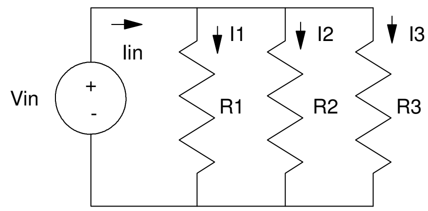
$$R_{net} = R_1 + R_2 + R_3$$

$$1000\Omega = 100\Omega + 200\Omega + R_3$$

$$\boxed{R_3 = 700\Omega}$$

Resistors in Parallel:

Find the net resistance:



Using Kirchoff's current law:

Iin = I1 + I2 + I3

$$I_{in} = \left(\frac{V_{in}}{R_1}\right) + \left(\frac{V_{in}}{R_2}\right) + \left(\frac{V_{in}}{R_3}\right)$$

$$I_{in} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) V_{in}$$

$$V_{in} = I_{in} \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

$$V_{in} = I_{in} \cdot R$$

Resistors in parallel combine as the sum of the inverses, inverted:

$$R = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\right)^{-1}$$

Problem 3: Let R1 = 100 Ohms, R2 = 200 Ohms, R3 = 300 Ohms. Find the total resistance.

Solution: Resistors in parallal add as the sum of the inverses inverted.

$$R_{net} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

$$R_{net} = \left(\frac{1}{100} + \frac{1}{200} + \frac{1}{300}\right)^{-1}$$

$$\boxed{R_{net} = 54.54\Omega}$$

Problem 4: Let R2 = 200 Ohms, R3 = 300 Ohms, and the total resistance be 100 Ohms. Find R1.

Solution:

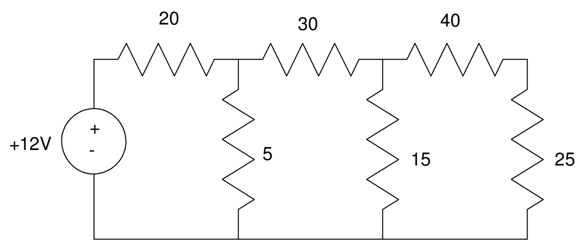
$$R_{net} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

$$100\Omega = \left(\frac{1}{R_1} + \frac{1}{200} + \frac{1}{300}\right)^{-1}$$

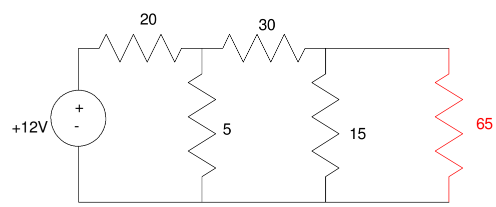
$$\frac{1}{100} = \frac{1}{R_1} + \frac{1}{200} + \frac{1}{300}$$

$$\boxed{R_1 = 600\Omega}$$

With this, you can simplify some circuits. For example, find the resistance seen by the voltage source:

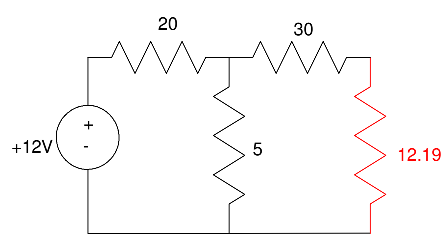


The 40 Ohm and 25 Ohm are in series. Add these to 65 Ohms.

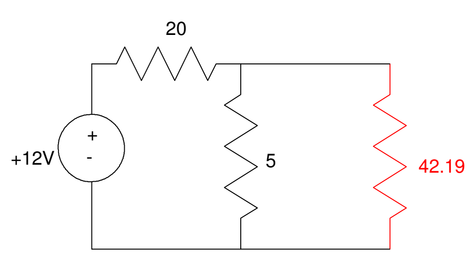


65 Ohms and 15 Ohms are in parallel. Add these to gether

$$R = \left(\frac{1}{15} + \frac{1}{65}\right)^{-1} = 12.19\Omega$$

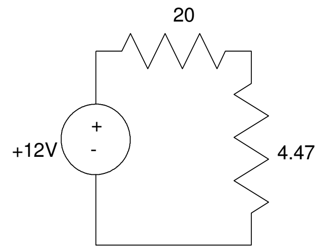


12.19 and 30 ohms are in series. Add thest together tp get 42/19 Ohms



42.19 is in parallel with 5 Ohms. Add these to gether to get

$$R = \left(\frac{1}{5} + \frac{1}{42.19}\right)^{-1} = 4.47\Omega$$



This is in series with 20 Ohms.

