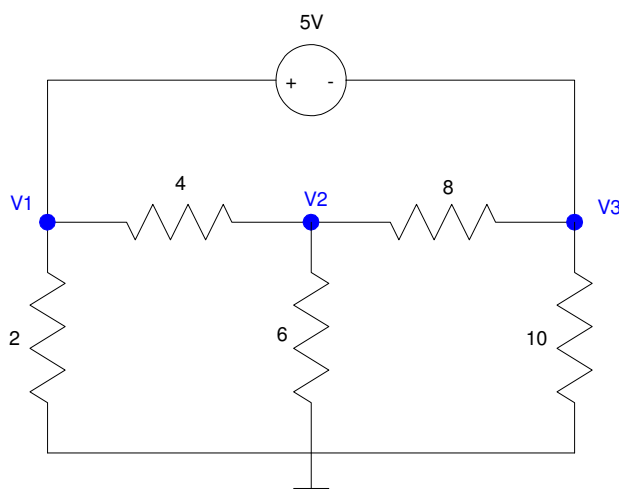


SuperNodes:

If a voltage source is connected to a node, you can't sum the currents to zero: the voltage source provides whatever current it takes to hold the voltage. In this case, there are two options:

- The voltage node is trivial: it is a constant set by the voltage source, or
- Draw a rectangle around the voltage source so that all connecting elements are resistors or current sources (things where you know the current). This is called a *supernode*. Sum the current from this supernode to zero.

For example, find the voltages for the following circuit:



Example 3: In this case we'll need a supernode.

Again, we have three unknowns, so we need to write three equations. Start with the voltage source (the easy equation)

$$V_1 - V_3 = 5 \quad (1)$$

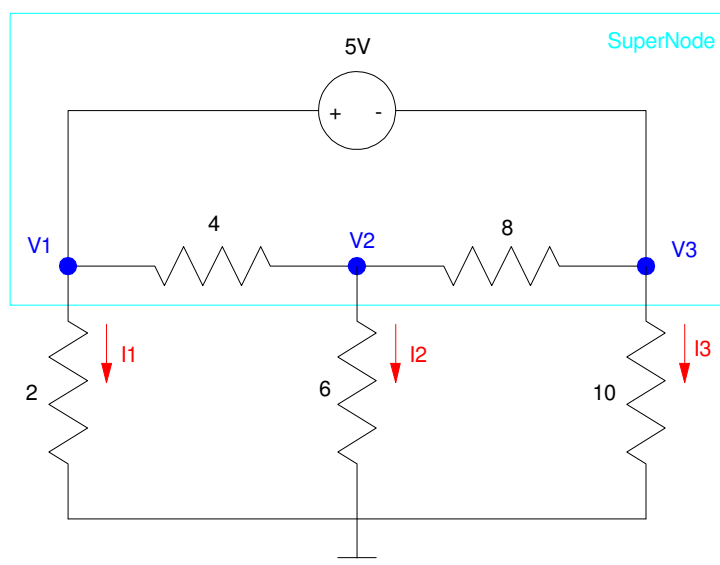
At node V2, sum the currents to zero

$$\left(\frac{V_2 - V_1}{4} \right) + \left(\frac{V_2}{6} \right) + \left(\frac{V_2 - V_3}{8} \right) = 0 \quad (2)$$

Now we're stuck.

- We can't sum the current to zero at node V1 since we don't know the current to the 5V source
- We can't sum the current to zero at node V3 since we don't know the current into the - terminal of the 5V source

Instead, draw a path that encloses the 5V source and only has resistors attached to it (shown in light blue). Note that there are several solutions:

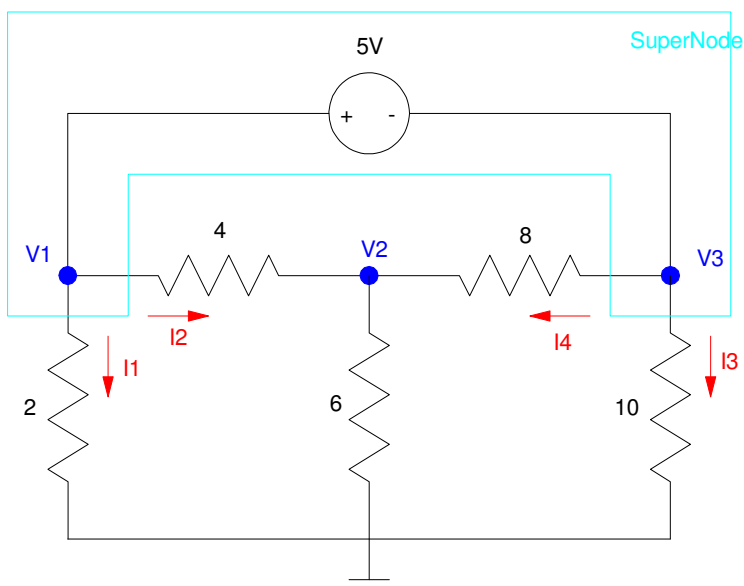


One possible definition of a supernode: the current from the supernode must add to zero

Now sum the currents from the supernode to zero for the third equation

$$\left(\frac{V_1}{2}\right) + \left(\frac{V_2}{6}\right) + \left(\frac{V_3}{10}\right) = 0 \quad (3)$$

Note that it's perfectly valid to write the third equation the following:



Another valid definition of the supernode.

$$\left(\frac{V_1}{2}\right) + \left(\frac{V_1 - V_2}{4}\right) + \left(\frac{V_3}{10}\right) + \left(\frac{V_3 - V_2}{8}\right) = 0 \quad (3)$$

Once you have three equations for three unknowns, solve.

Grouping terms:

$$V_1 - V_3 = 5 \quad (1)$$

$$-\left(\frac{1}{4}\right)V_1 + \left(\frac{1}{4} + \frac{1}{6} + \frac{1}{8}\right)V_2 - \left(\frac{1}{8}\right)V_3 = 0 \quad (2)$$

$$\left(\frac{V_1}{2}\right) + \left(\frac{V_2}{6}\right) + \left(\frac{V_3}{10}\right) = 0 \quad (3)$$

Place in matrix form:

$$\begin{bmatrix} 1 & 0 & -1 \\ -0.25 & 0.5417 & -0.125 \\ 0.5 & 0.1666 & 0.1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

```
>> A = [1,0,-1 ; -0.25,0.5417,-0.125 ; 0.5,0.16666,0.1]
```

```
A =
```

```
    1.0000         0   -1.0000
   -0.2500    0.5417   -0.1250
    0.5000    0.1667    0.1000
```

```
>> B = [5;0;0]
```

```
B =
```

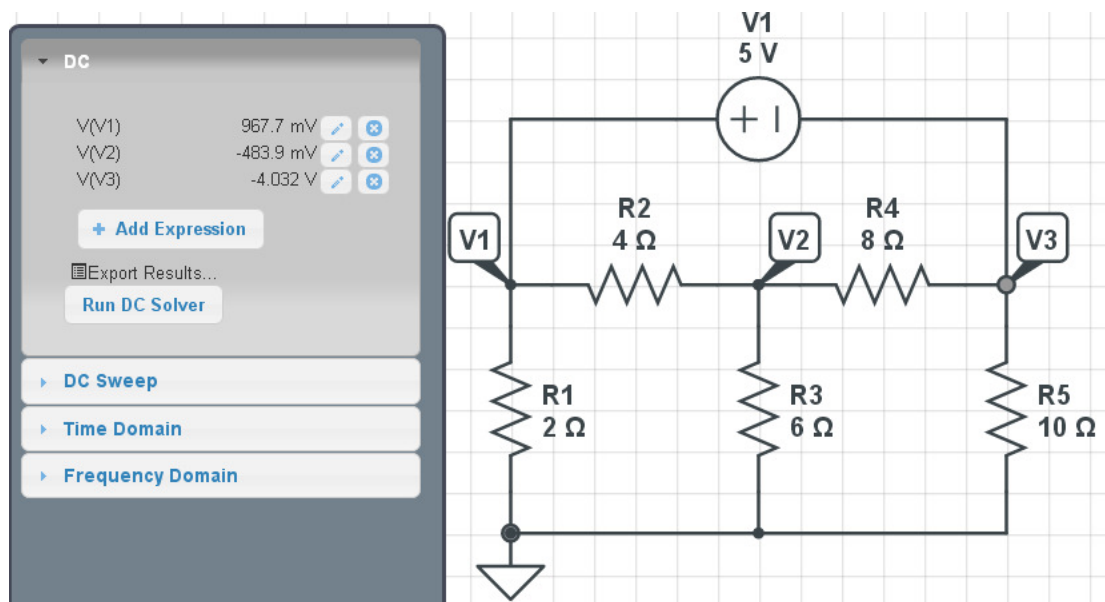
```
    5
    0
    0
```

```
>> V = inv(A)*B
```

```
V =
```

```
    0.9677
   -0.4838
   -4.0323
```