The resistance seen by the 12V source is 24.47 Ohms

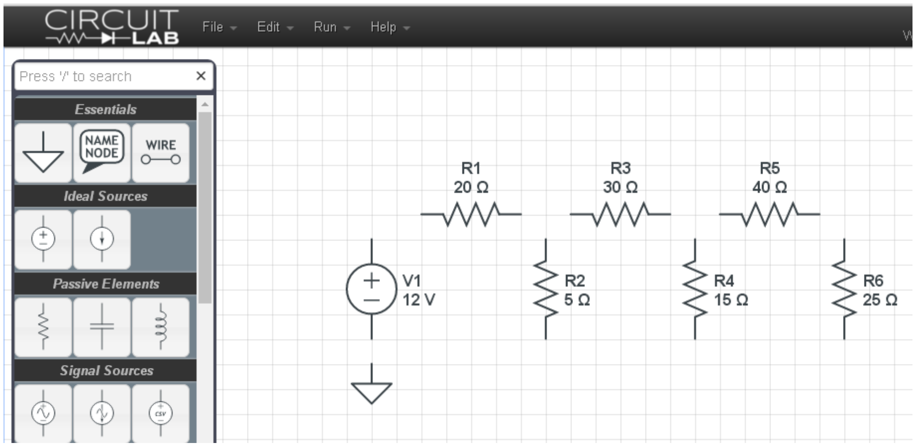
CircuitLab (www.CircuitLab.com)

You can check your answer in CircuitLab. This is a free (!) circuit simulator for NDSU students

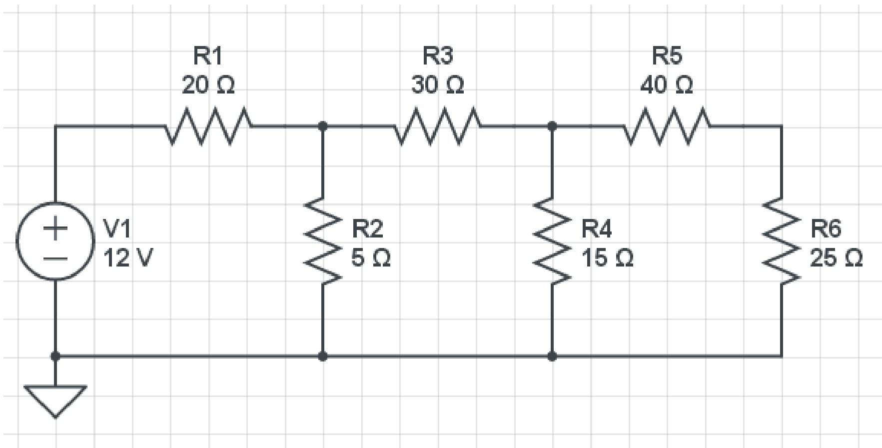
- Sign up for an account using your NDSU.edu email address
- You cans save your circuits for future reference
- **Please** keep your circuit names clean and avoid profanity.

Step 1: Add the components

- Drag and drop components
- R rotates the component
- Double click allows you to change the value



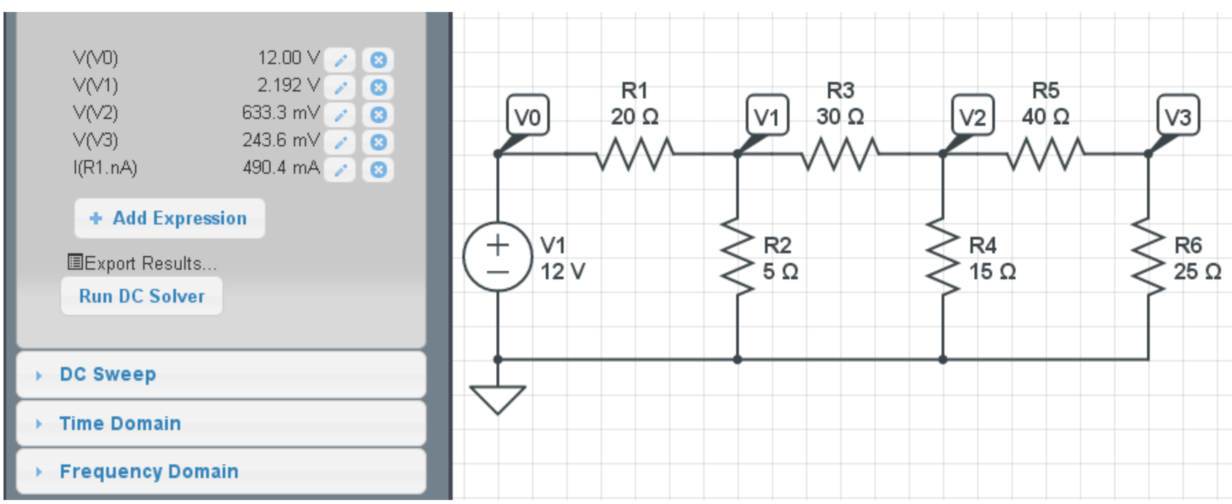
Step 2: Connect the components using drag and drop. **Note: you should see a dot when wires are connected. No dot means there's no connection**.



Step 3: Click Run, DC Bias

- Add labels for the voltages you want to look at
- Add Expression and click on a voltage (V0) to see that, click on a wire to see the current

Click Run DC Solver to get answers



From V = IR, you can compute the resistance seen by the 12V source

$$R = \frac{V}{I}$$

$$R = \frac{12V}{0.4904A} = 24.4698\Omega$$

which matches our computed resistance of 24.47 Ohms.