Checking in Circuitlab:



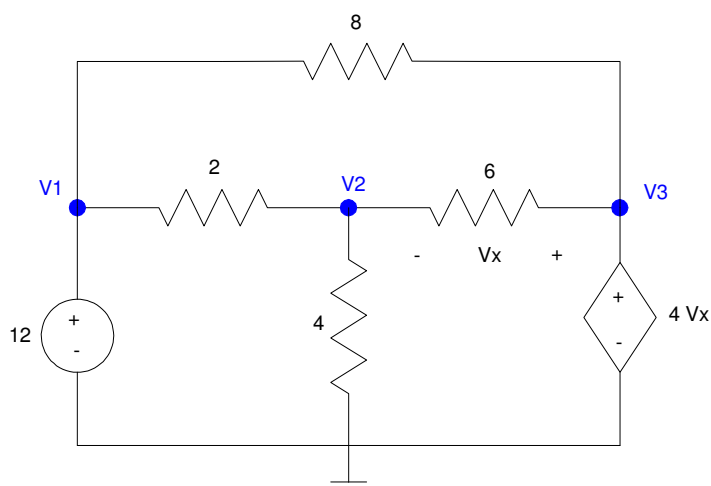
Circuitlab agrees with our computed results.

Voltage Nodes with Dependent Sources

If you have a dependent source in your circuit, the way you handle the node(s) with the dependent source depend upon whether it's a current or a voltage source.

- If it's a current source, sum the currents to zero as normal.
- If it's a voltage source, you can't sum the current to zero at that node. The source provides whatever current it takes to hold the voltage. Instead, replace that node voltage with its voltage as defined by the controlled voltage source.

For example, consider the following circuit



Circuit with a Dependent Source

Start with writing V_x in terms of what you know (V_1 , V_2 , V_3)

$$V_x = V_3 - V_2 \quad (1)$$

Now write three more equations so you can solve N equations for N unknowns. Start with the easy ones, the voltage sources

$$V_1 = 12 \quad (2)$$

$$V_3 = 4V_x \quad (3)$$

Now we need one more equation: sum the current from V2 to zero

$$\left(\frac{V_2 - V_1}{2}\right) + \left(\frac{V_2}{4}\right) + \left(\frac{V_2 - V_3}{6}\right) = 0 \quad (4)$$

This gives 4 equations for 4 unknowns. (Note: you could simplify this into 3 equations for 3 unknowns, but I prefer not to do so. With Matlab, it's easy to solve N equations for N unknowns. What's most important is that you get the equations right.)

To solve in Matlab, group terms

$$V_x - V_3 + V_2 = 0 \quad (1)$$

$$V_1 = 12 \quad (2)$$

$$V_3 - 4V_x = 0 \quad (3)$$

$$\left(\frac{-1}{2}\right)V_1 + \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6}\right)V_2 + \left(\frac{-1}{6}\right)V_3 = 0 \quad (4)$$

Placing in matrix form

$$\begin{bmatrix} 0 & 1 & -1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -4 \\ -0.5 & 0.9167 & -0.1666 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_x \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

```
>> A = [0,1,-1,1 ; 1,0,0,0 ; 0,0,1,-4 ; -0.5,0.9167,-0.1666,0]
```

```

      0      1.0000     -1.0000      1.0000
    1.0000         0         0         0
      0         0      1.0000     -4.0000
   -0.5000     0.9167    -0.1666         0
```

```
>> B = [0;12;0;0]
```

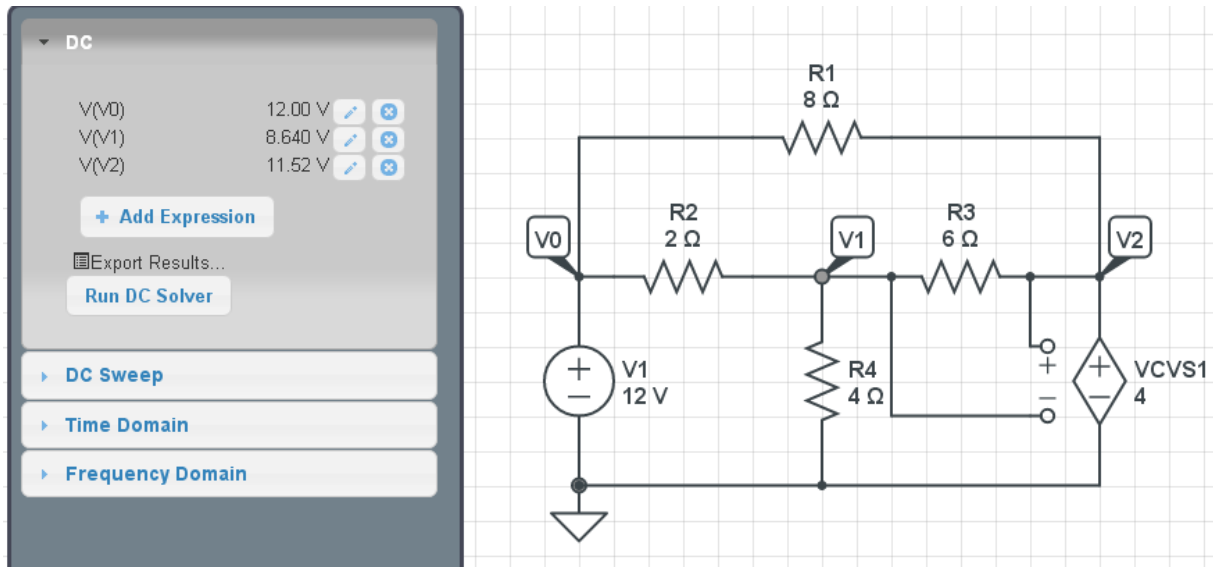
```

      0
     12
      0
      0
```

```
>> V = inv(A)*B
```

V1 12.0000
V2 8.6385
V3 11.5180
Vx 2.8795

Checking in Circuitlab - the voltages match

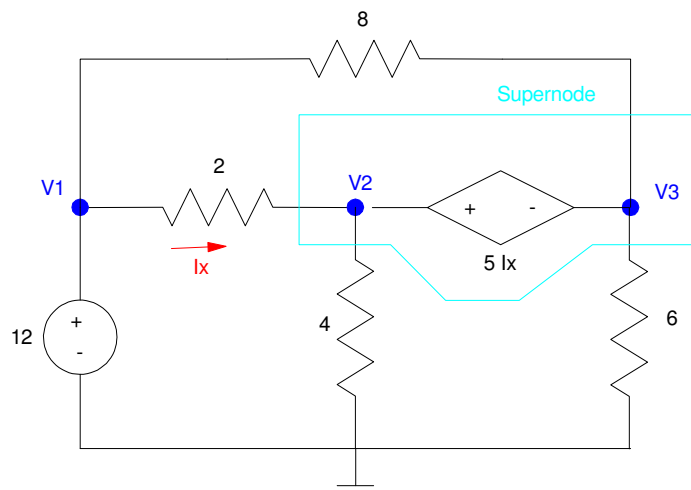


Circuitlab Results: Voltages match our calculations

SuperNodes and Dependent Sources

Supernodes apply to dependent sources as well. If you can't determine a certain current, create a supernode so that you only have resistors and current sources connecting to the supernode.

For example, determine the voltages for the following circuit:



Circuit with a dependent source. For this circuit, we'll need to use a supernode.

Start with the controlled source

$$I_x = \left(\frac{V_1 - V_2}{2} \right) \quad (1)$$

Write the easy equations (the voltage sources)

$$V_1 = 12 \quad (2)$$

$$V_2 - V_3 = 5I_x \quad (3)$$

Now we're stumped: we can't write the node equation at V2 or V3 due to now knowing the current through the controlled voltage source. So, create a super-node (show in light blue)

$$\left(\frac{V_2 - V_1}{2} \right) + \left(\frac{V_2}{4} \right) + \left(\frac{V_3}{6} \right) + \left(\frac{V_3 - V_1}{8} \right) = 0 \quad (4)$$

Now that we have 4 equations with 4 unknowns, we can solve. Grouping terms:

$$V_1 - V_2 - 2I_x = 0 \quad (1)$$

$$V_1 = 12 \quad (2)$$

$$V_2 - V_3 - 5I_x = 0 \quad (3)$$

$$\left(\frac{-1}{2} + \frac{-1}{8}\right)V_1 + \left(\frac{1}{2} + \frac{1}{4}\right)V_2 + \left(\frac{1}{6} + \frac{1}{8}\right)V_3 = 0 \quad (4)$$

Place in matrix form

$$\begin{bmatrix} 1 & -1 & 0 & -2 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -5 \\ -0.625 & 0.75 & 0.2917 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ I_x \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

```
>> A = [1,-1,0,-2 ; 1,0,0,0 ; 0,1,-1,-5 ; -0.625,0.75,0.2917,0]
```

```
    1.0000    -1.0000         0    -2.0000
    1.0000         0         0         0
         0     1.0000    -1.0000    -5.0000
   -0.6250     0.7500     0.2917         0
```

```
>> B = [0;12;0;0]
```

```
    0
   12
    0
    0
```

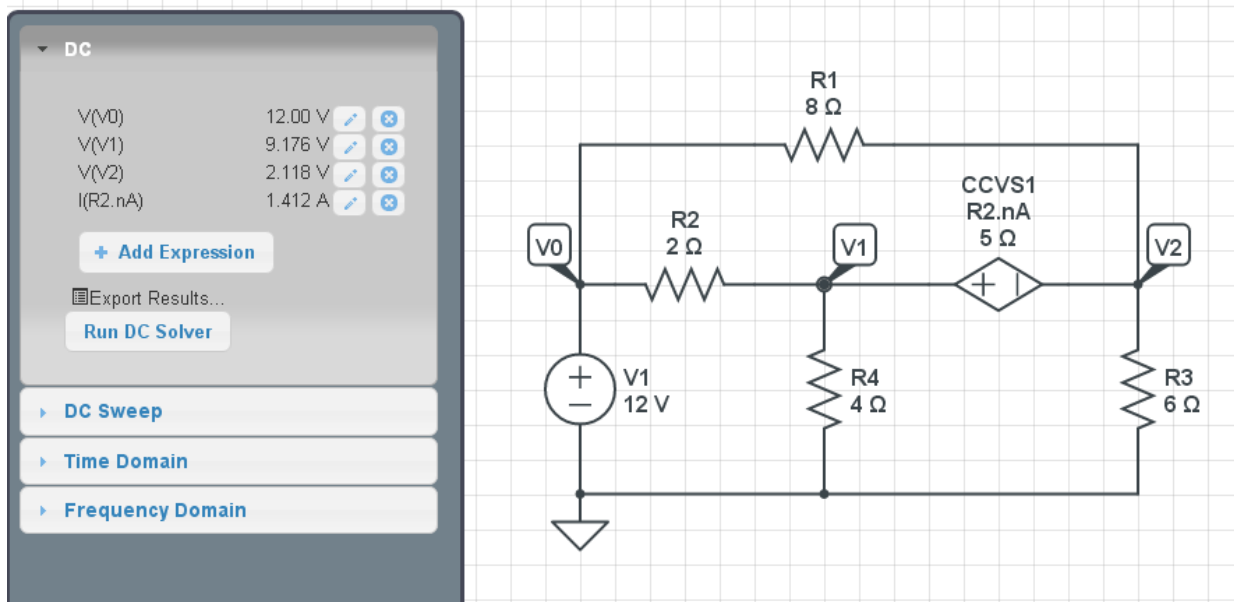
```
>> V = inv(A)*B
```

```
V1    12.0000
V2     9.1764
V3     2.1175
Ix     1.4118
```

Verifying in Circuitlab

Matlab Values (repeated)

```
V1    12.0000
V2     9.1764
V3     2.1175
Ix     1.4118
```



Circuitlab verification: voltages match our calculations.